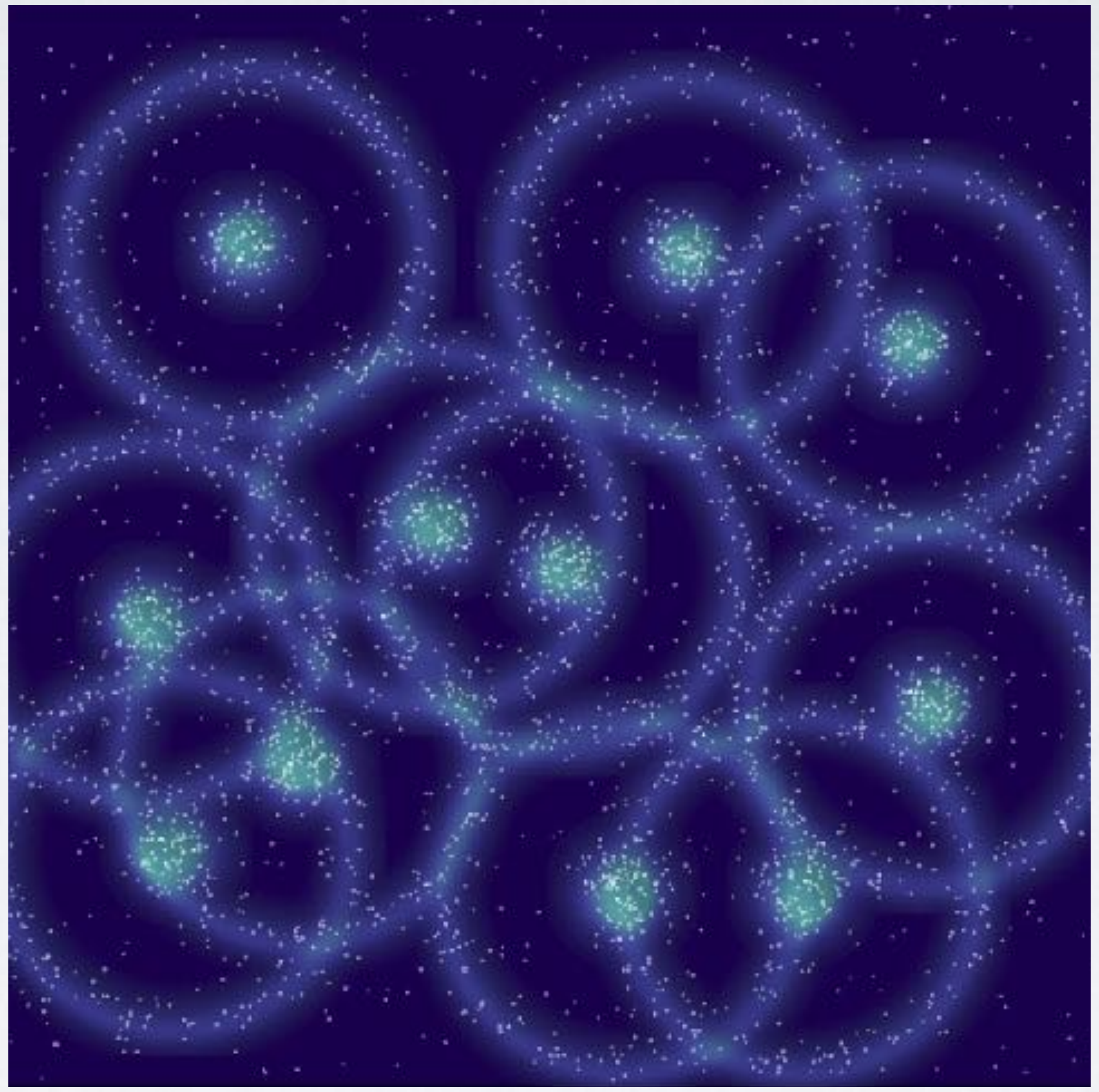
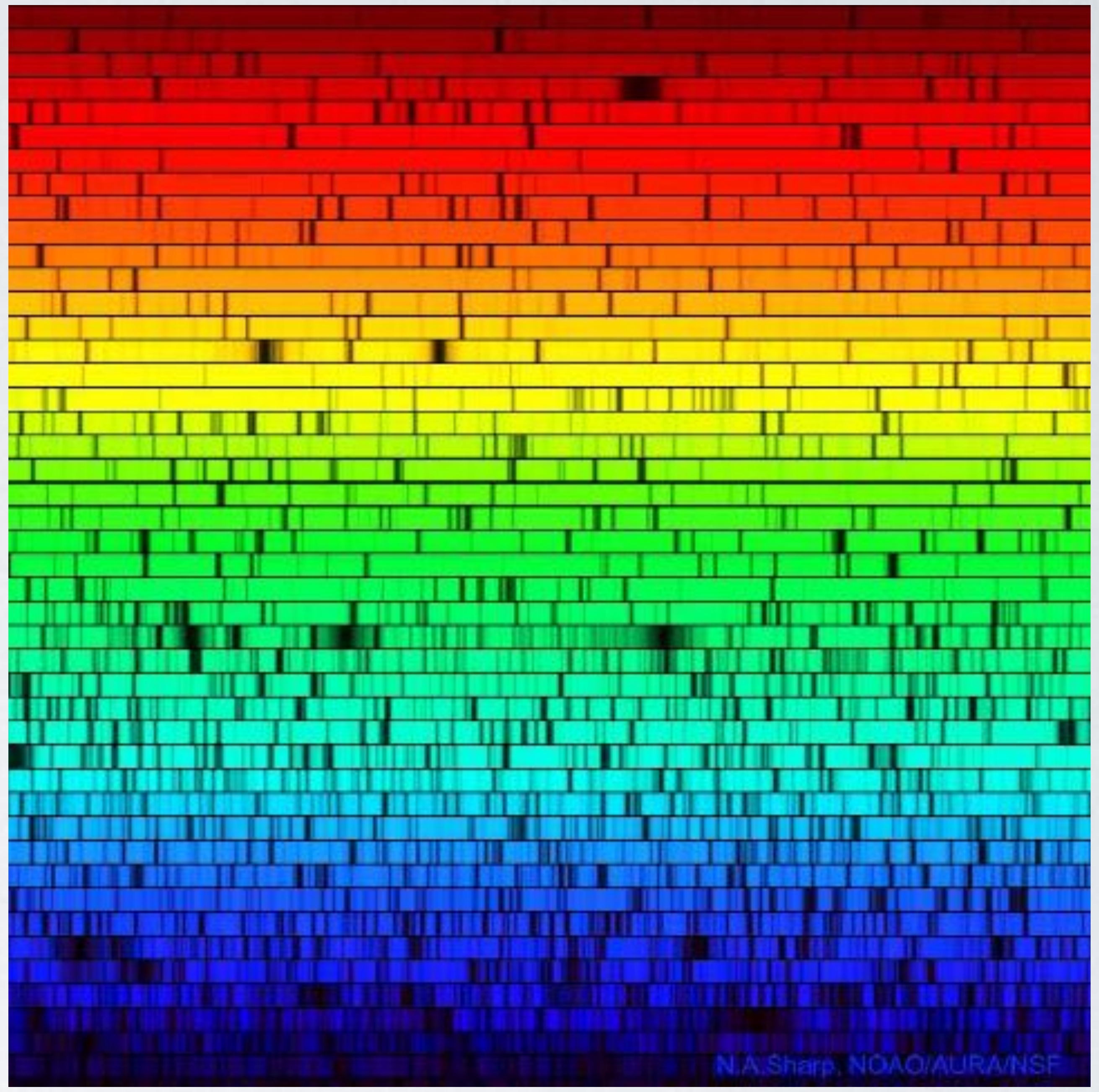




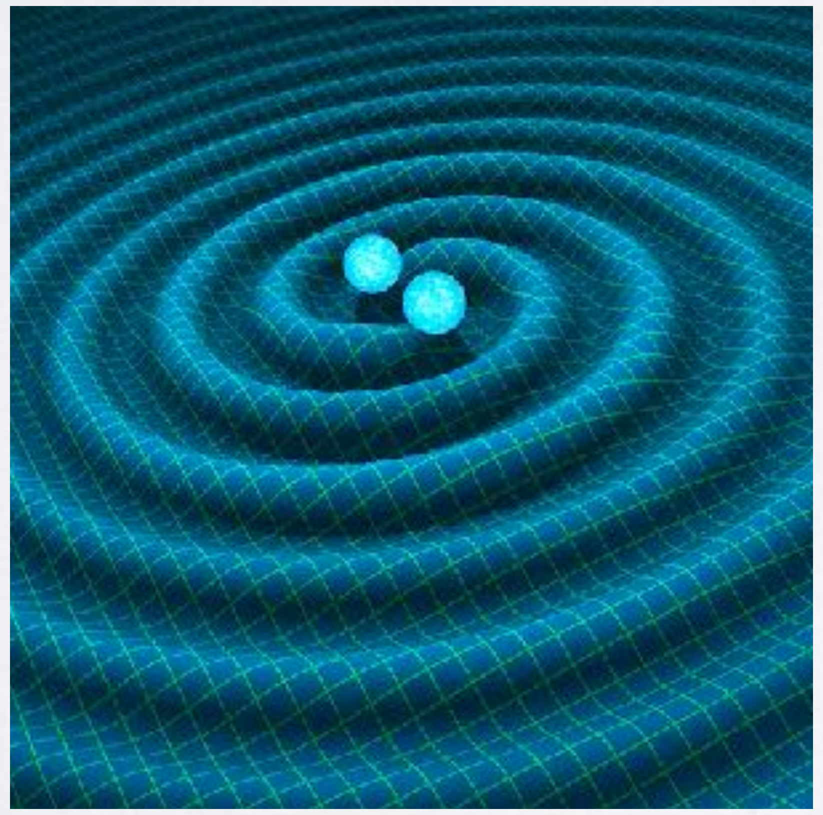
CANDLES, RULERS, AND REDSHIFTS

A theoretical look at H_0

LOCAL / GLOBAL … OR … CANDLES / RULERS?

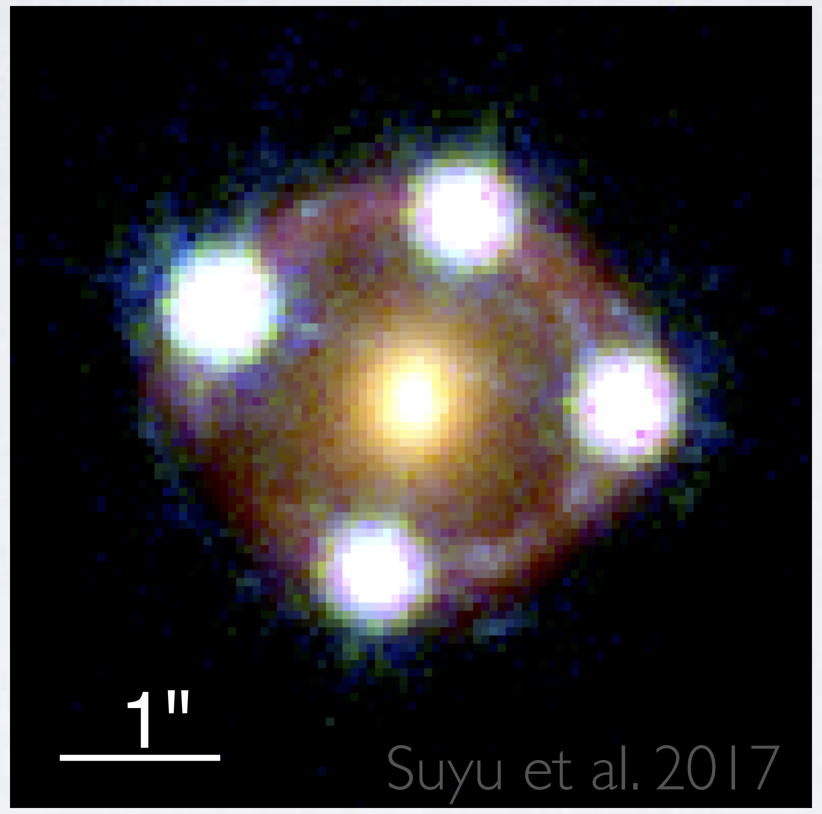
$$\tilde{D} = \begin{cases} R_0 \sin(\chi) & \text{closed} \\ R_0 \chi & \text{flat} \\ R_0 \sinh(\chi) & \text{open} \end{cases}$$

Candles



$$D_L = \tilde{D}(1+z)$$

Clocks/rulers



angular diameter distances

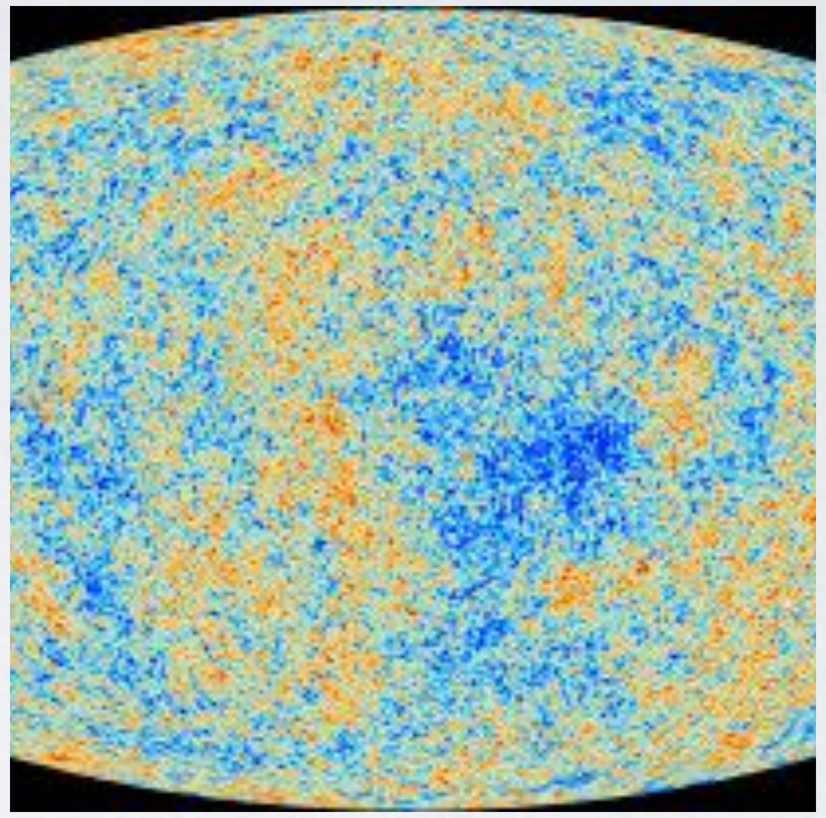
l = lens

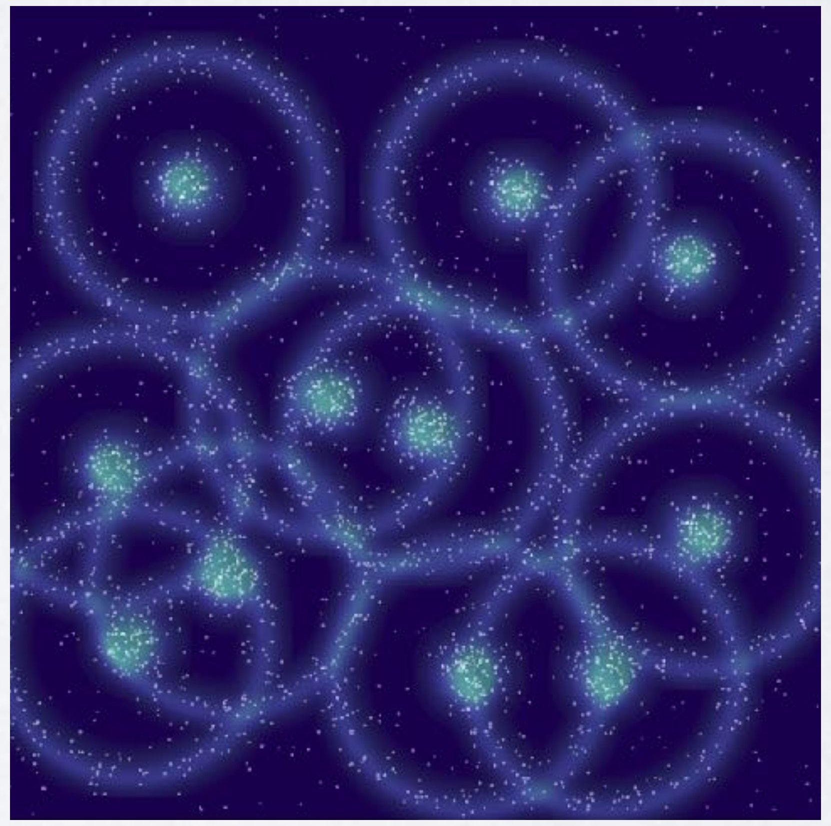
s = source

$$D_{\Delta t} = (1+z_{\rm l})\frac{D_{\rm l} D_{\rm s}}{D_{\rm ls}}$$

$$D_{\Delta t} = \frac{\tilde{D}_{\rm l}\tilde{D}_{\rm s}}{\tilde{D}_{\rm ls}}/(1+z_{\rm l})$$

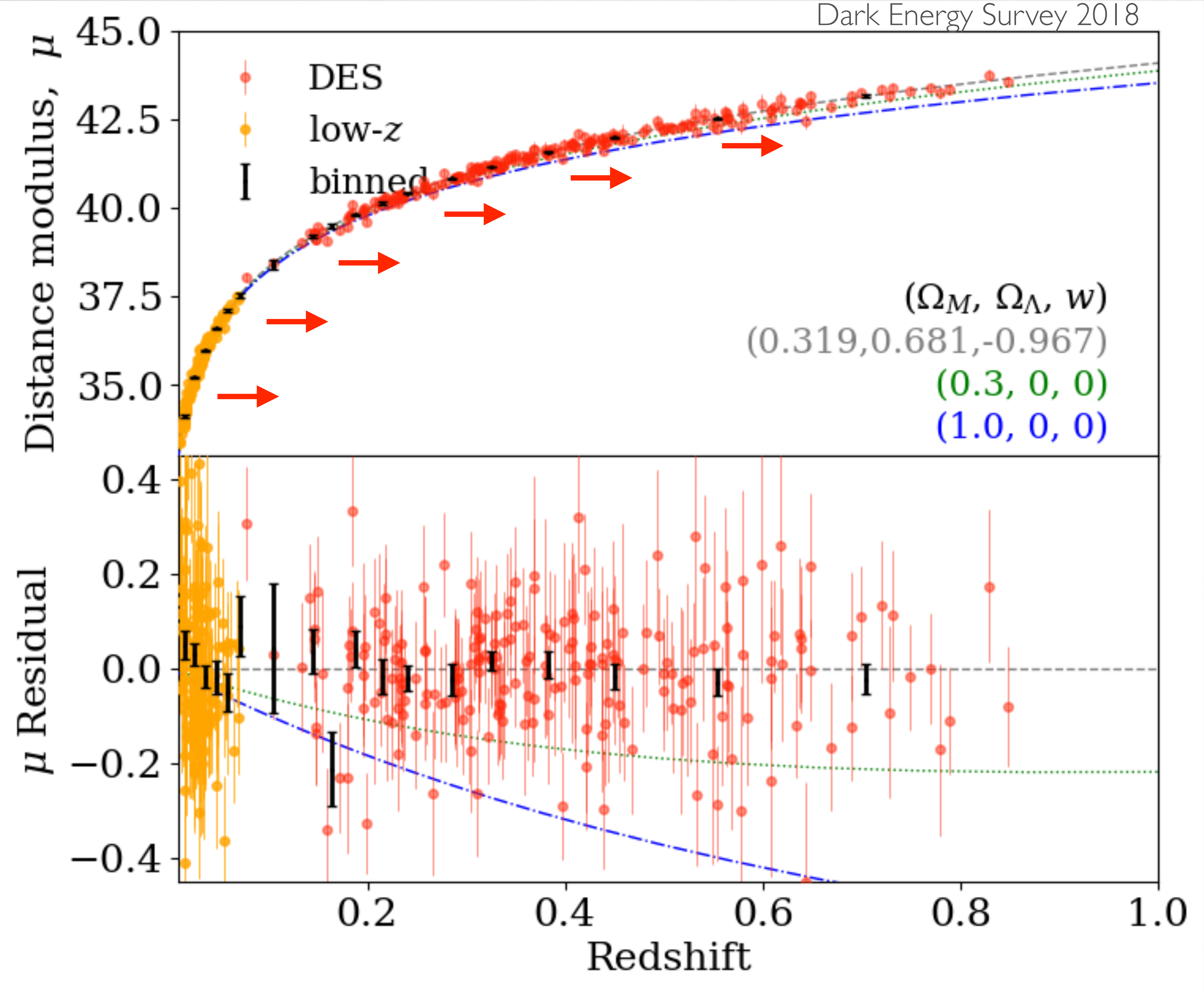
Rulers

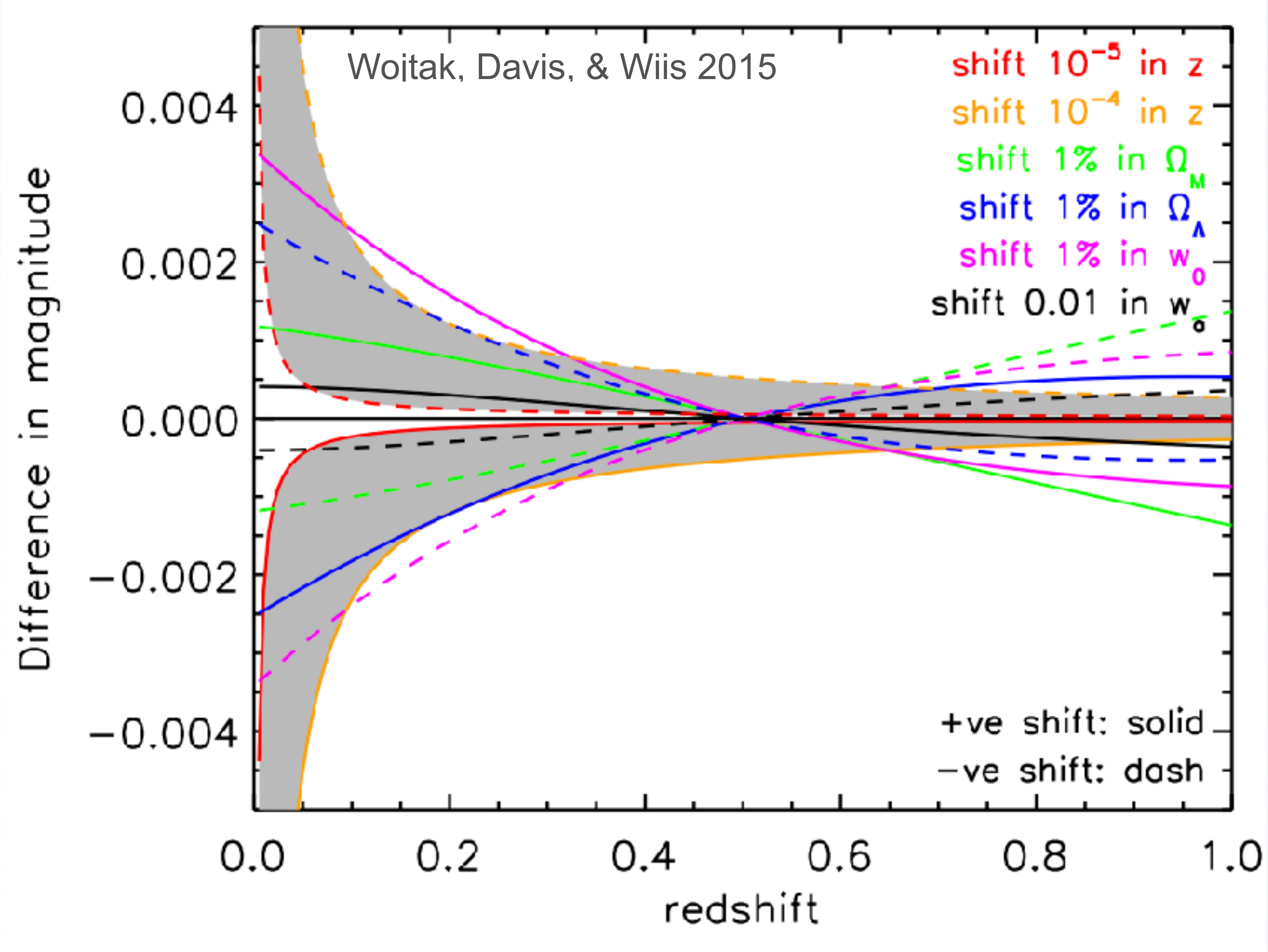




$$D_A = \tilde{D}/(1+z)$$

DO WE NEED TO WORRY ABOUT REDSHIFTS?





DERIVING H_0 FROM CANDLES

$$D_0(z) = \frac{c}{H_0} \int \frac{dz}{E(z)}$$

$$v_0 = c \int \frac{dz}{E(z)} \qquad E(z) = H(z)/H_0$$

$$v_0 \approx \frac{cz}{1+z} \left(1 + \frac{1}{2}[1 - q_0] z - \frac{1}{6}\left[1 - q_0 - 3q_0^2 + j_0\right] z^2\right)$$

$$H_0 = \frac{v_0}{D_0}$$

$$H_0 = \frac{v_0(1+z)}{D_{L,0}}$$

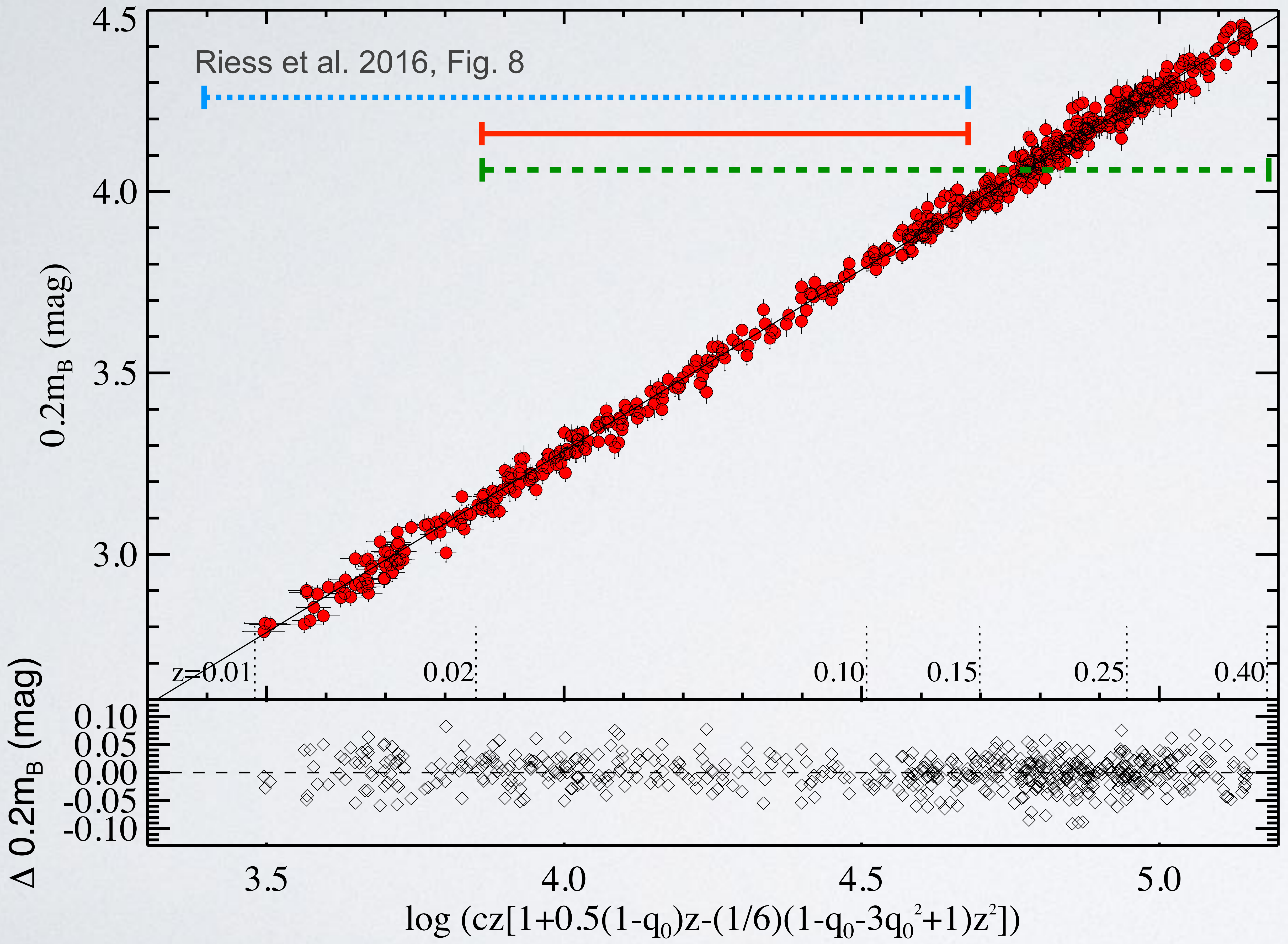
$$\mu = m - M = 5 \log_{10} D_L(\mathrm{Mpc}) + 25$$

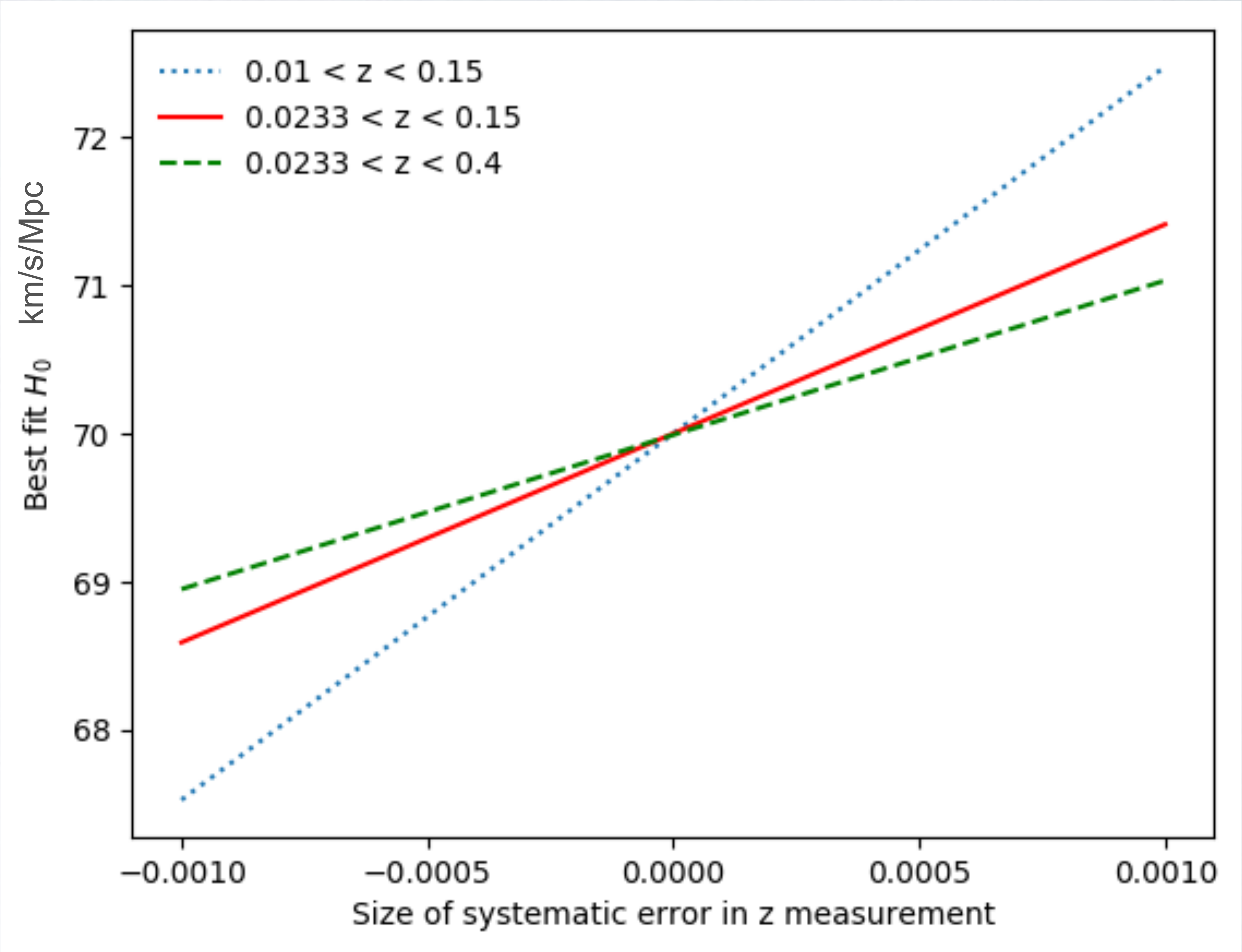
$$\log_{10} D_L = \frac{m - M - 25}{5} = 0.2m - \frac{M+25}{5}$$

$$\begin{aligned}\log_{10} H_0 &= \log_{10}[v_0(1+z)] - \log_{10} D_{L,0} \\ &= \log_{10}[v_0(1+z)] - 0.2m + \frac{M+25}{5} \\ &= a_x + \frac{M+25}{5} \\ &= \frac{5a_x + M + 25}{5}\end{aligned}$$

$$a_x \equiv \log_{10}[v_0(1+z)] - 0.2m$$

HOW LARGE A REDSHIFT ERROR WOULD SIGNIFICANTLY CHANGE H_0?



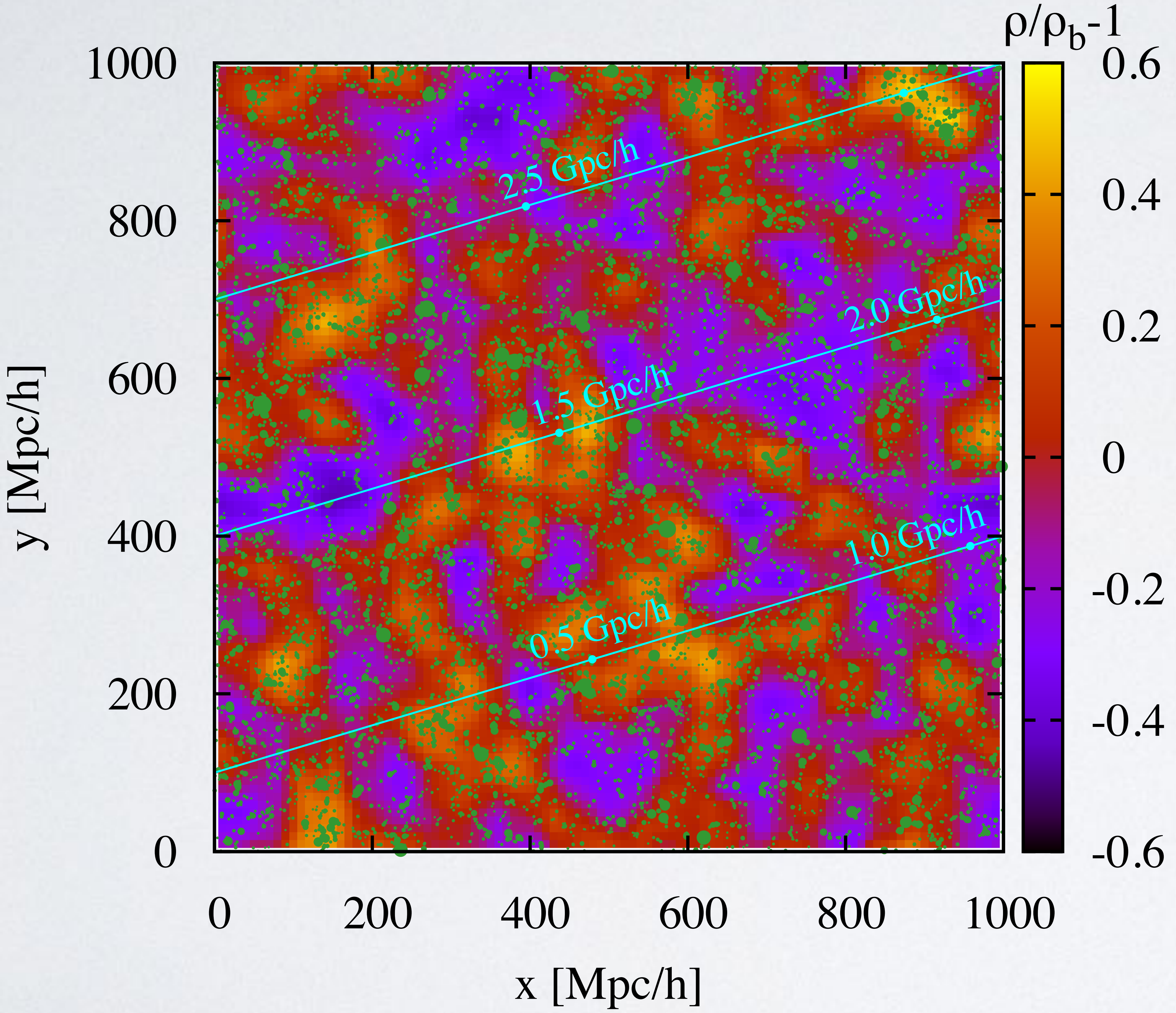


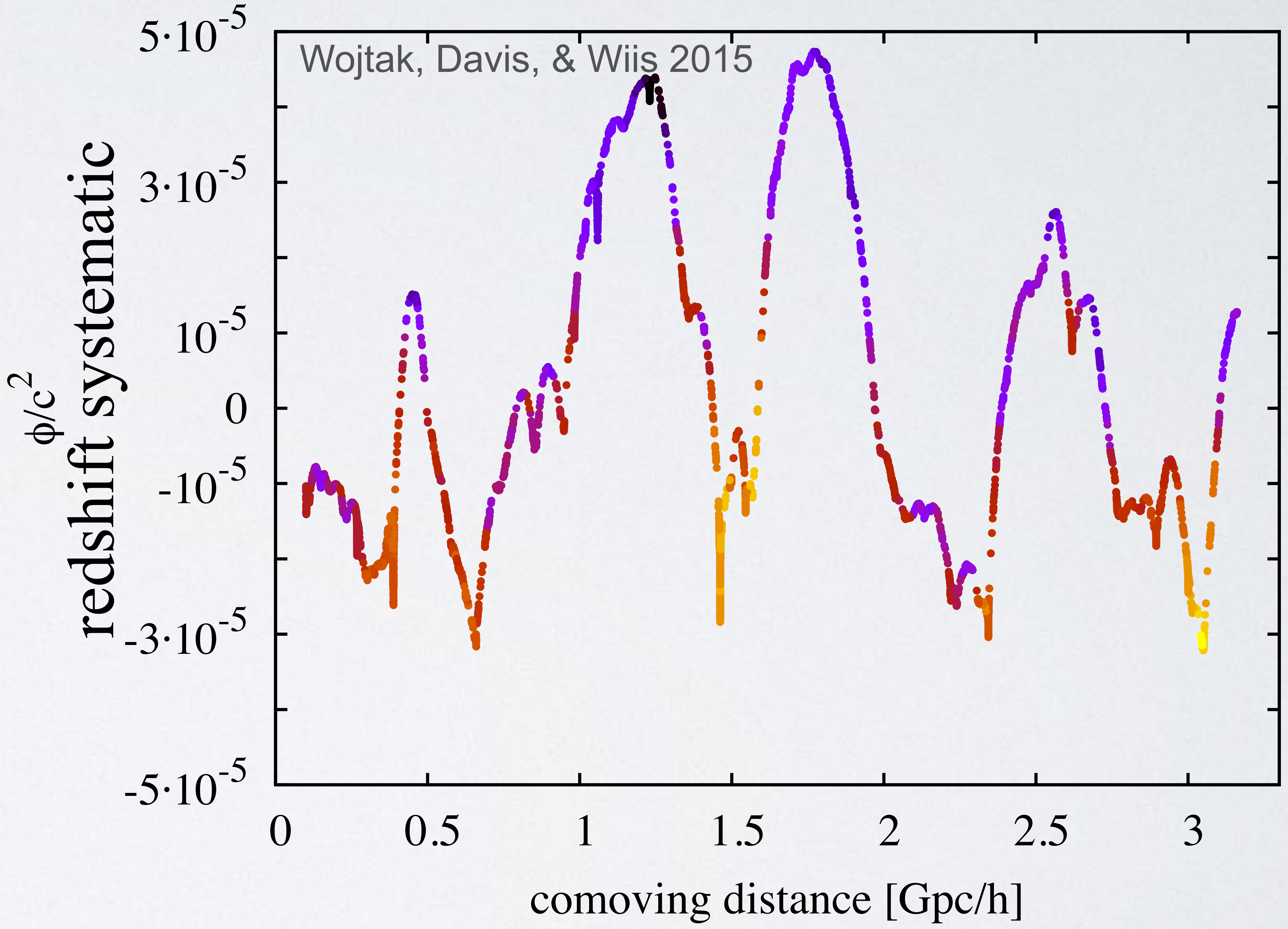
HOW LARGE COULD OUR REDSHIFT BIAS BE?

- Gravitational z (local density fluct.)
- Observational error
- Heliocentric correction
- Using (1+z) factors incorrectly

HOW LARGE COULD OUR REDSHIFT BIAS BE?

- **Gravitational z (local density fluct.)**

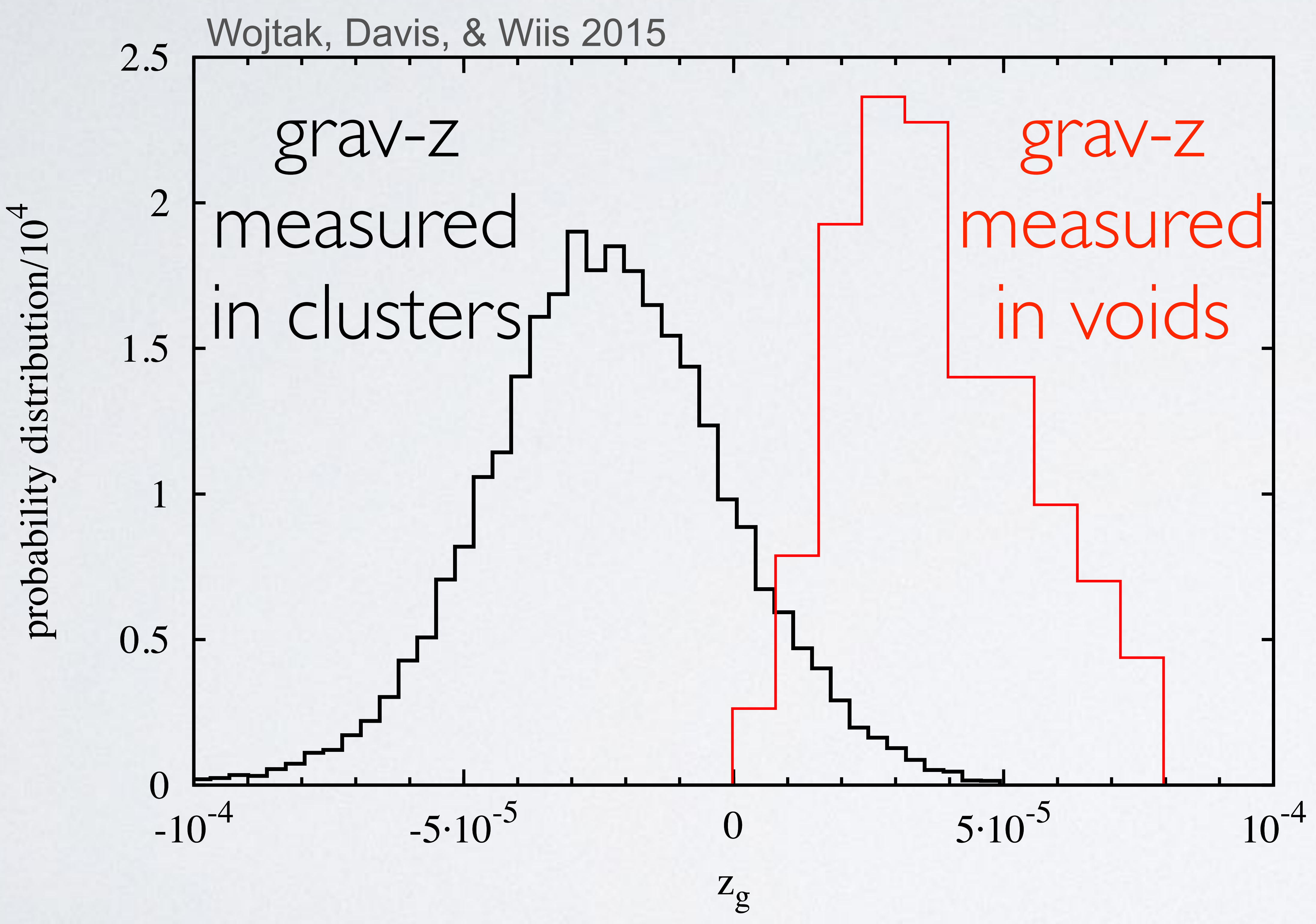




Sim from MultiDark database:
$\Omega_m = 0.27$, $\Omega_\Lambda = 0.73$, $\sigma_8 = 0.82$

HOW LARGE COULD OUR REDSHIFT BIAS BE?

- **Gravitational z (local density fluct.)**



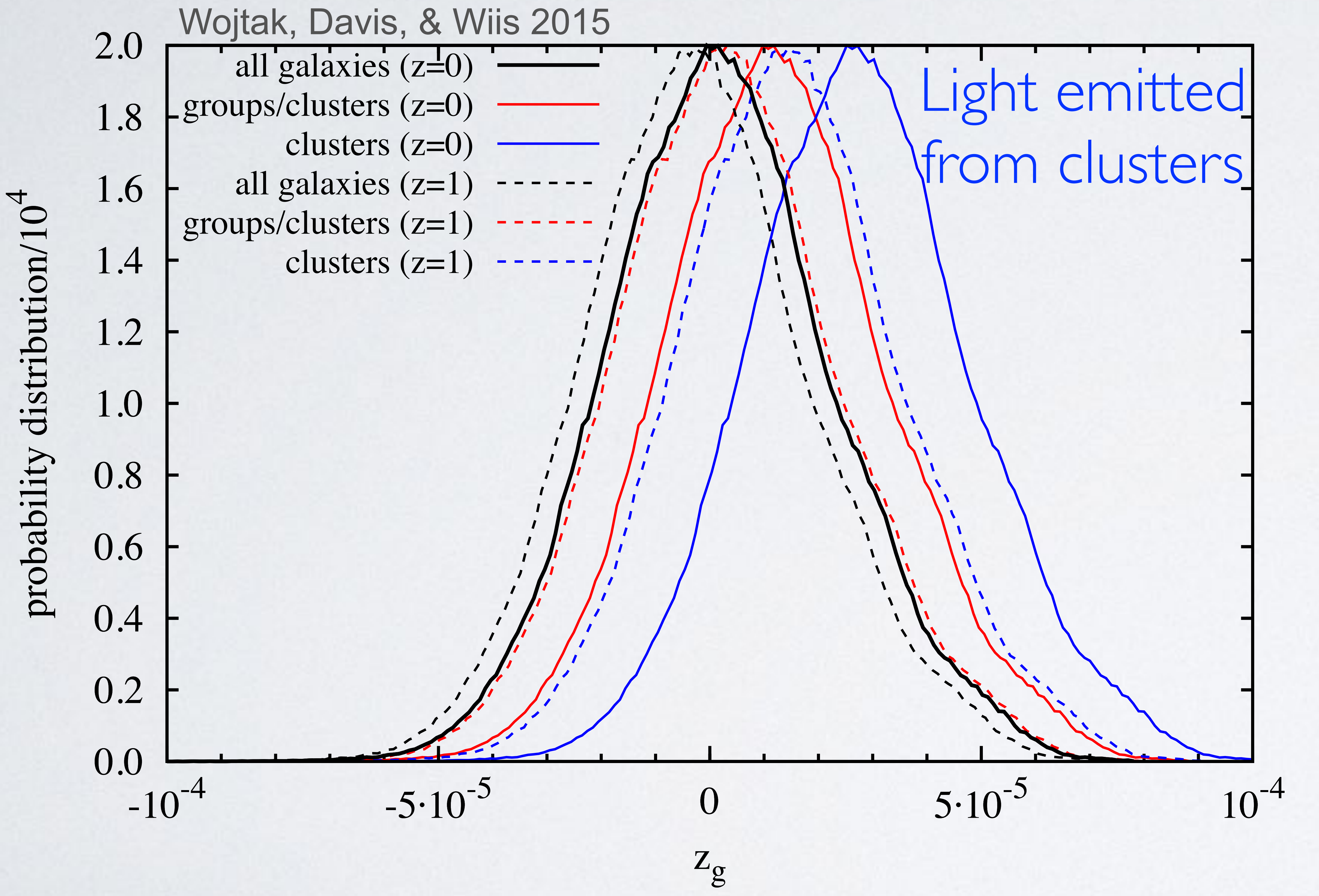
Probability distribution of the gravitational redshift measured by observers in clusters or voids at z = 0.

Observers in **underdense** environments tend to measure a positive signal (gravitational **redshift**),

whereas those in galaxy **clusters** tend to observe a negative signal (gravitational **blueshift**).

HOW LARGE COULD OUR REDSHIFT BIAS BE?

- **Gravitational z (local density fluct.)**

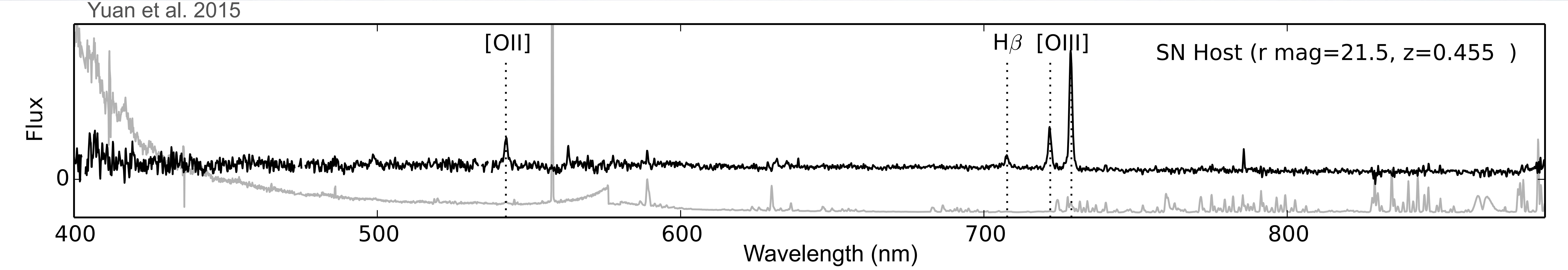


Probability distributions of gravitational redshift at positions of Milky-Way-like galaxies.

Gravitational redshifts are smaller for light emitted from high-redshift galaxies, because structure was less clustered at the time of emisison.

HOW LARGE COULD OUR REDSHIFT BIAS BE?

- **Observational error**

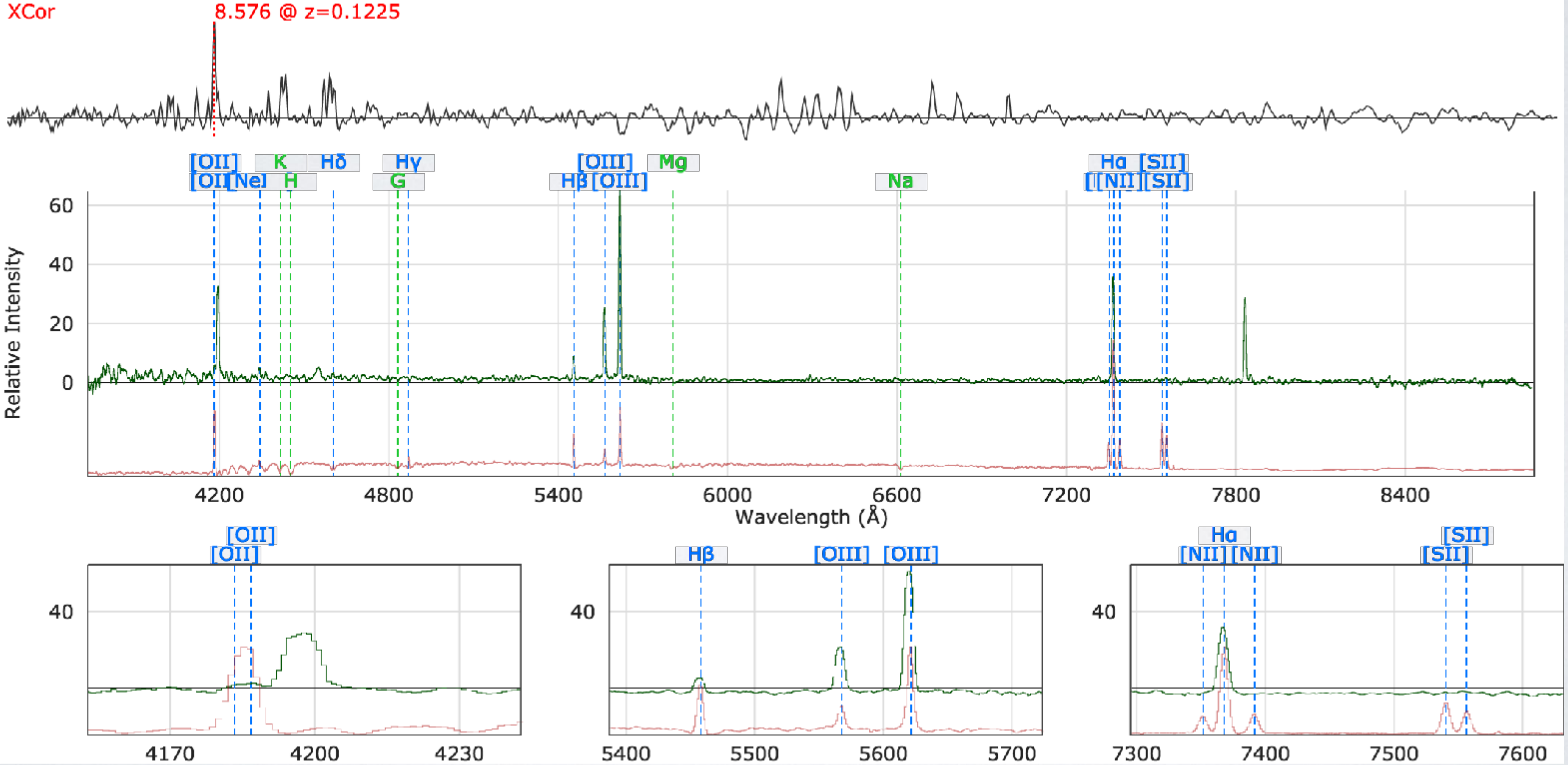


OzDES spectrum of a supernova host galaxy (an extremely pretty one)

OzDES spectrum of a supernova host galaxy (an extremely ugly one)

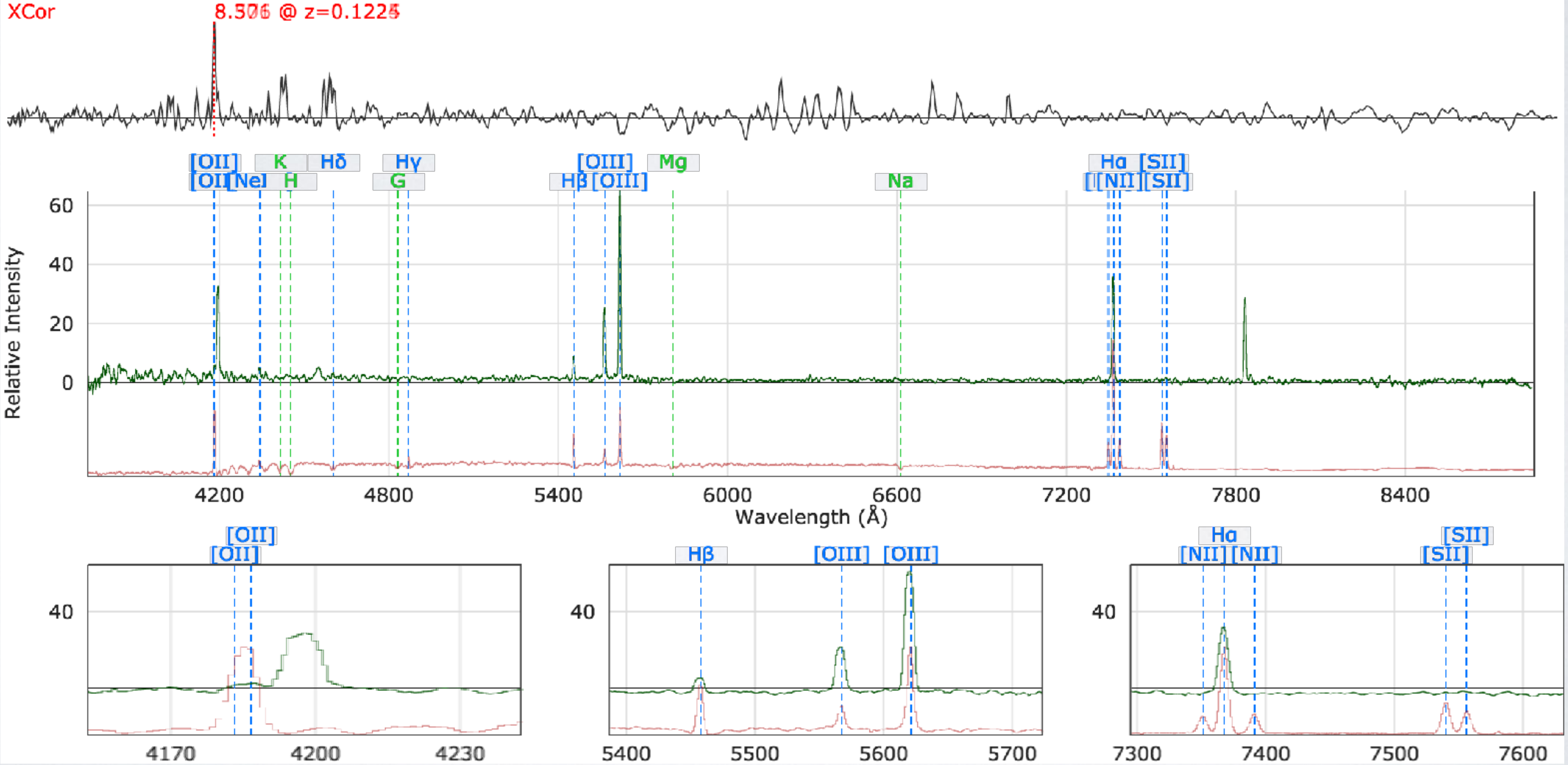
HOW LARGE COULD OUR REDSHIFT BIAS BE?

- **Observational error**



HOW LARGE COULD OUR REDSHIFT BIAS BE?

- **Observational error**



HOW LARGE COULD OUR REDSHIFT BIAS BE?

- **Heliocentric correction**

```
helio_corr.f
*-----------------------------------------------------------
* New subroutine to calculate helio-centric correction using SLALIB
* routines which are more robust (and correct!) than the previous
* versions here.  Returns heliocentric velocity correction in km/s.
* this version corrects for the annual motion of the earth around the
* sun (max correction of ~30km/s) but does not correct for earth rotation
* (<0.5km/s) or other weaker effects.  A quick cross-check with IRAF
* rvcorrect gave agreement with the annual correction to better
* than 0.1 km/s.

      subroutine helio_corr (cenra,cendec,actmjd,HCV) ! returns hcv in km/s.
*
* written by SMC (14/09/09)
      implicit none
…
```

velocity	redshift
30km/s	10^{-4}
0.5km/s	$\sim 10^{-6}$

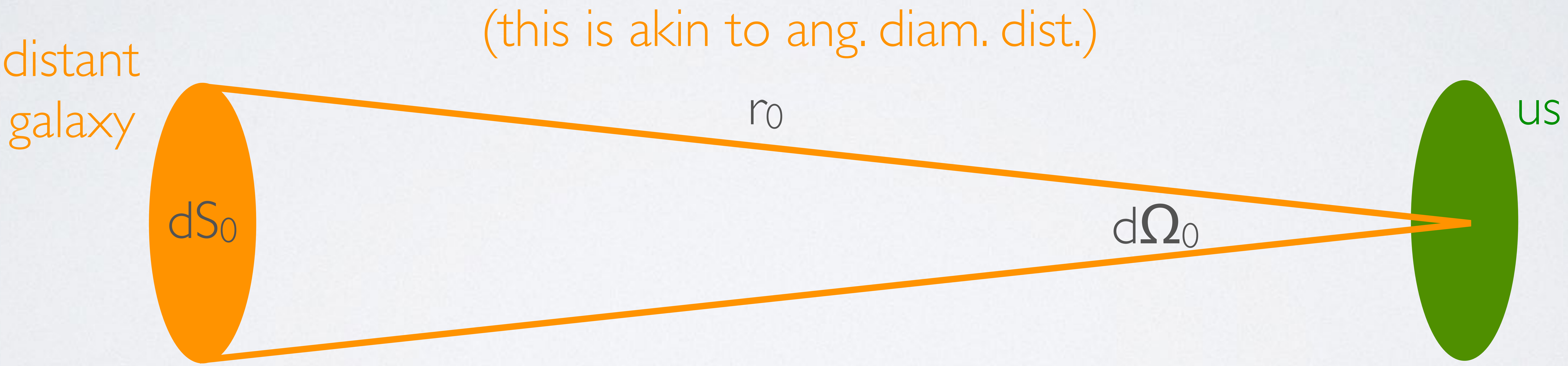
HOW LARGE COULD OUR REDSHIFT BIAS BE?

- **Using (1+z) factors incorrectly**

Reciprocity relation (distance duality)

$$D_L = \tilde{D}(1+z)$$
$$D_A = \tilde{D}/(1+z)$$

$$\tilde{D} = \begin{cases} R_0 \sin(\chi) & \text{closed} \\ R_0 \chi & \text{flat} \\ R_0 \sinh(\chi) & \text{open} \end{cases}$$



$$dS_0 = r_0^2 \, d\Omega_0$$

HOW LARGE COULD OUR REDSHIFT BIAS BE?

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Reciprocity relation (distance duality)

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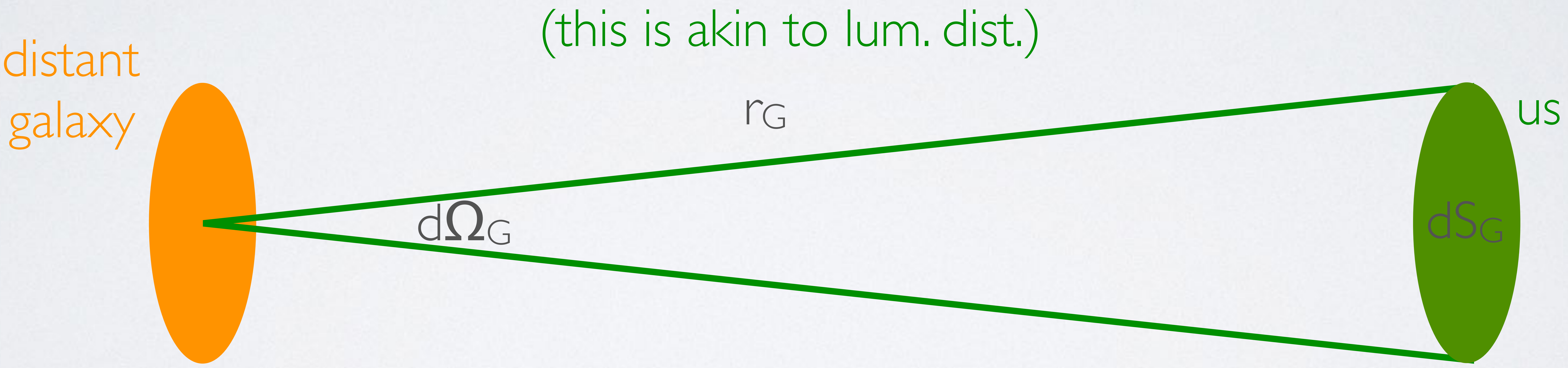


$dS_G = r_G^2 \, d\Omega_G$

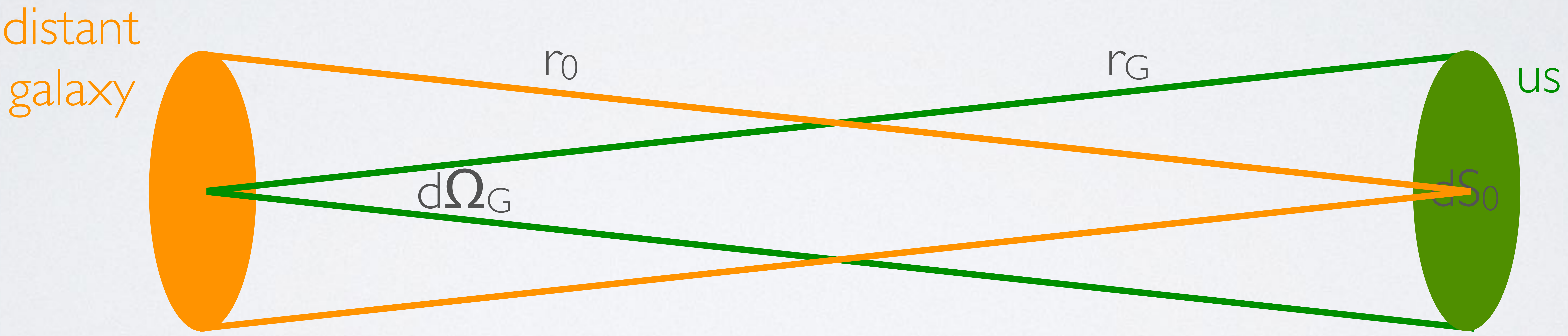
HOW LARGE COULD OUR REDSHIFT BIAS BE?

- **Using (1+z) factors incorrectly**

Reciprocity relation (distance duality)
Etherington 1933

$$D_L = \tilde{D}(1+z)$$
$$D_A = \tilde{D}/(1+z)$$

$$\tilde{D} = \begin{cases} R_0 \sin(\chi) & \text{closed} \\ R_0 \chi & \text{flat} \\ R_0 \sinh(\chi) & \text{open} \end{cases}$$



But which redshifts should we use?

HOW LARGE COULD OUR REDSHIFT BIAS BE?

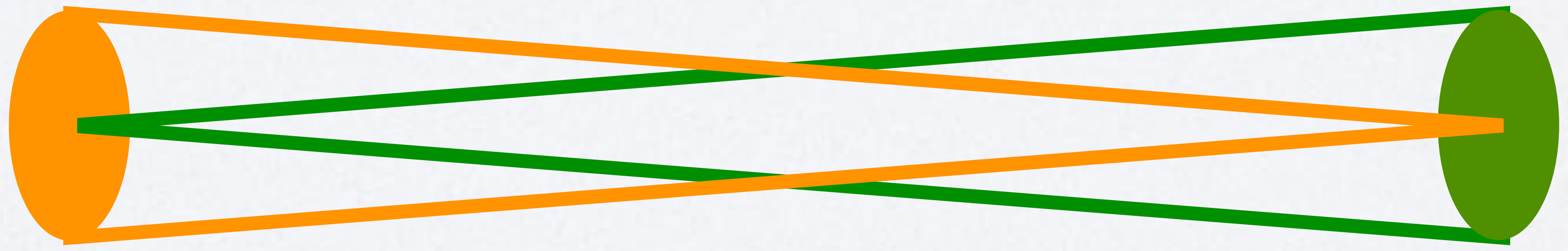
- **Using (1+z) factors incorrectly**

But which redshifts should we use?

$$D_L(\bar{z}, z_{\rm obs}) = \tilde{D}(\bar{z})(1 + z_{\rm obs})$$
$$D_A(\bar{z}, z_{\rm obs}) = \tilde{D}(\bar{z})/(1 + z_{\rm obs})$$

CMB frame
(cosmological) redshift

observed
redshift



1. Ellis, G.F.R.: Relativistic Cosmology. In: Sachs, R.K. (ed.) General Relativity and Cosmology, Proc Int School of Physics "Enrico Fermi" (Varenna), Course XLVII, pp. 104–179. Academic Press, New York (1971)

2. Weinberg, S.W.: Gravitation and Cosmology:Principles and applications of the general theory of relativity. Wiley, New York (1972)

HOW LARGE COULD OUR REDSHIFT BIAS BE?

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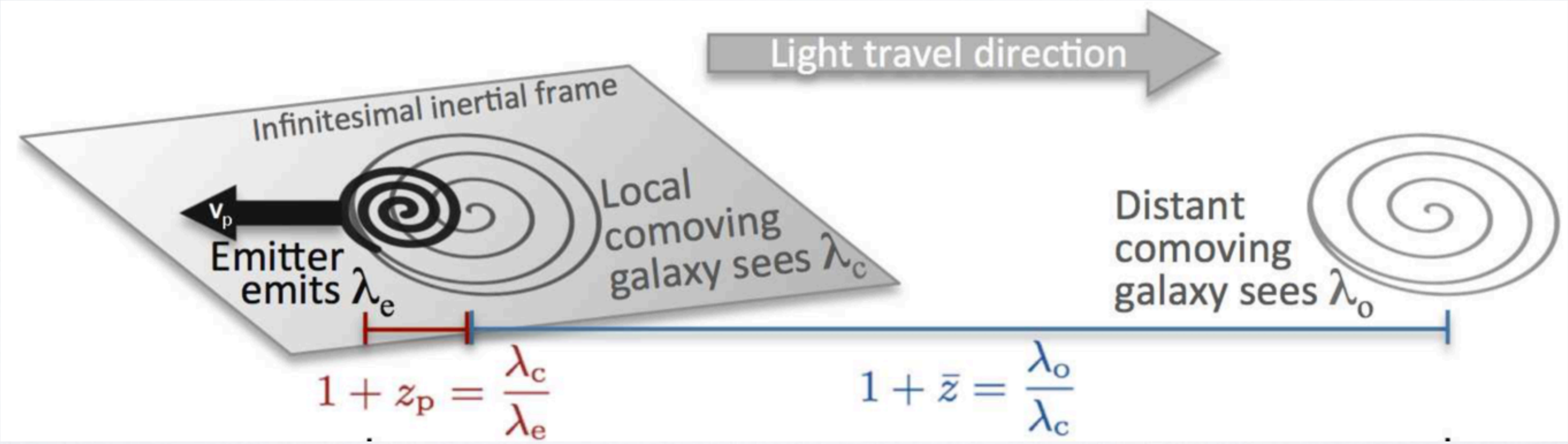
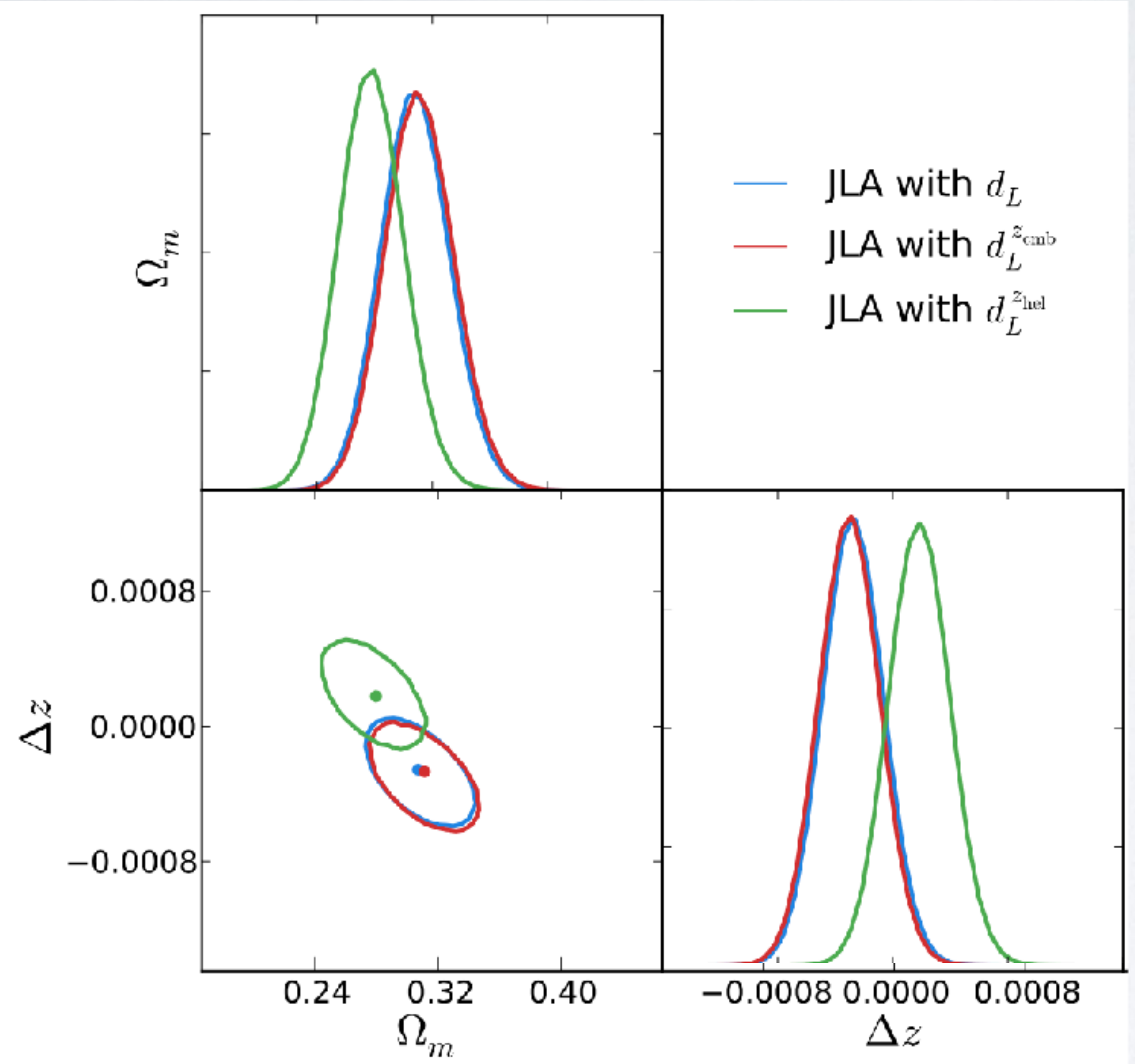


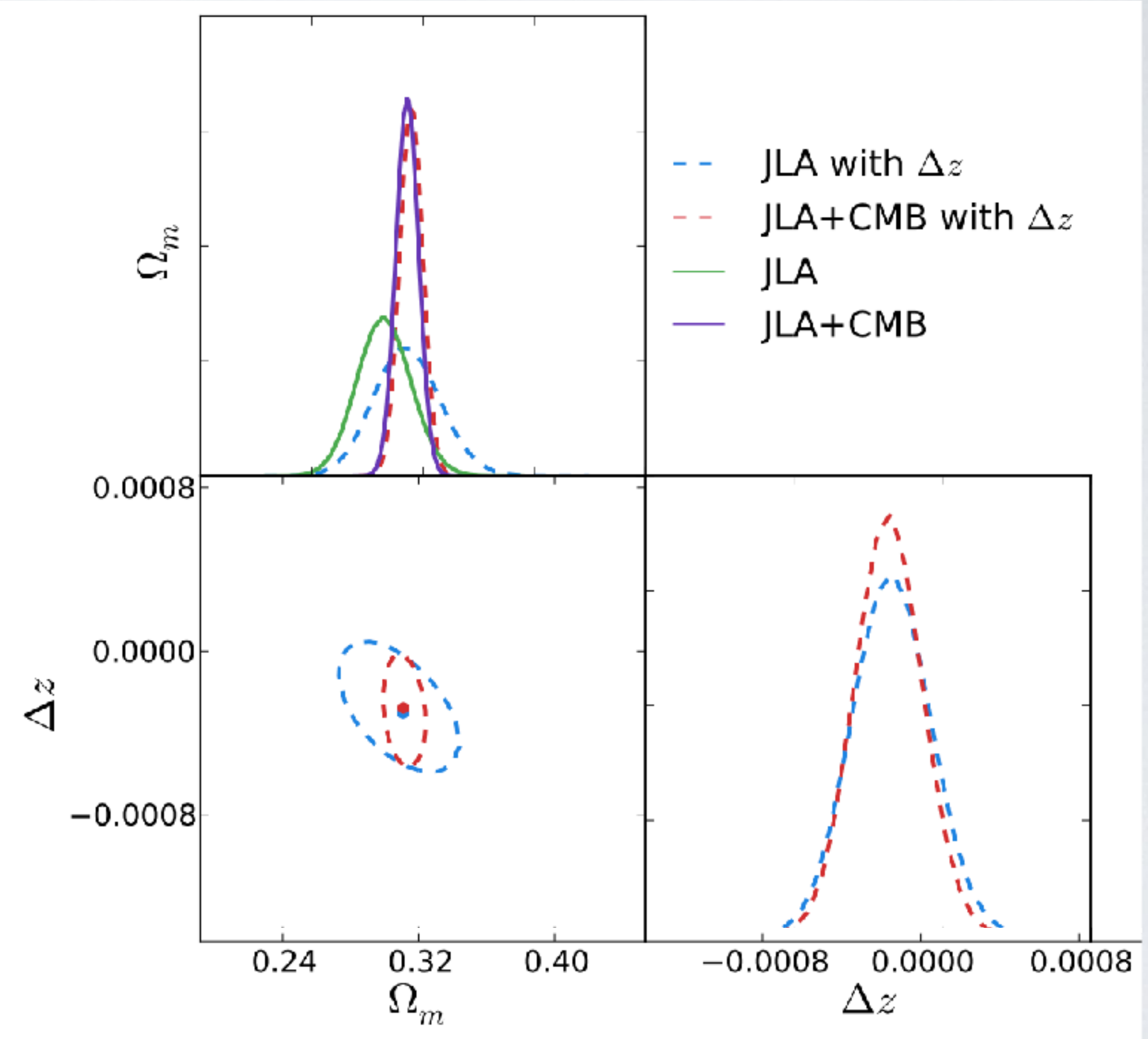
$$(1+z) = \frac{\lambda_o}{\lambda_e} = \frac{\lambda_o}{\lambda_c}\frac{\lambda_c}{\lambda_e} = (1+\bar{z})(1+z_p)$$

Fig from Davis & Scrimgeour, 2015: arXiv:1405.0105

DOES HELIOCENTRIC CORRECTION MATTER?



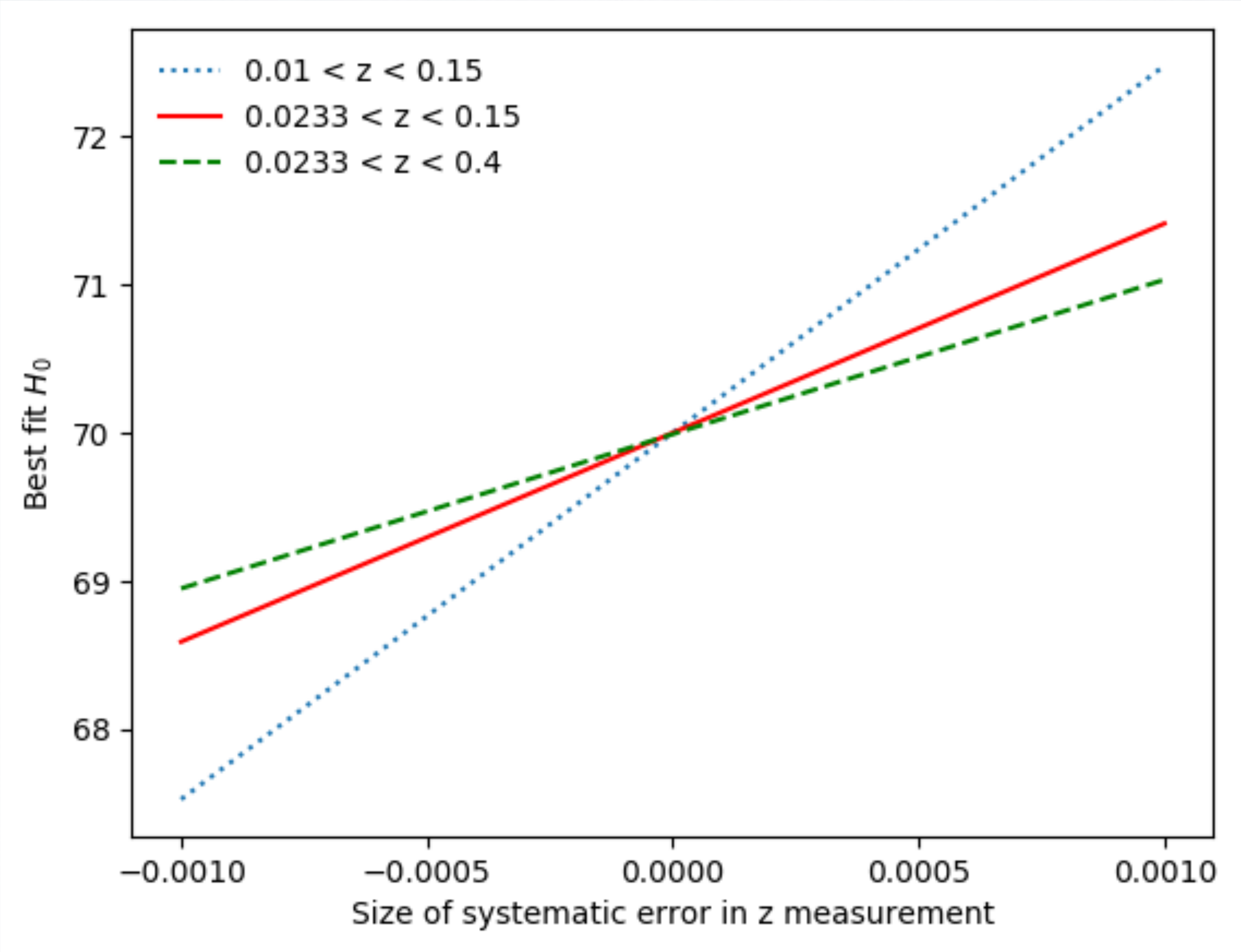
IS THERE EVIDENCE FOR A REDSHIFT SHIFT?



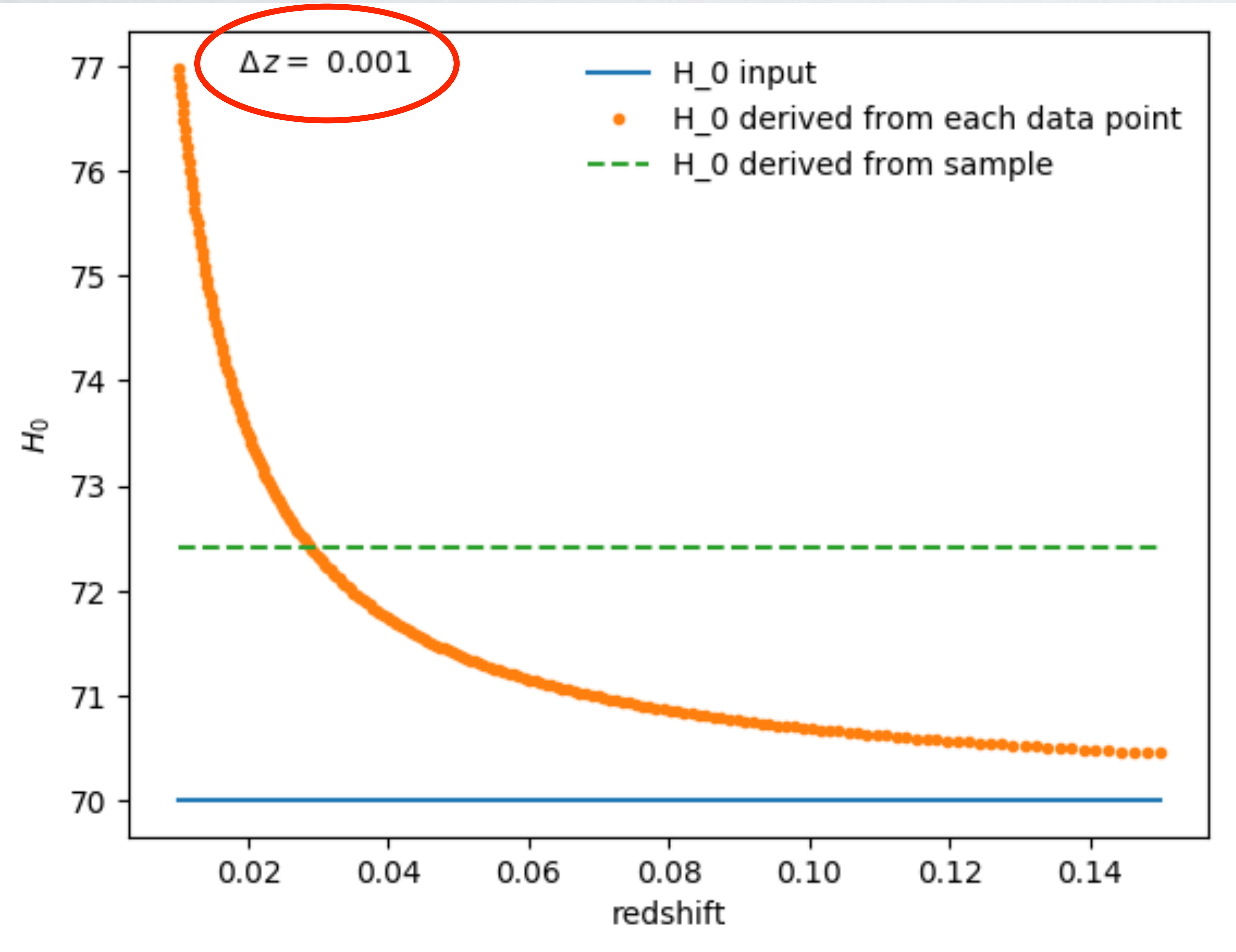
$$d_L = \begin{cases} (1+z_{\rm hel})d(z_{\rm cmb}) \\ (1+z_{\rm hel}+\Delta z)d(z_{\rm cmb}+\Delta z) \end{cases}$$

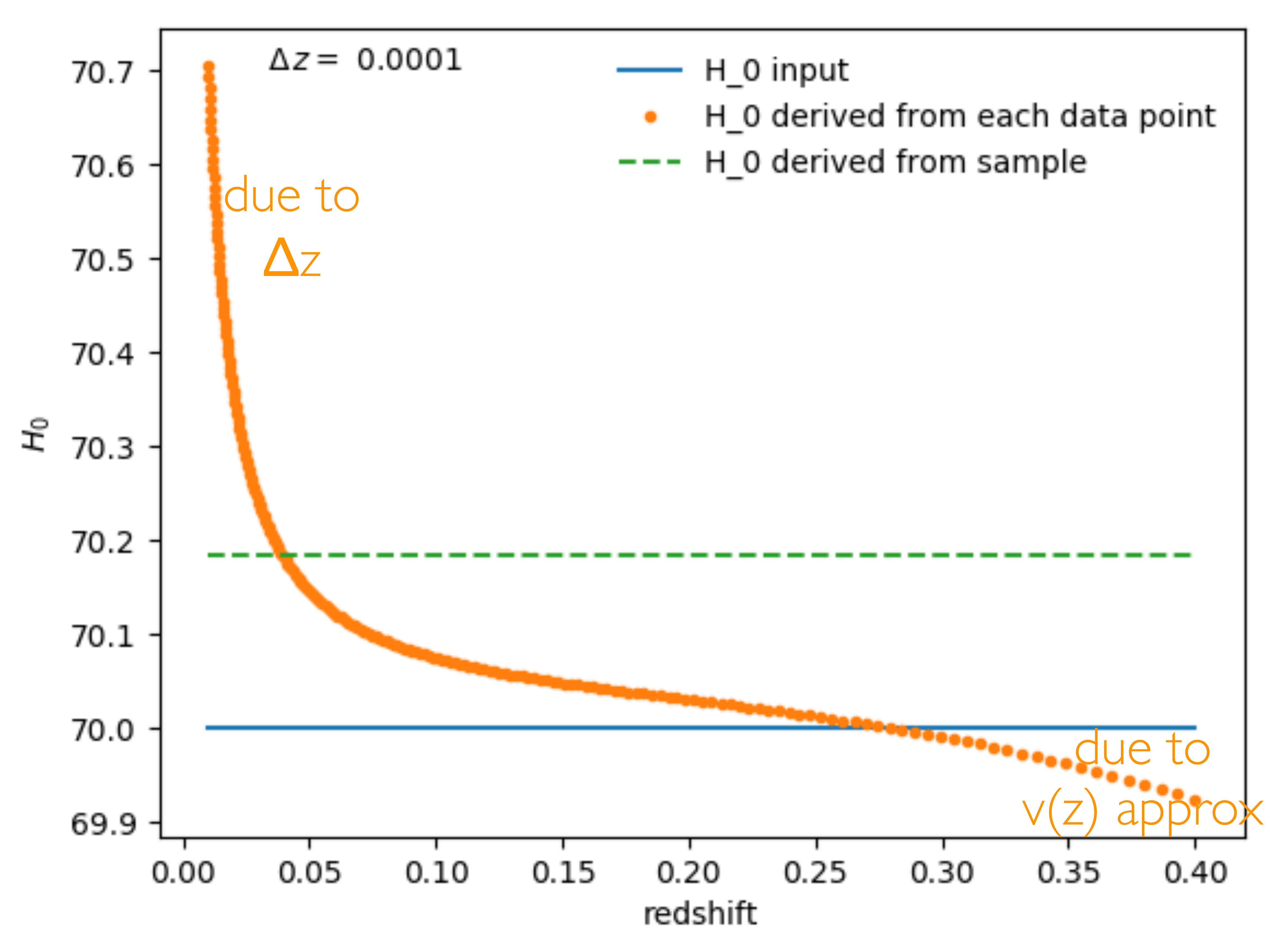
Calcino et al. 2017 (arXiv:1610.07695) + honours thesis

HOW LARGE A REDSHIFT ERROR WOULD SIGNIFICANTLY CHANGE H_0?

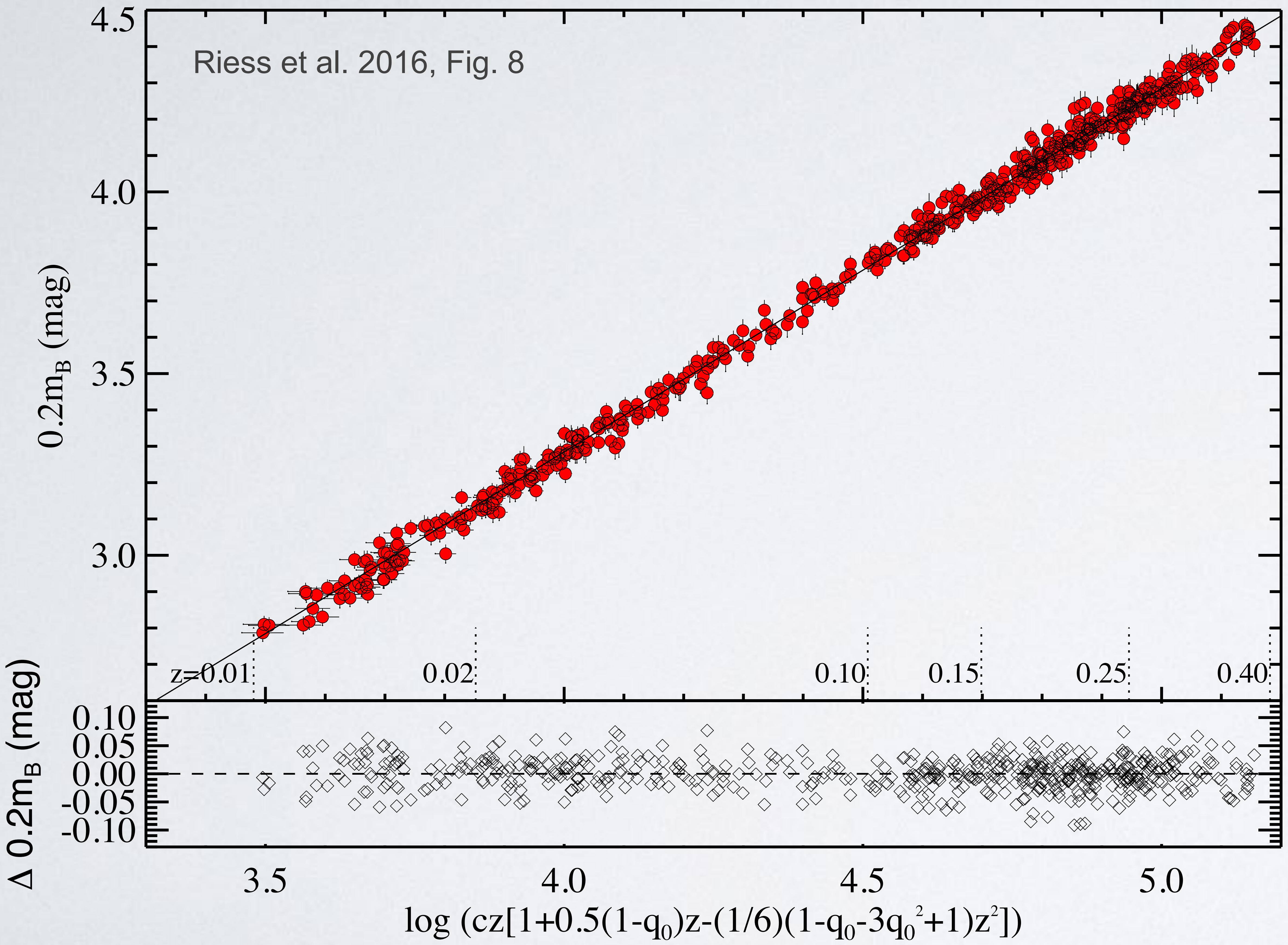


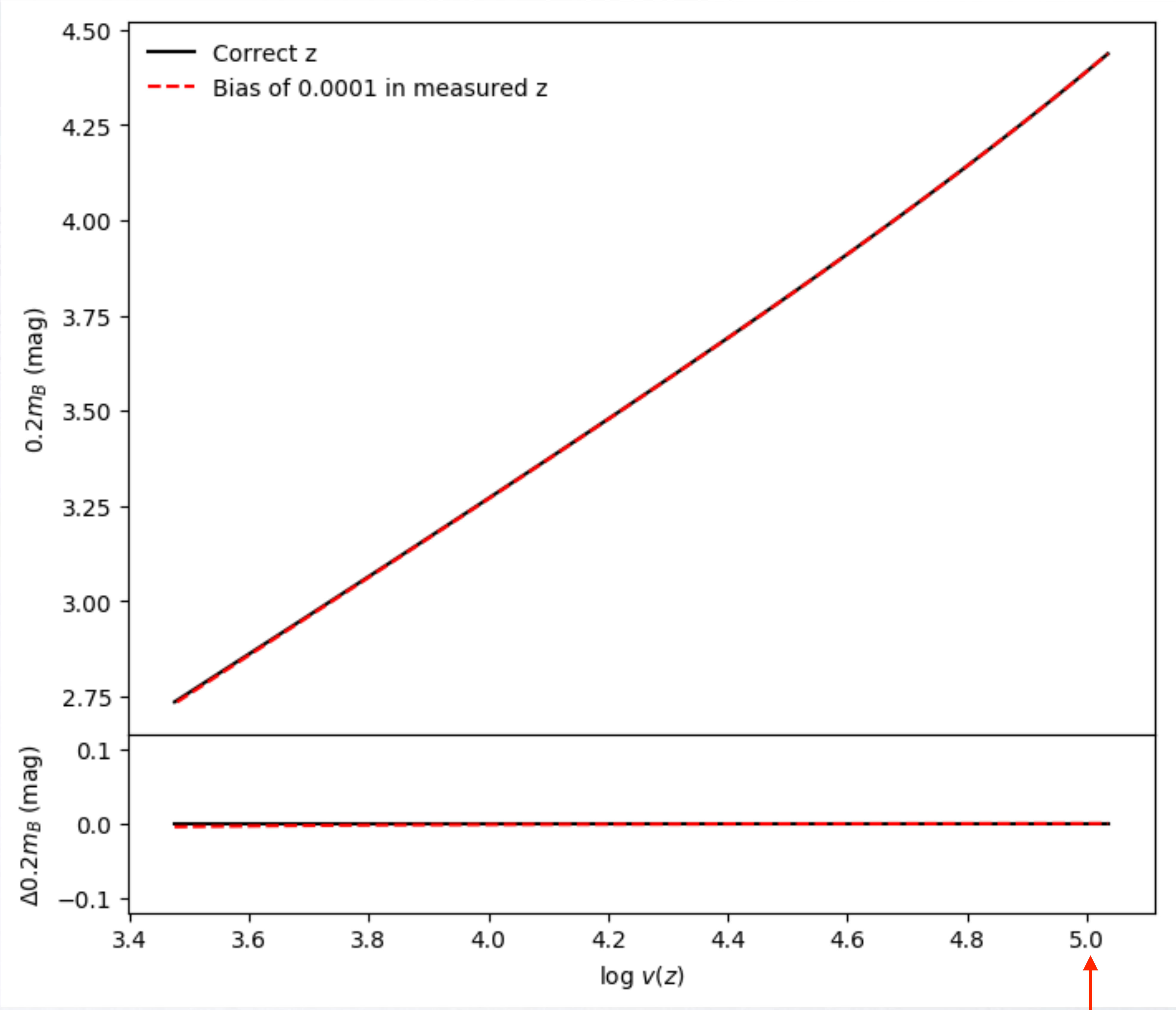
HOW LARGE A REDSHIFT ERROR WOULD SIGNIFICANTLY CHANGE H_0?





SURELY WE'D HAVE NOTICED THAT, RIGHT?

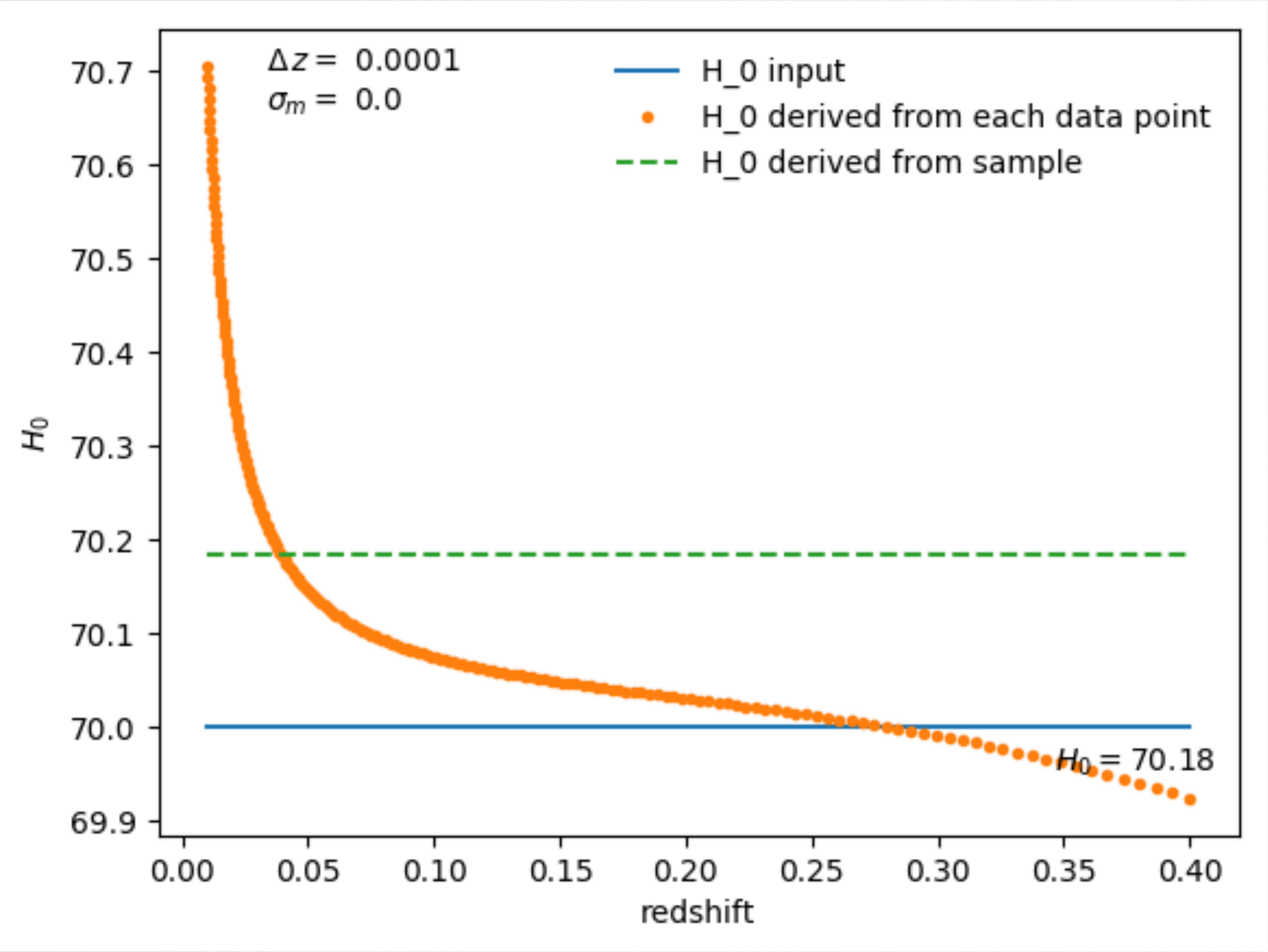


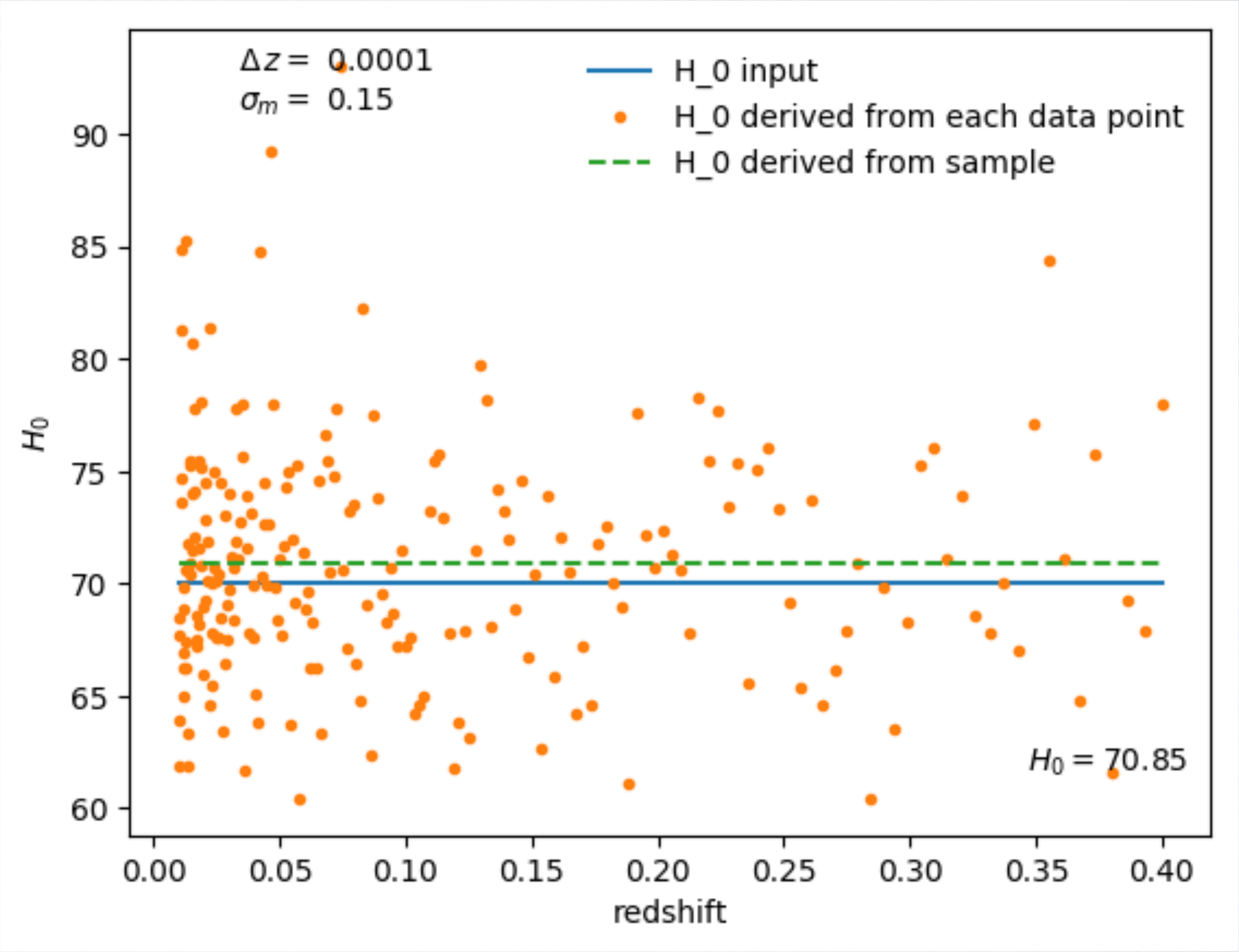


10^5 km/s

∴ small % errors in velocity matter

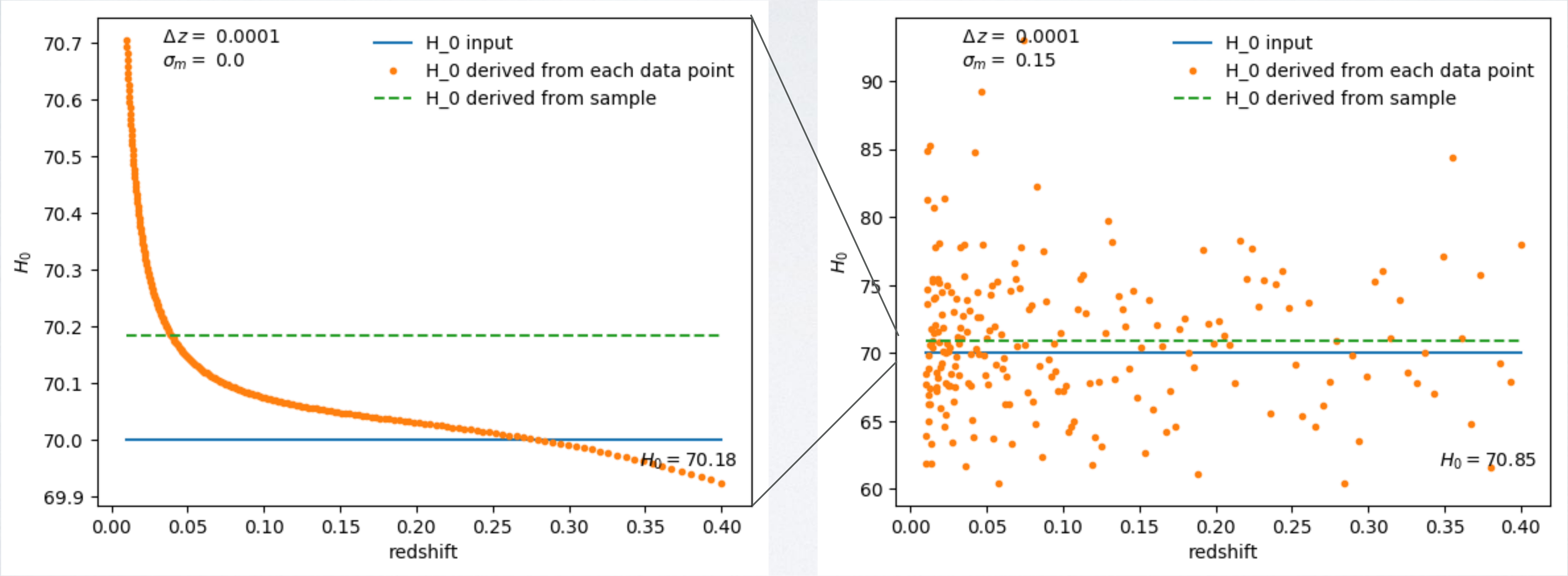
SCATTER VERSION OF H_0 VS Z





magnitude error of 0.15 mag

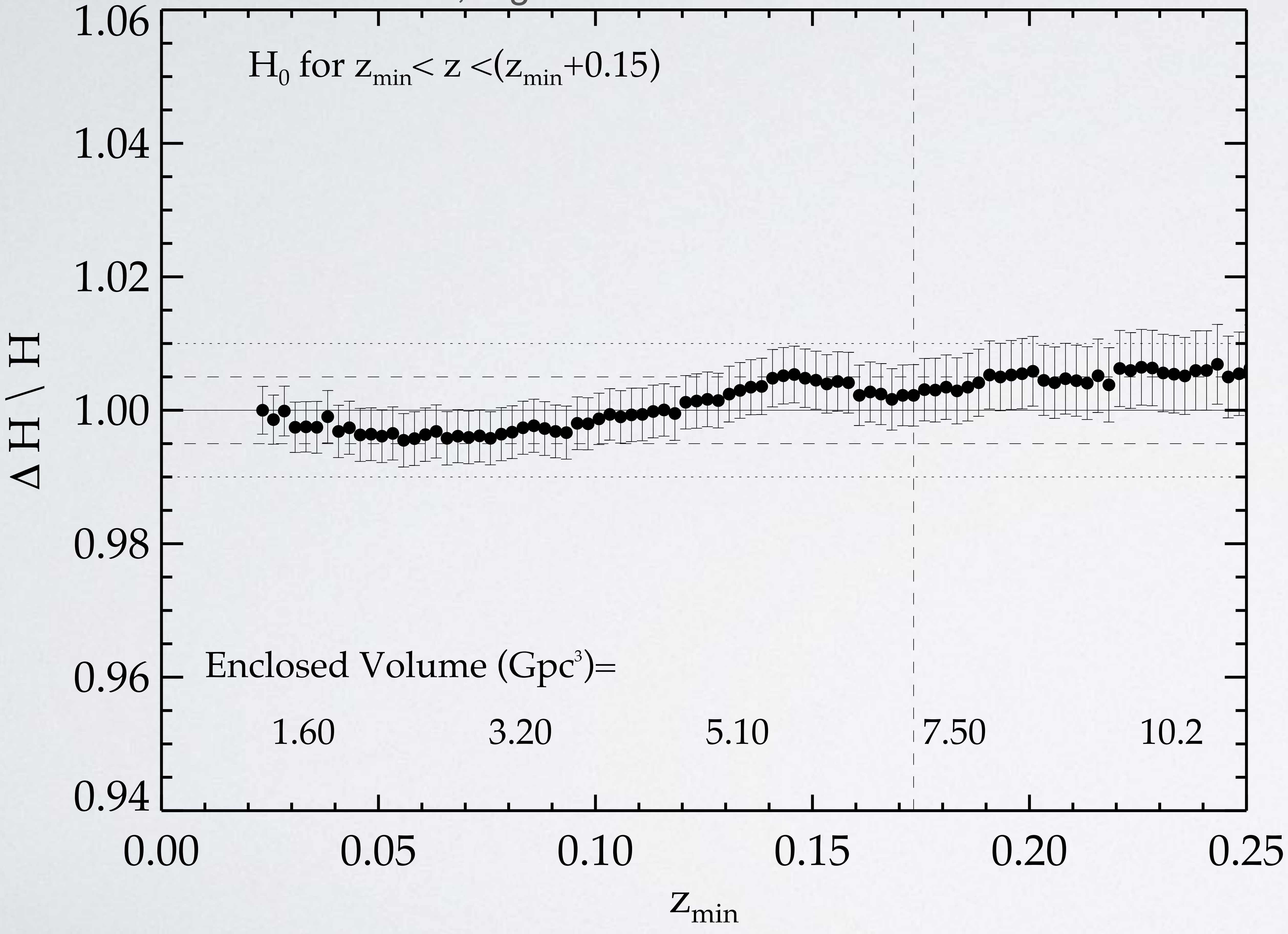
SCATTER VERSION OF H_0 VS Z

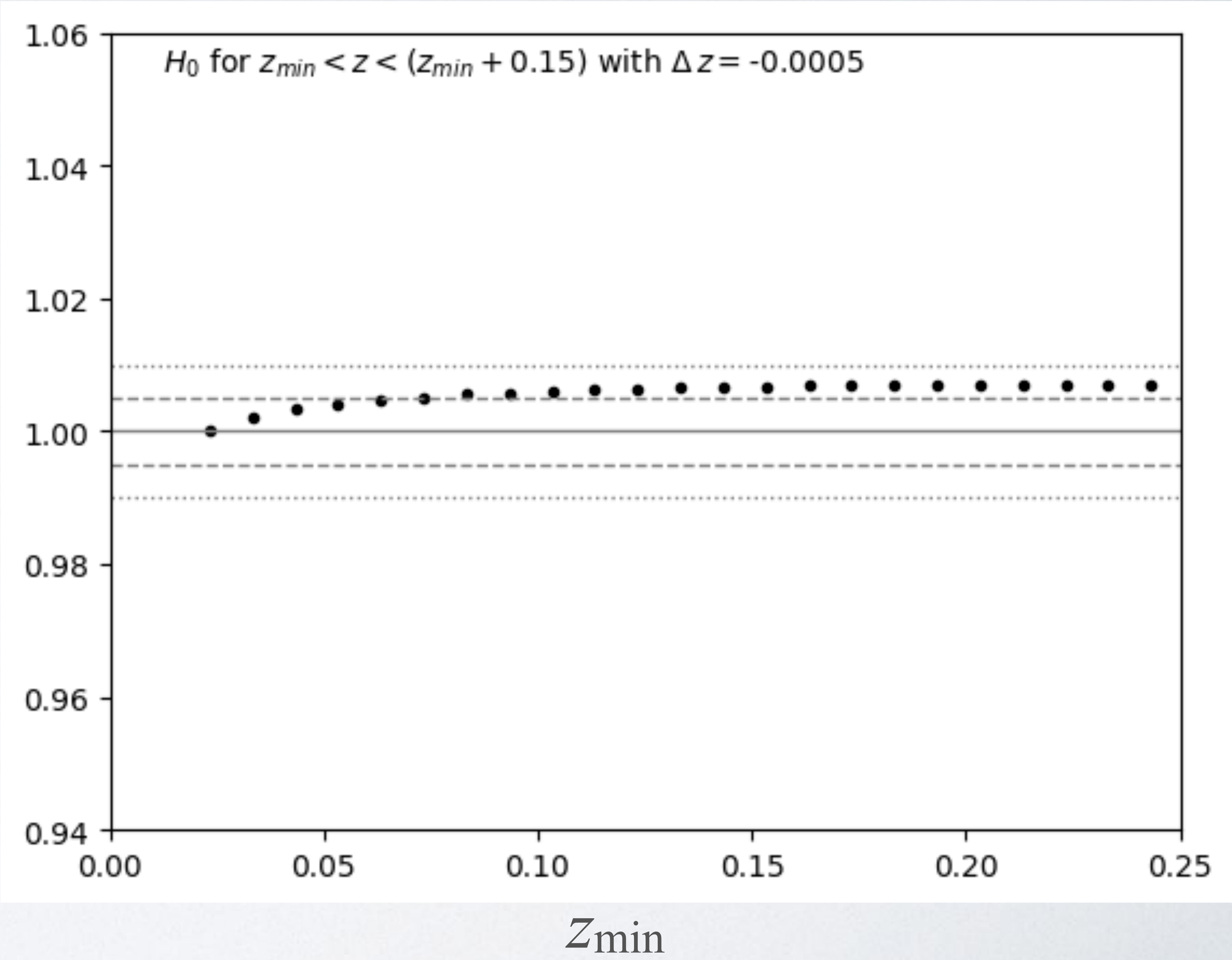


magnitude error of 0.15 mag

HOW LARGE A REDSHIFT ERROR WOULD SIGNIFICANTLY CHANGE H_0?

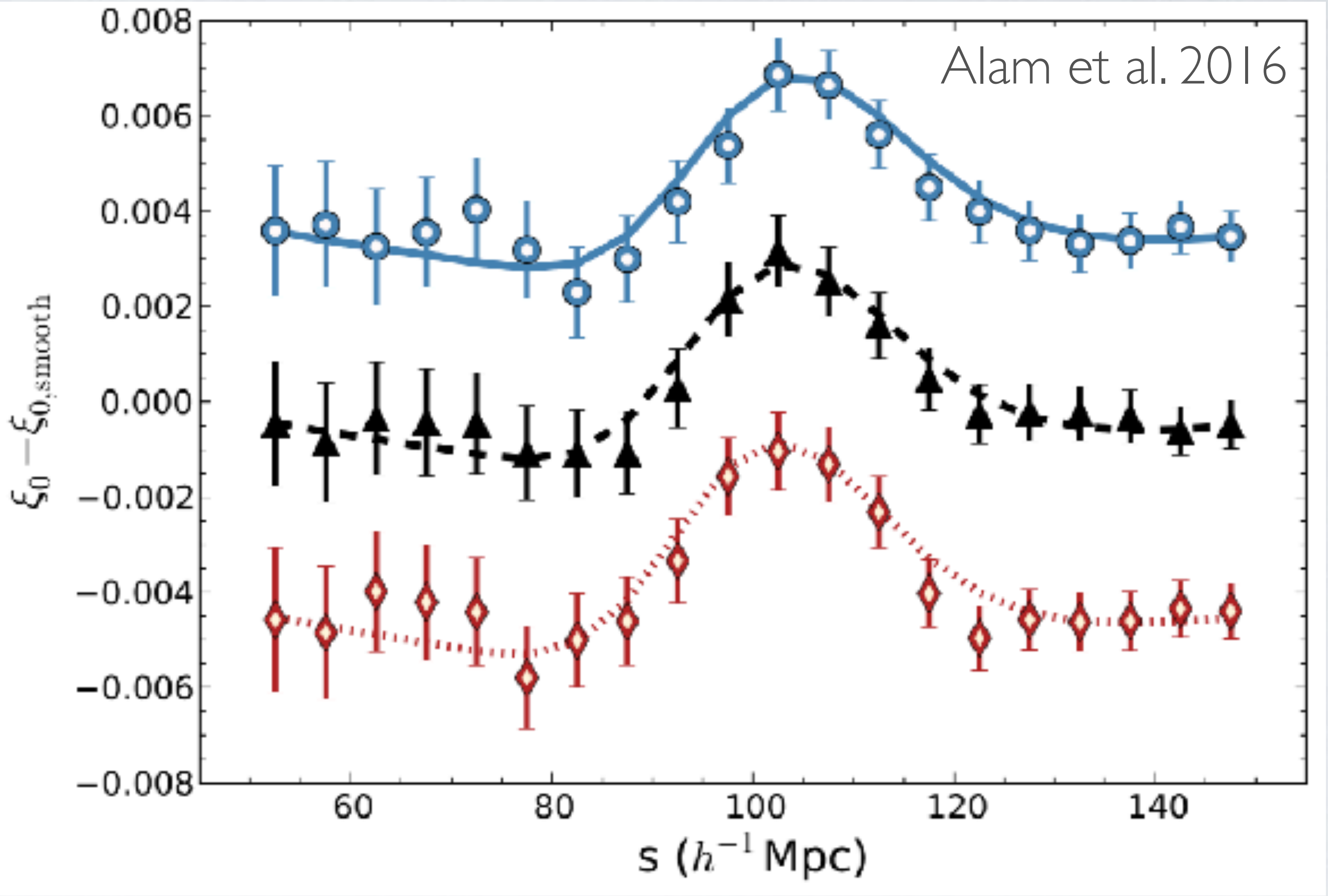
Riess et al. 2016, Figure 12





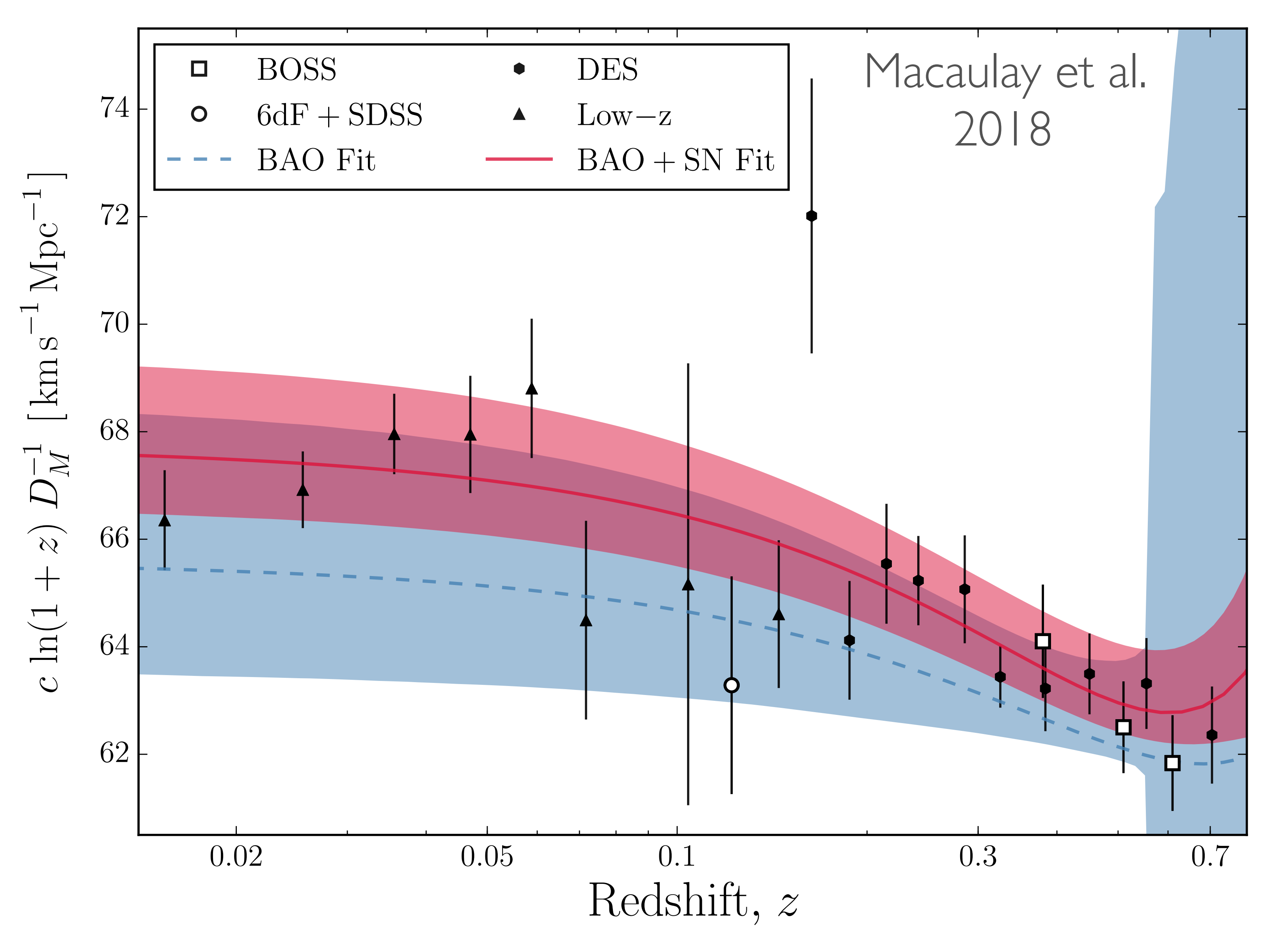
MEASURING H_0 WITH BAO - TWO METHODS

- Fit a cosmological model to the BAO

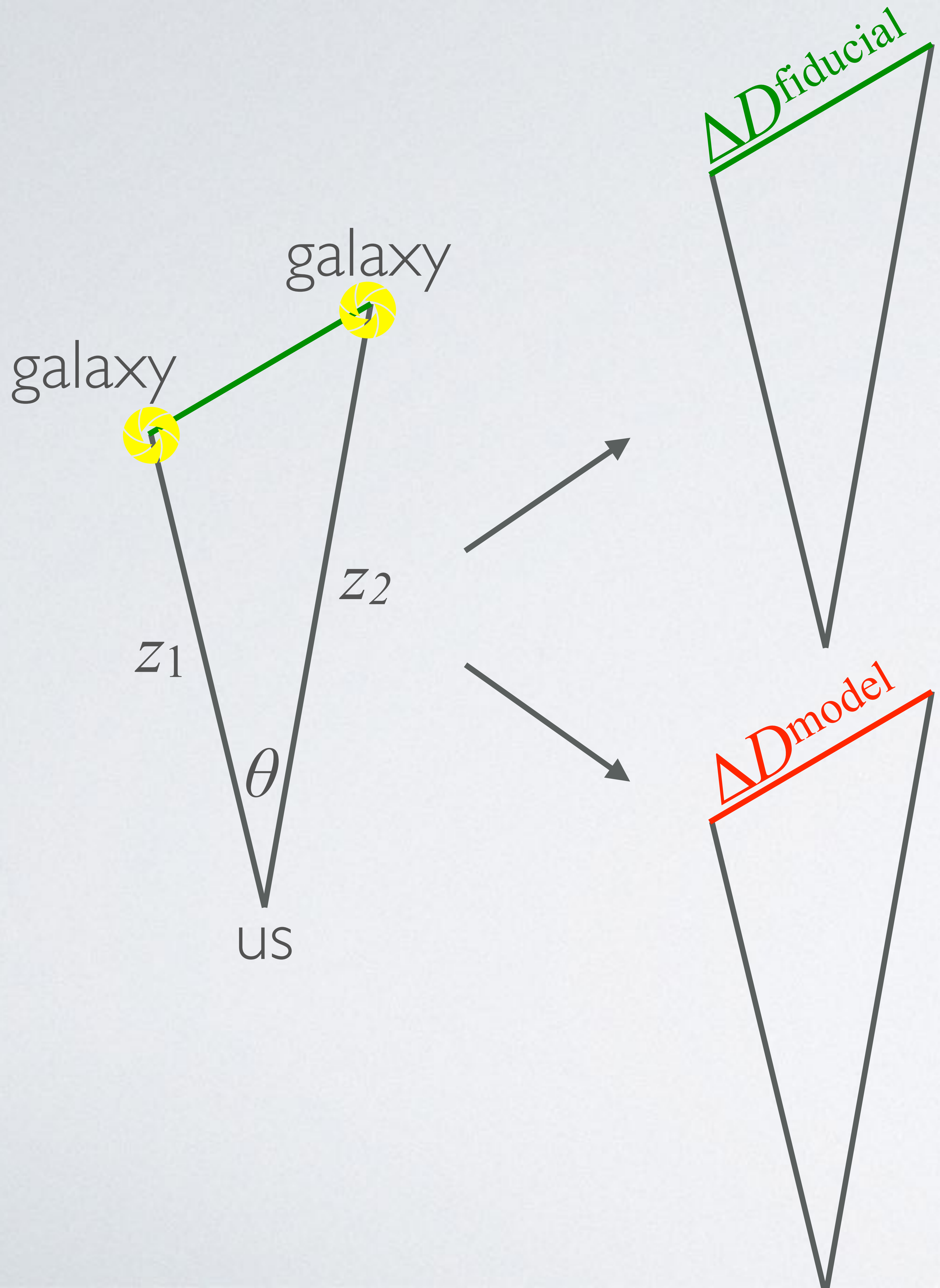


- Use an "inverse distance ladder"

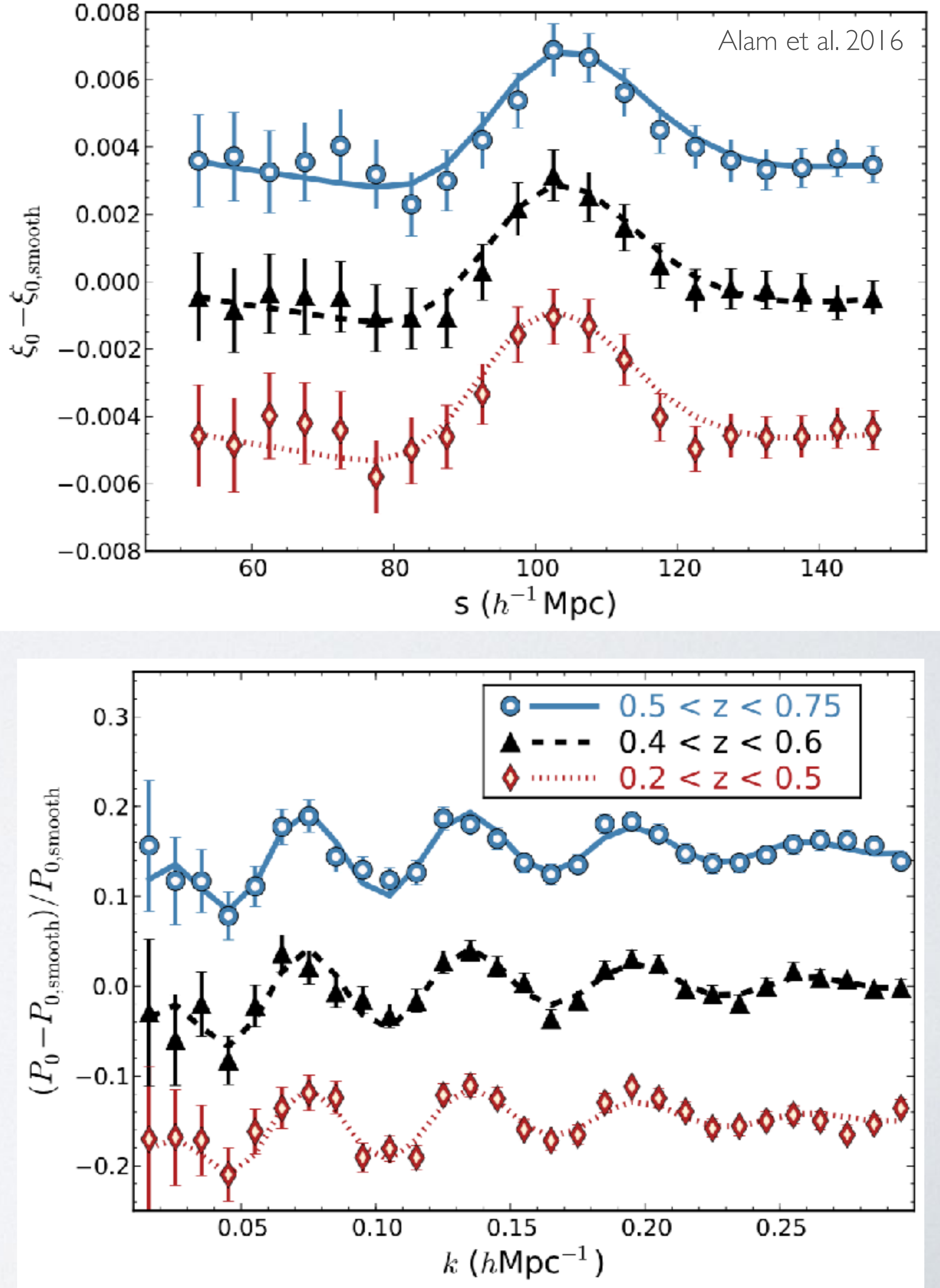
(shares ruler with CMB)



DERIVING H_0 FROM BAO



$$\alpha = \frac{\Delta D^{\text{fiducial}}}{\Delta D^{\text{model}}}$$



REDSHIFT EFFECTS IN BAO

- What is the **redshift** of the standard ruler?

$$0.2 < z < 0.5 \rightarrow z_{\rm eff} = 0.38,$$
$$0.4 < z < 0.6 \rightarrow z_{\rm eff} = 0.51,$$
$$0.5 < z < 0.75 \rightarrow z_{\rm eff} = 0.61.$$

$$w_i = \frac{1}{1 + n_i P_0},$$

$$\bar{z}_{\rm pair} = \frac{z_1 + z_2}{2}.$$

$$z_{\rm eff} = \frac{\sum_{i=1}^{n} \bar{z}_{{\rm pair},i} w_i}{\sum_{i=1}^{n} w_i}.$$

(Blake et al. 2011)

$$z_{\rm eff} = \frac{\sum_i^{N_{\rm gal}} w_{\rm FKP}(\vec{x}_i) z_i}{\sum_i^{N_{\rm gal}} w_{\rm FKP}(\vec{x}_i)},$$

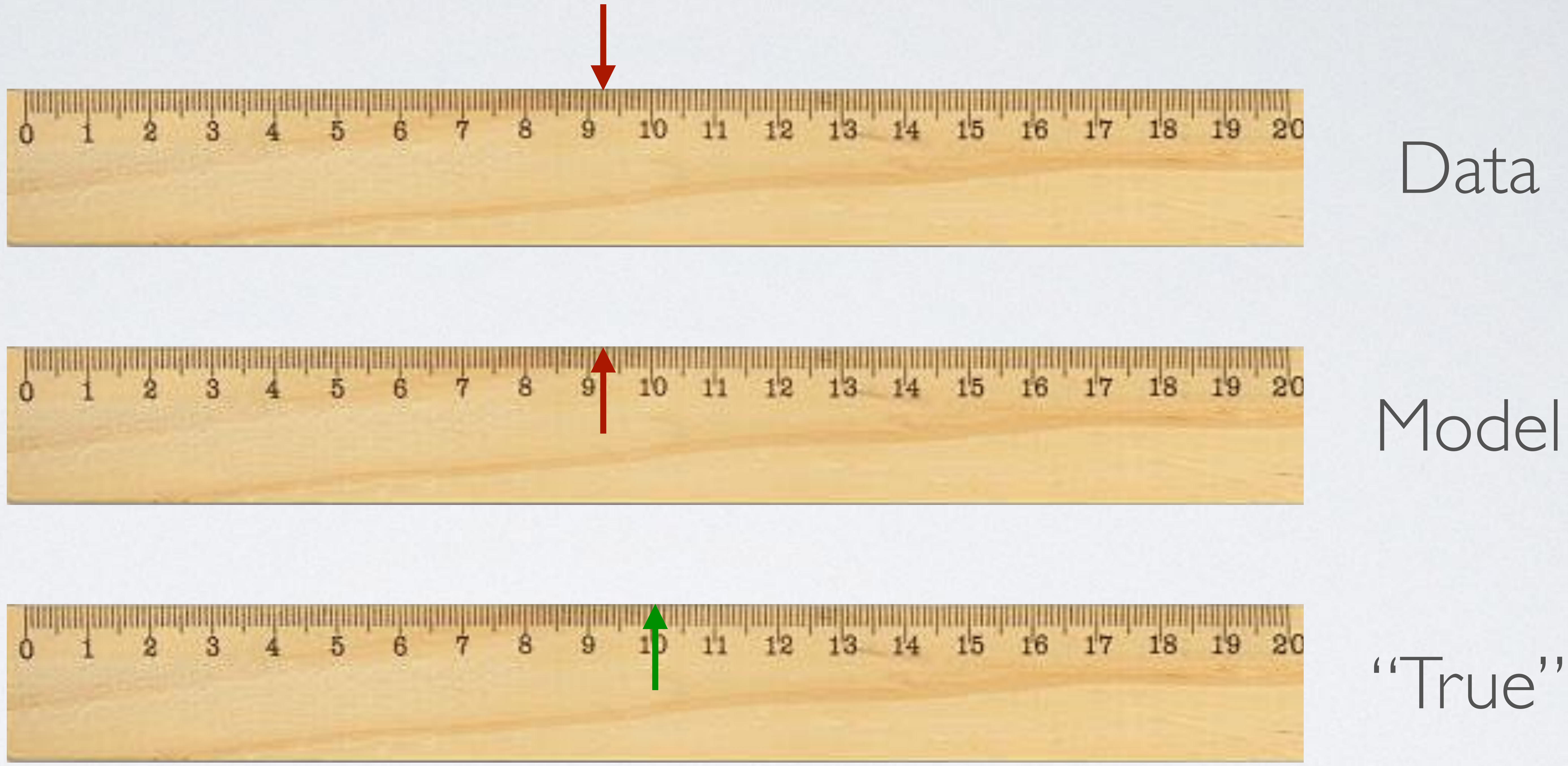
$$w_{\rm FKP}(\vec{x}) = \frac{1}{1 + \frac{n'_g(\vec{x}) P_0}{w_{\rm sys}(\vec{x})}}.$$

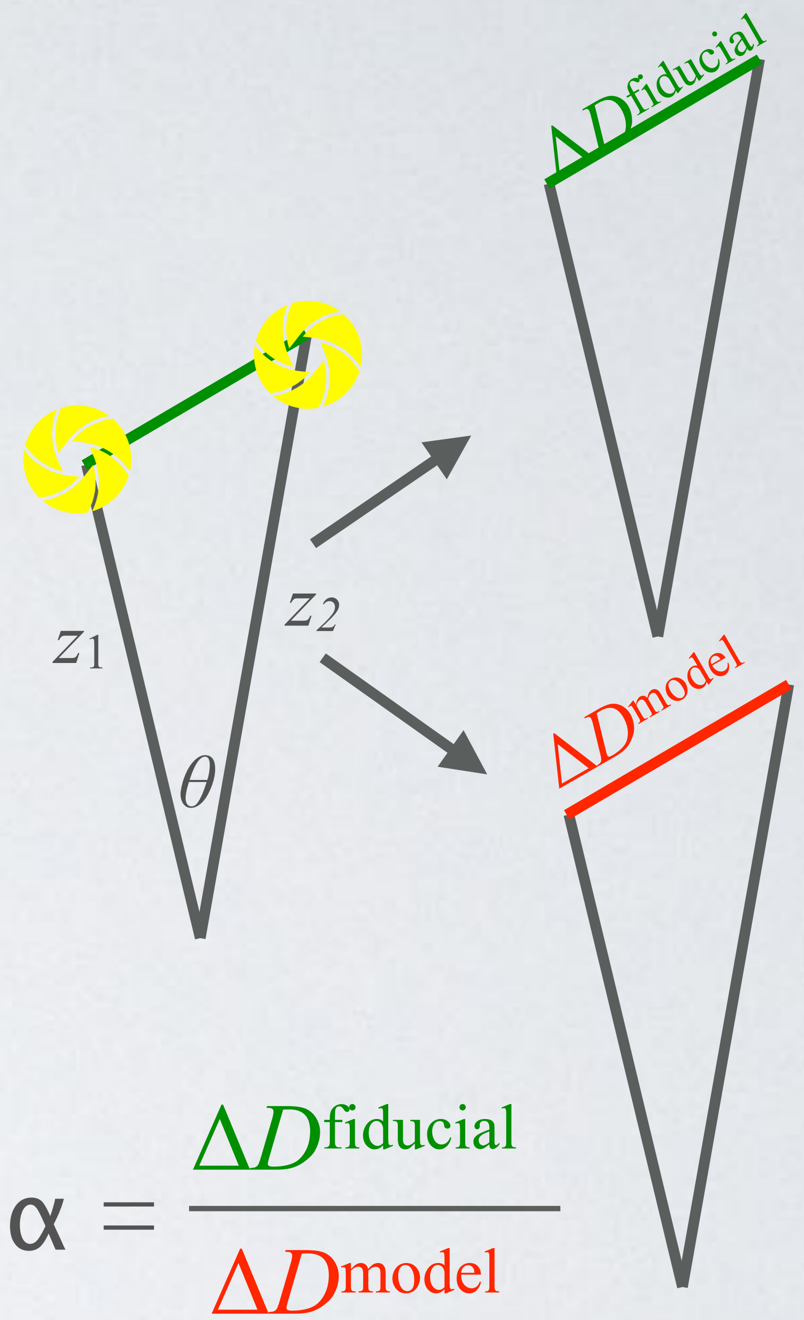
(Beutler et al. 2017)

But the weighted average redshift
is not the weighted average distance…

TWO WRONGS CAN MAKE A RIGHT

If you use the wrong calibration on both the data and the model, you're okay.



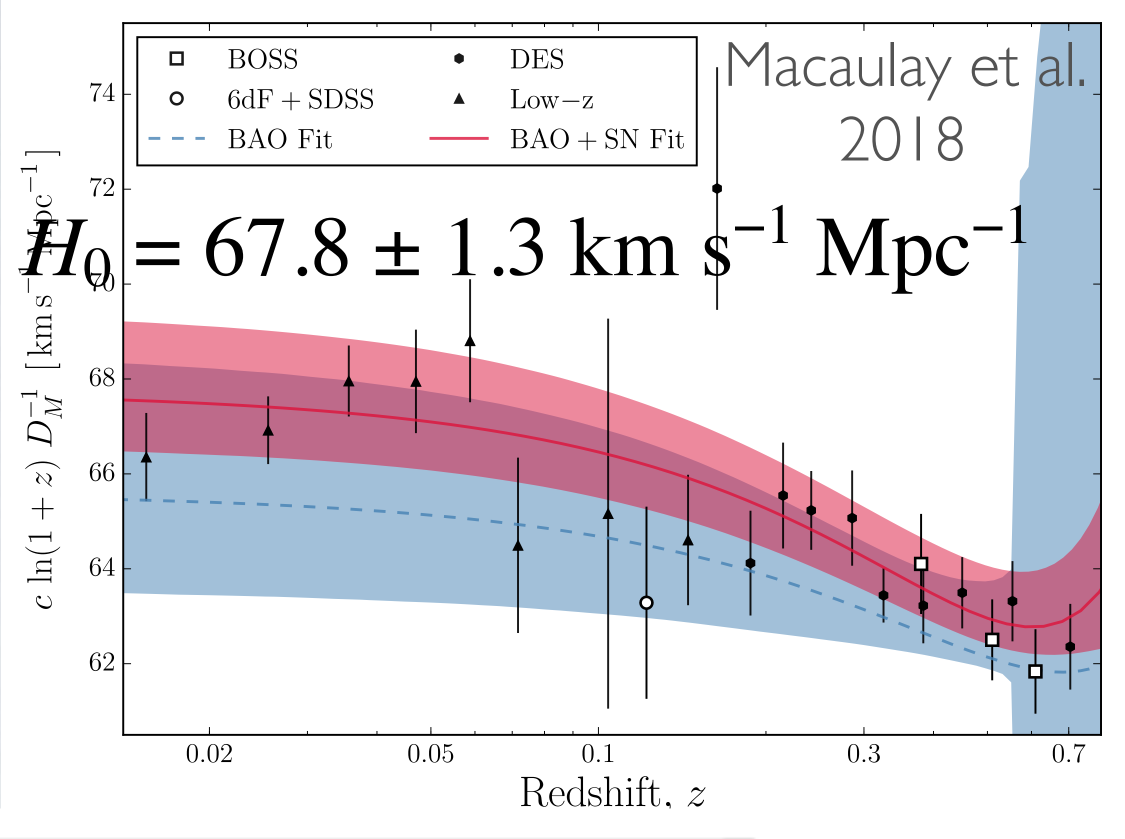


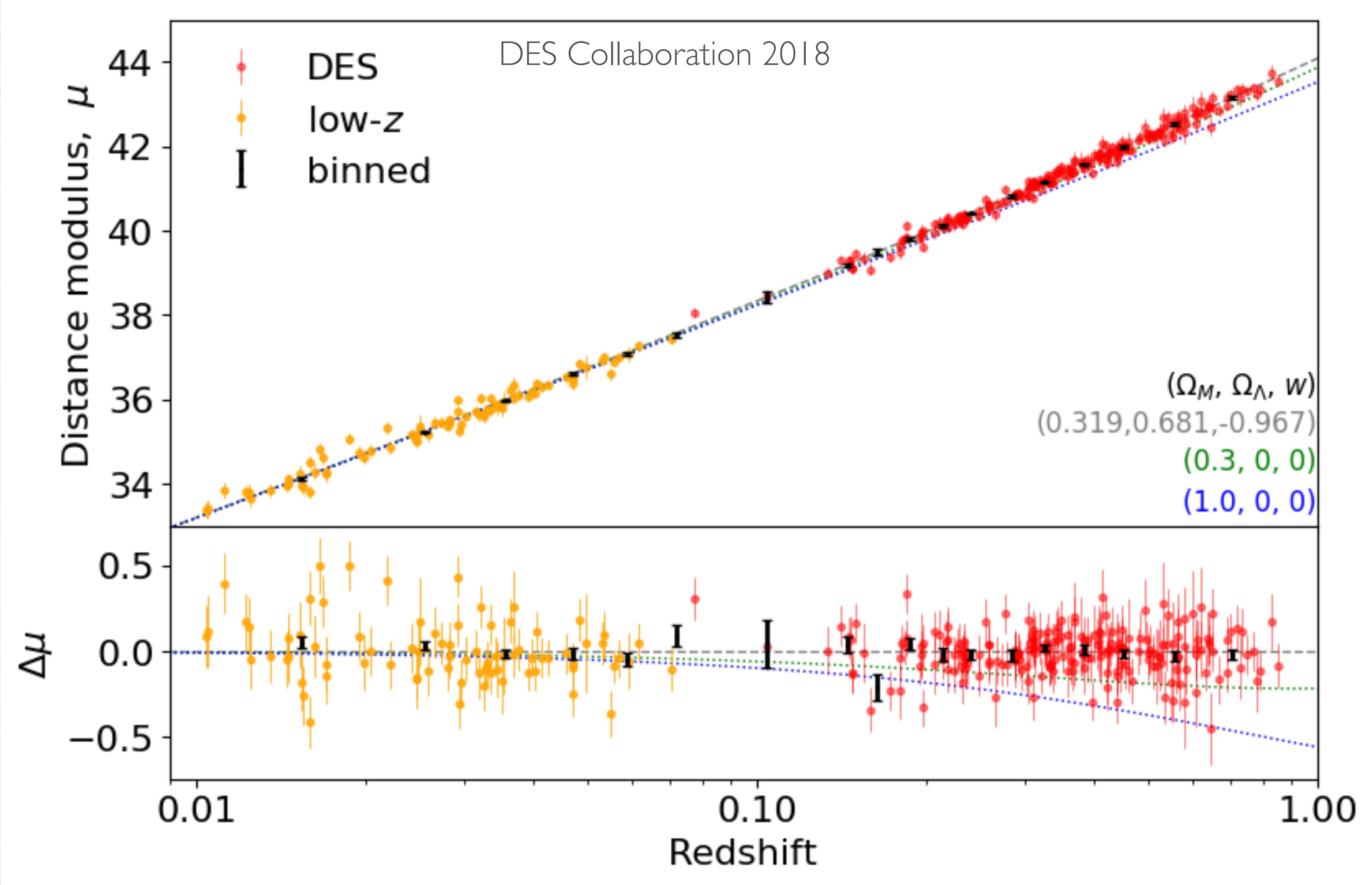
$$\alpha = \frac{\Delta D^{\text{fiducial}}}{\Delta D^{\text{model}}}$$

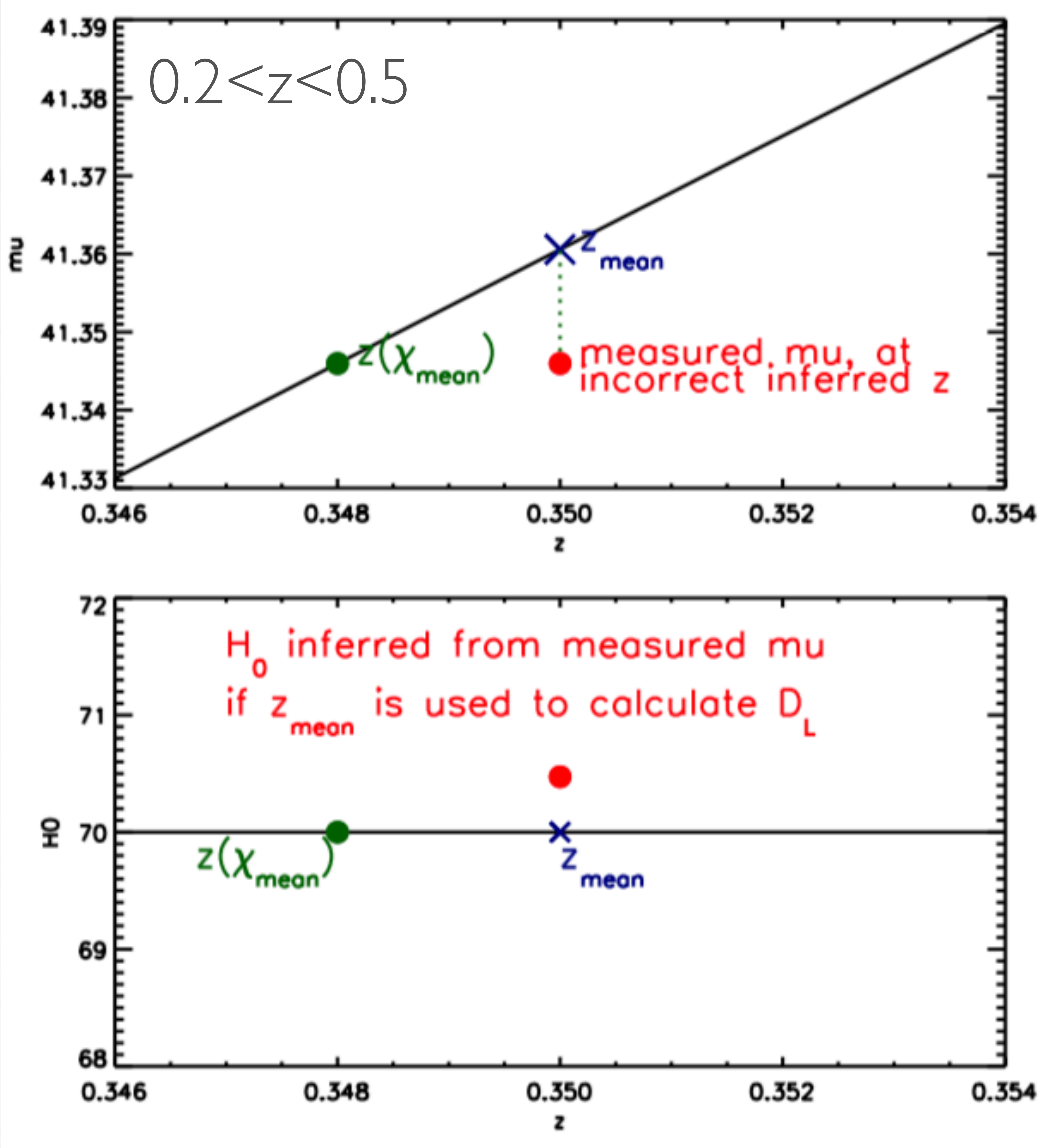
- But if you want an **absolute** distance, the correct z does matter.

HOW MUCH WILL H_0 SHIFT?

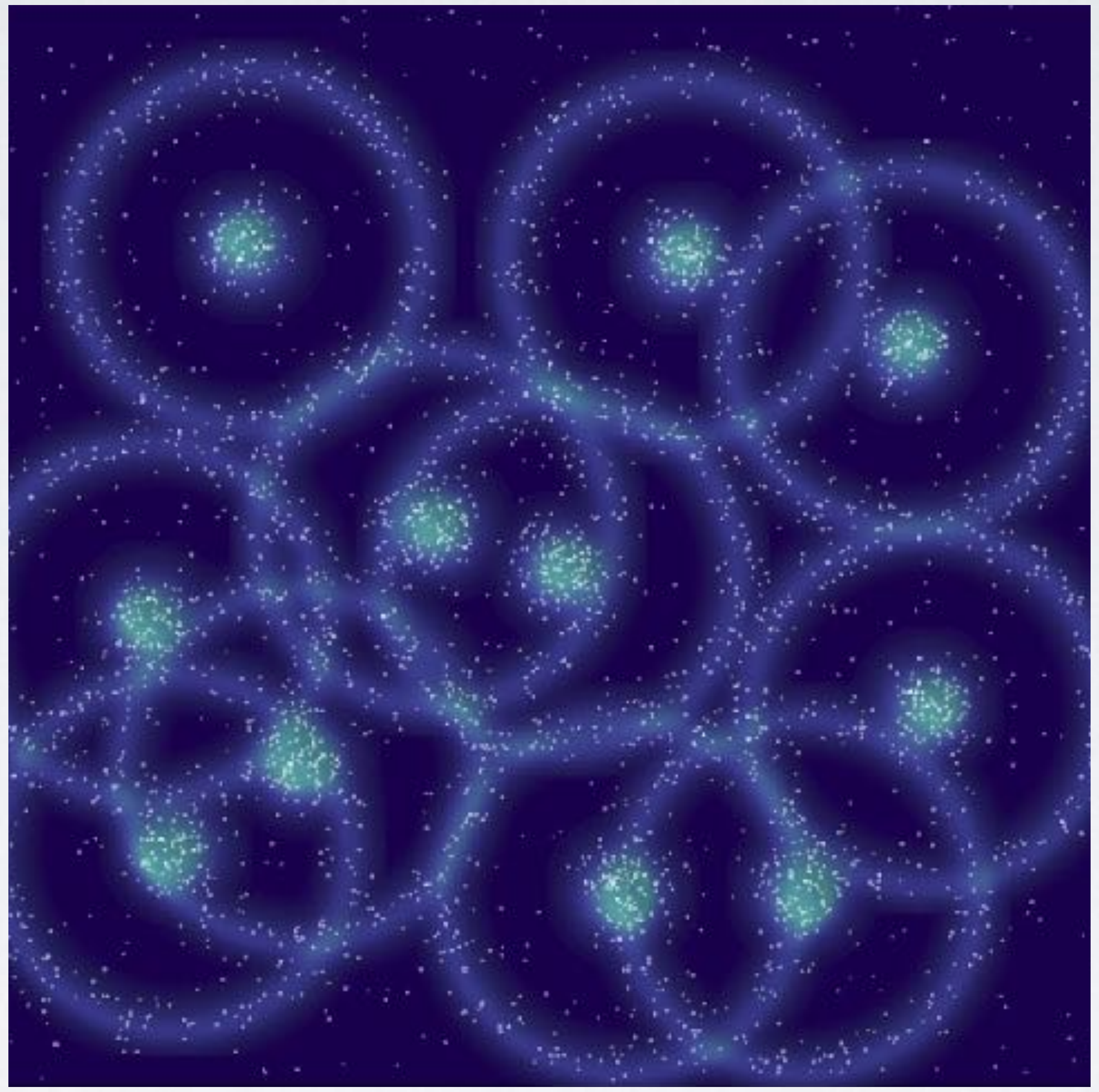
- Use an "inverse distance ladder"

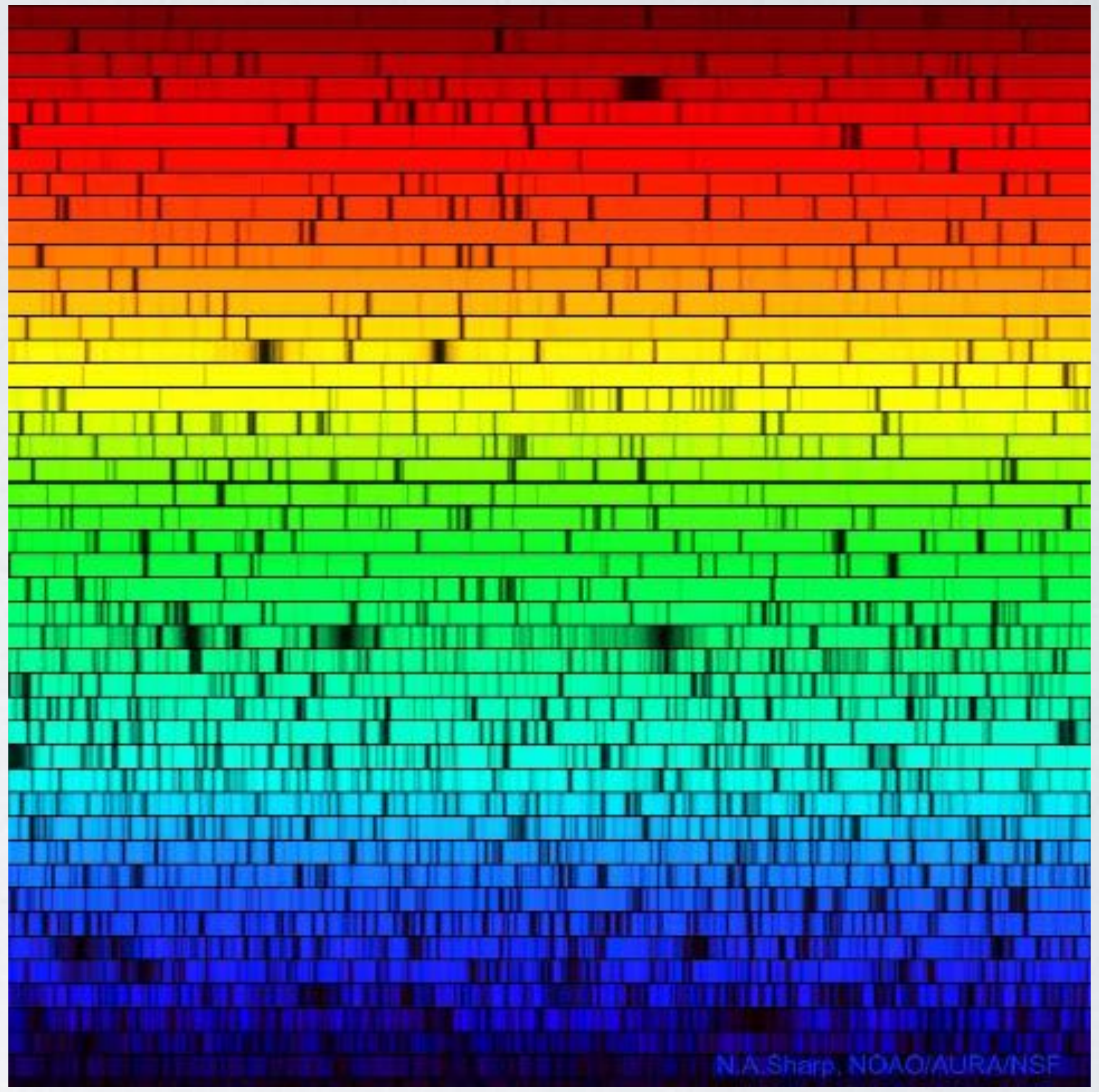






CANDLES, RULERS, AND REDSHIFTS





Maybe the H_0 tension arises between standard candles and standard rulers, rather than local vs global measurements.

Maybe we should double check our redshifts.

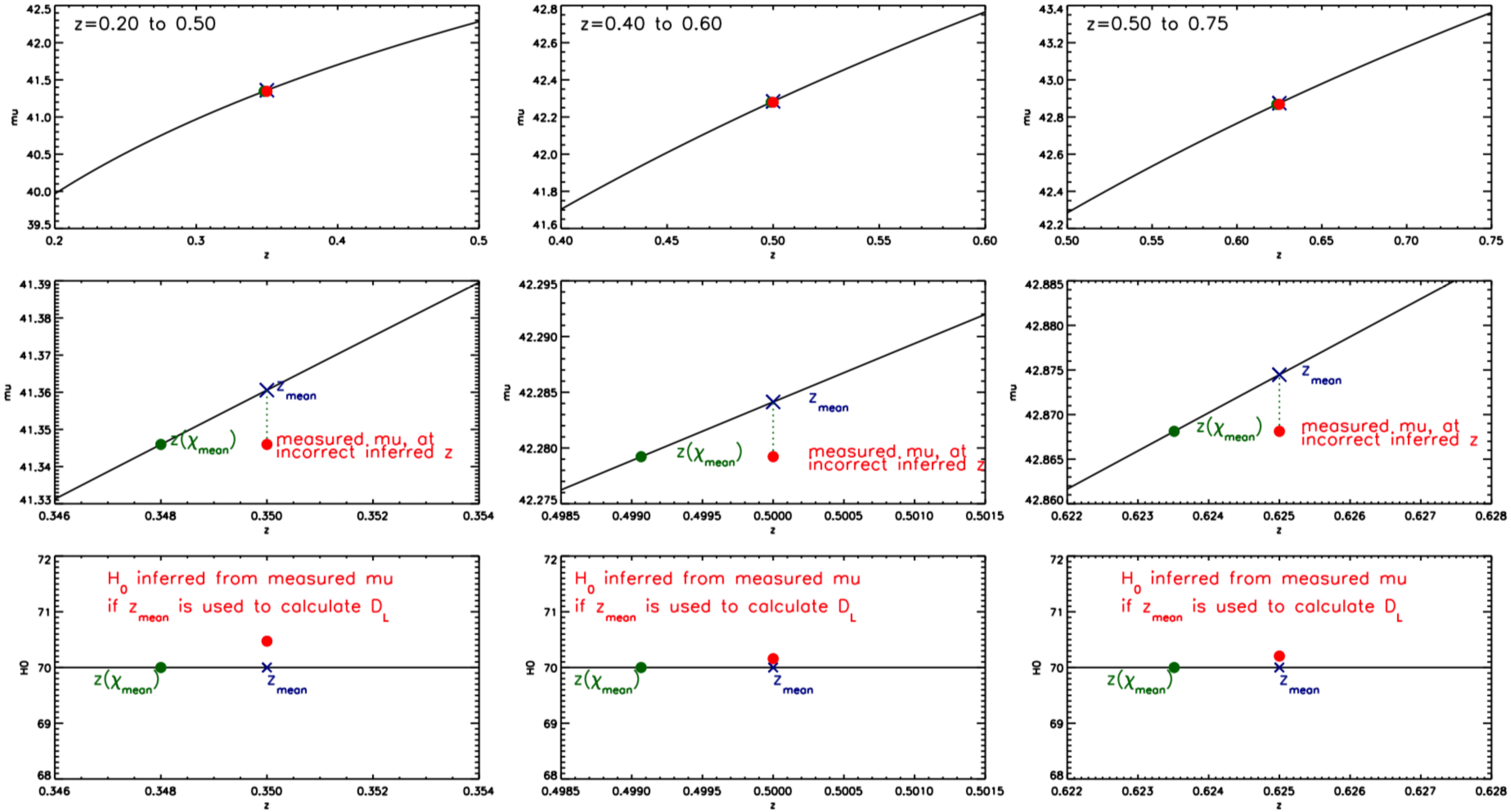
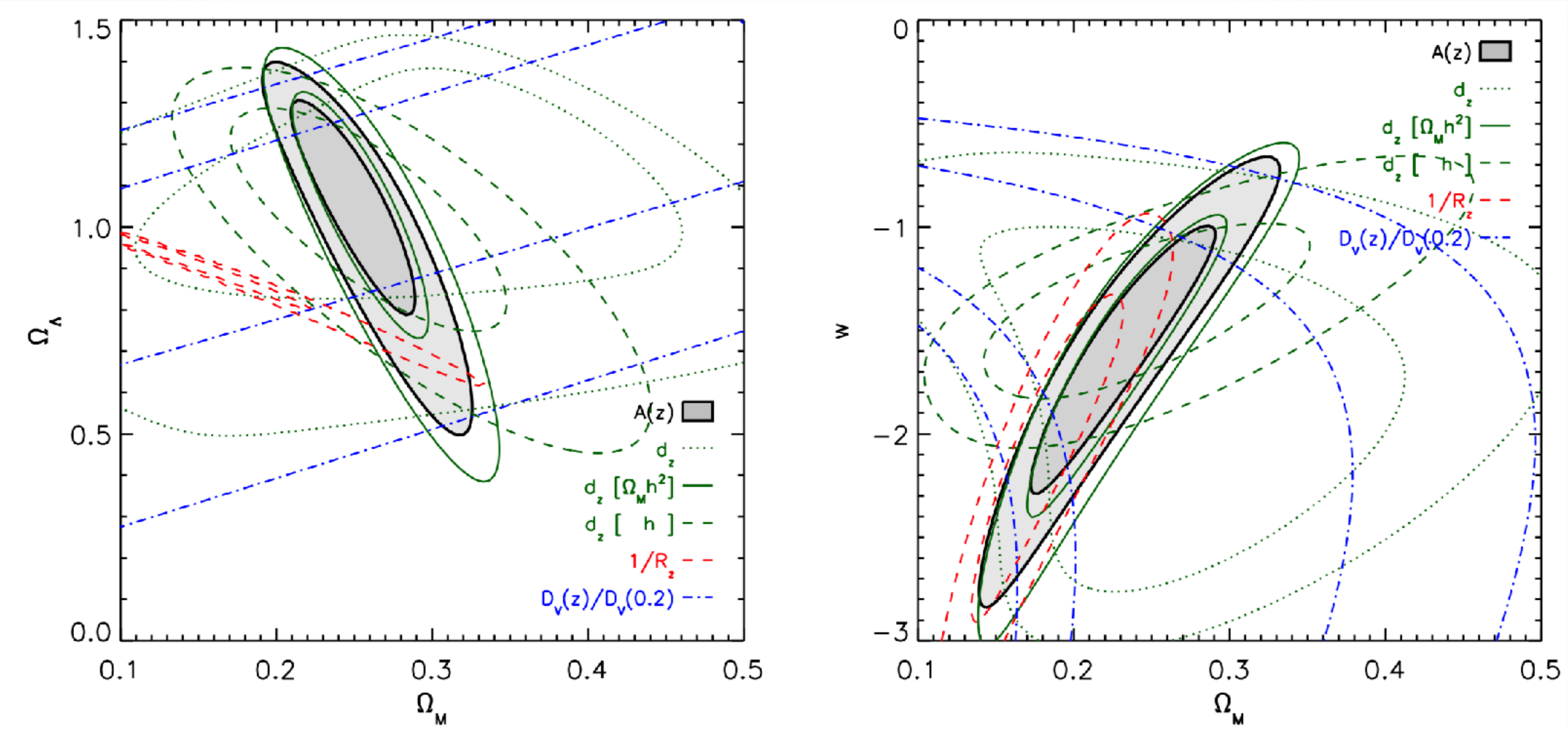


Figure 1: Each column shows a different redshift bin, as labelled at top left. The upper row shows the distance modulus vs redshift plot for each bin. The second row shows the same, but zoomed in on the central redshift region to show the difference between the mean redshift, $z_{\rm mean}$, and the redshift corresponding to the mean comoving distance, $z(\chi_{\rm mean})$. For this example each bin is evenly populated in redshift (this will not be the case in real data). In the lower panel I show the Hubble constant inferred from assuming the measurement was at $z_{\rm mean}$ when it was actually at $z(\chi_{\rm mean})$. The model used to generate the fake data was $(h, \Omega_m, \Omega_\Lambda) = (0.70, 0.27, 0.73)$ (to do the calculation of H_0 I used the same model, but without the $h = H_0/100{\rm km\,s^{-1}Mpc^{-1}}$ input).


1.5
1.0
0.5
0.0
0.1
0.2
0.3
0.4
0.5
Ω_Λ
Ω_M
A(z)
d_z
d_z [$\Omega_M h^2$]
d_z [h]
$1/R_z$
$D_V(z)/D_V(0.2)$
0
−1
−2
−3
w