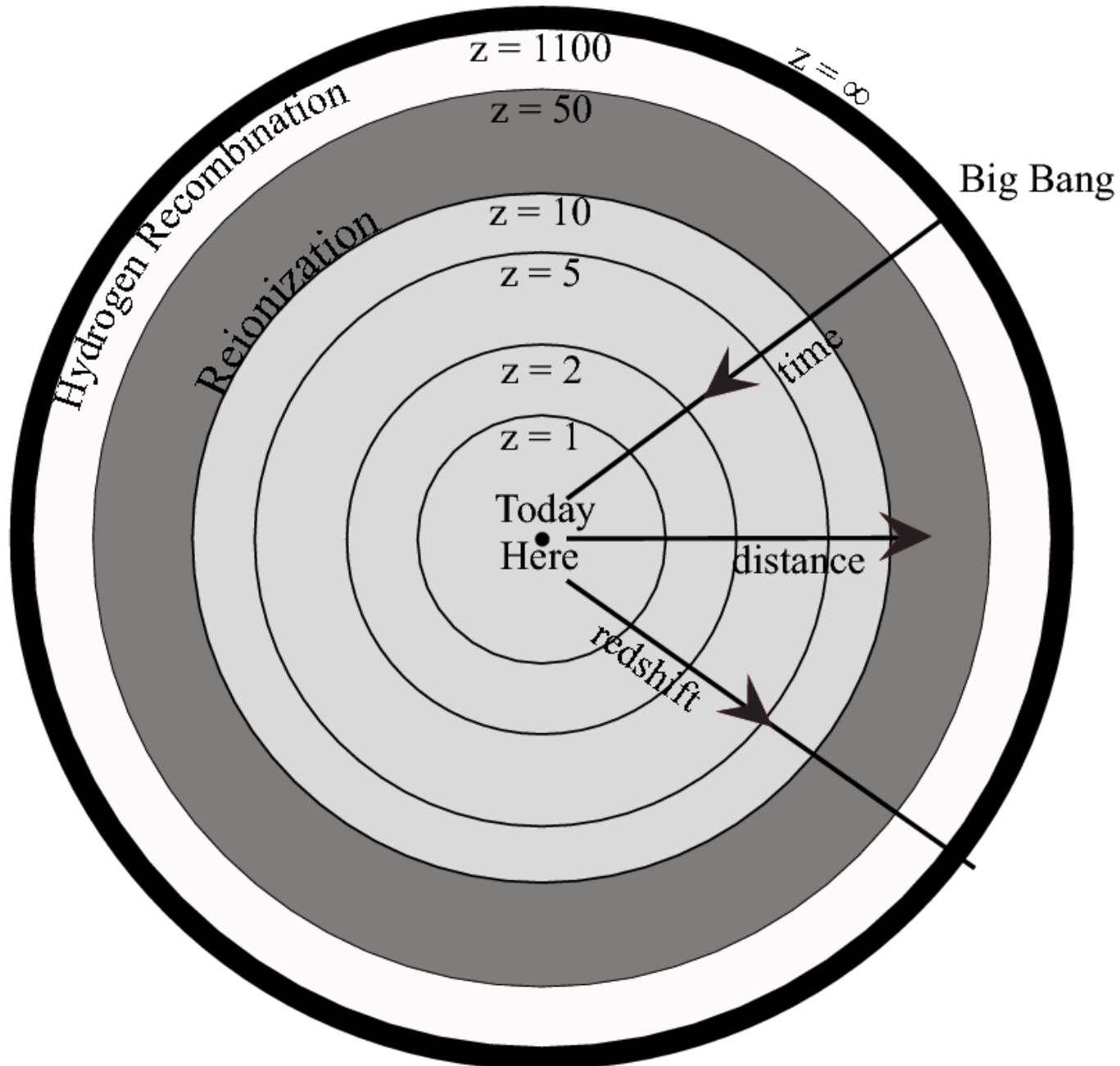


The Visible Universe



- *Cosmological
Implications of
the H_0 Discrepancy*

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Good news: the controversy was much worse when I started to work in Cosmology ($h=0.5$ versus 1 at the same redshift, $z \sim 0!$)

There is clearly progress in the field.

Standard Cosmological Model

- On large scales: homogeneous and isotropic

$$ds^2 = c^2 dt^2 - dl^2$$

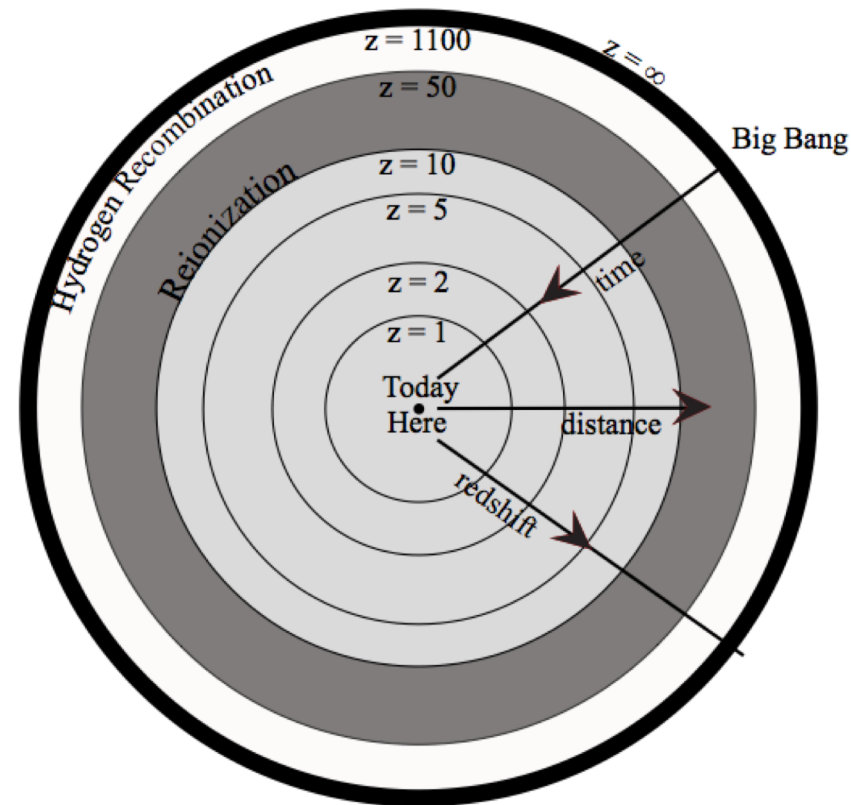
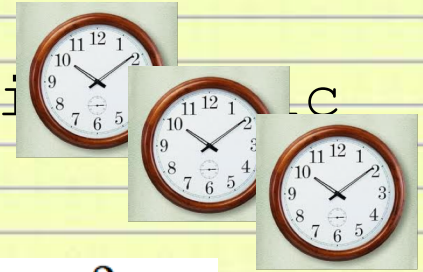
$$dl^2 = a(t)^2(dx^2 + dy^2 + dz^2) = a^2(t)(dr^2 + r^2 d\Omega)$$

- Hubble expansion

$$v = dR/dt = \dot{a}r = (\dot{a}/a)R$$

$$H = \dot{a}/a$$

$$a = 1/(1 + z)$$



Standard Expansion

History

- Gravitating mass density $\rho_{\text{grav}} = (\rho + 3p/c^2)$

$$p_{\text{rad}}/c^2 = \frac{1}{3}\rho_{\text{rad}}$$

$$\rho_{\text{grav}} = 2\rho_{\text{rad}}$$

$$p_{\text{vac}}/c^2 = -\rho_{\text{vac}}$$

$$\rho_{\text{grav}} = (\rho_{\text{vac}} + 3p_{\text{vac}}/c^2) = -2\rho_{\text{vac}}$$

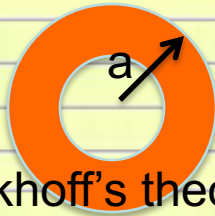
$$\rho_{\text{matter}} \propto a^{-3}$$

$$\rho_{\text{rad}} c^2 \propto a^{-4}$$



$$= \Delta E_{\text{vac}} = -p_{\text{vac}} \Delta V$$

- Acceleration



Birkhoff's theorem

$$\frac{d^2 a}{dt^2} = -\frac{GM_{\text{grav}}}{a^2}$$

$$M_{\text{grav}} = \rho_{\text{grav}} V$$

$$V = \frac{4\pi}{3} a^3$$

$$d(\rho c^2 V) = -p dV$$

$$-3pa\dot{a}/c^2 = a^2 \dot{\rho} + 3\rho a \dot{a}$$

$$E = \frac{1}{2} \dot{a}^2 - \frac{GM}{a}$$

$$M = \rho V$$

Expansion History

$$\frac{E}{\dot{a}^2/2} = 1 - \Omega,$$

where $\Omega = \rho/\rho_c$, with

$$\rho_c = \frac{3H^2}{8\pi G} = 9.2 \times 10^{-30} \frac{\text{g}}{\text{cm}^3} \left(\frac{H}{70 \text{ km s}^{-1} \text{ Mpc}^{-1}} \right)^2$$

- Hubble expansion rate

a flat universe with $E = 0$ satisfies

$$\frac{H(t)}{H_0} = \left[\frac{\Omega_m}{a^3} + \Omega_\Lambda + \frac{\Omega_r}{a^4} \right]^{1/2}$$

where we define H_0 and $\Omega_0 = (\Omega_m + \Omega_\Lambda + \Omega_r) = 1$ to be the present-day values of H and Ω , respectively.

Vacuum domination: $H \equiv (\dot{a}/a) = \text{const} \rightarrow a \propto \exp\{Ht\}$

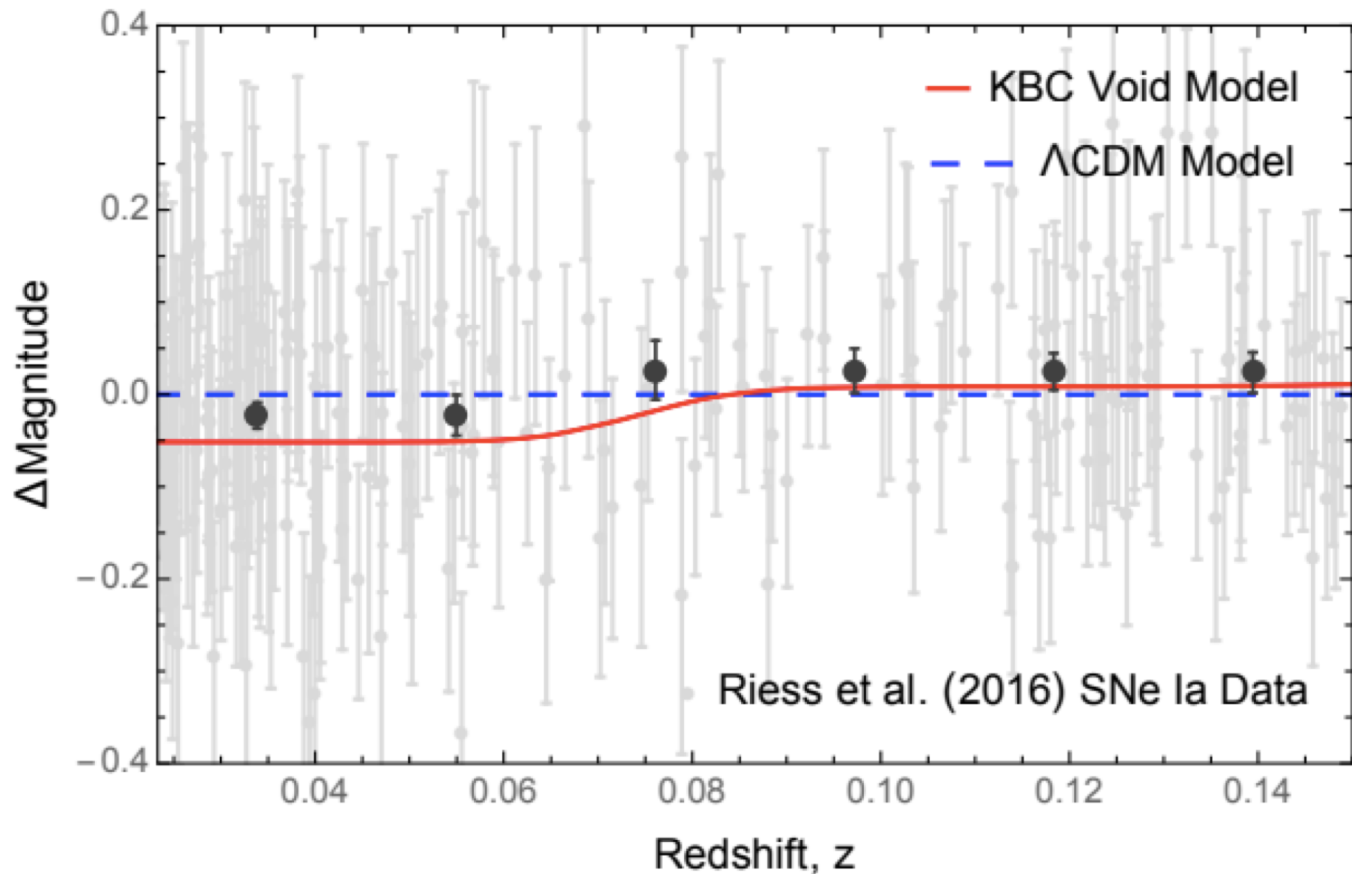
Radiation domination: $H \equiv (\dot{a}/a) \propto a^{-2} \rightarrow a \propto t^{1/2}$

Matter domination: $H \equiv (\dot{a}/a) \propto a^{-3/2} \rightarrow a \propto t^{2/3}$

Simplest Source of Low-Redshift Deviation from Global Expansion

Rate: Local Void

But effect too small to explain away CMB discrepancy
(reducing tension from 3.4 to 2.75 sigma)



Hoschett and Barger 2018

Expansion Affects Growth Rate of Linear Perturbations

$$\frac{\partial \delta}{\partial t} + \frac{1}{a} \nabla \cdot [(1 + \delta) \mathbf{u}] = 0$$
$$\frac{\partial \mathbf{u}}{\partial t} + H \mathbf{u} + \frac{1}{a} (\mathbf{u} \cdot \nabla) \mathbf{u} = -\frac{1}{a} \nabla \phi - \frac{1}{a \bar{\rho}} \nabla (\delta p)$$

$$\nabla^2 \phi = 4\pi G \bar{\rho} a^2 \delta$$

Pressure: zero for cold dark matter; finite for gas

$$\frac{\partial^2 \delta}{\partial t^2} + 2H \frac{\partial \delta}{\partial t} = 4\pi G \bar{\rho} \delta - \frac{c_s^2 k^2}{a^2} \delta$$

Growth factor during matter domination (without pressure):

$$D(t) \propto \frac{(\Omega_\Lambda a^3 + \Omega_m)^{1/2}}{a^{3/2}} \int_0^a \frac{a'^{3/2} da'}{(\Omega_\Lambda a'^3 + \Omega_m)^{3/2}} \propto a \quad (z \gg 1)$$

Distances

- Luminosity distance: observed flux

$$f = \frac{L}{4\pi d_L^2} \times$$

$$f = \frac{L dt_{\text{em}} / (1+z)}{4\pi r_{\text{em}}^2 dt_{\text{obs}}} = \frac{L}{4\pi r_{\text{em}}^2 (1+z)^2}$$

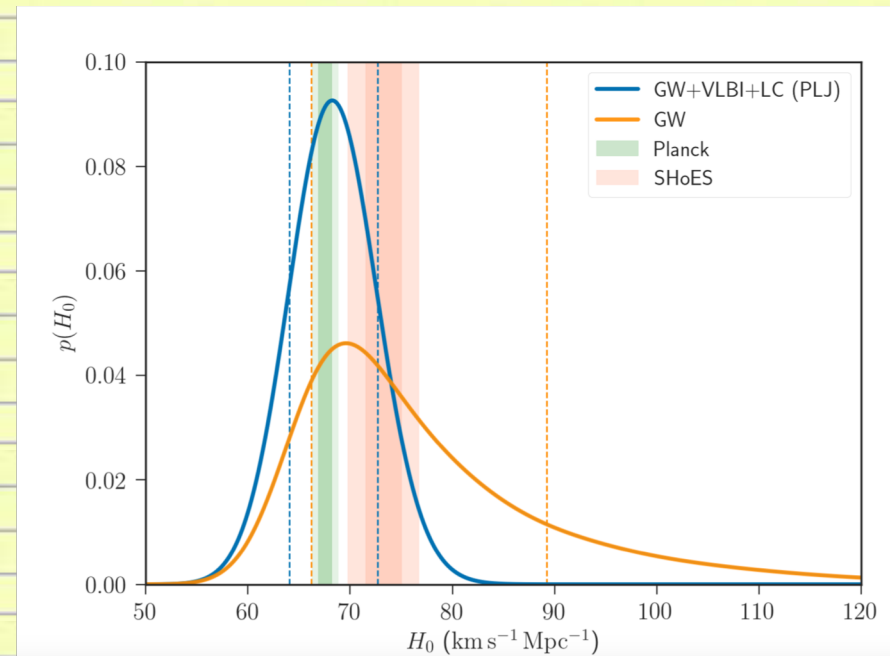
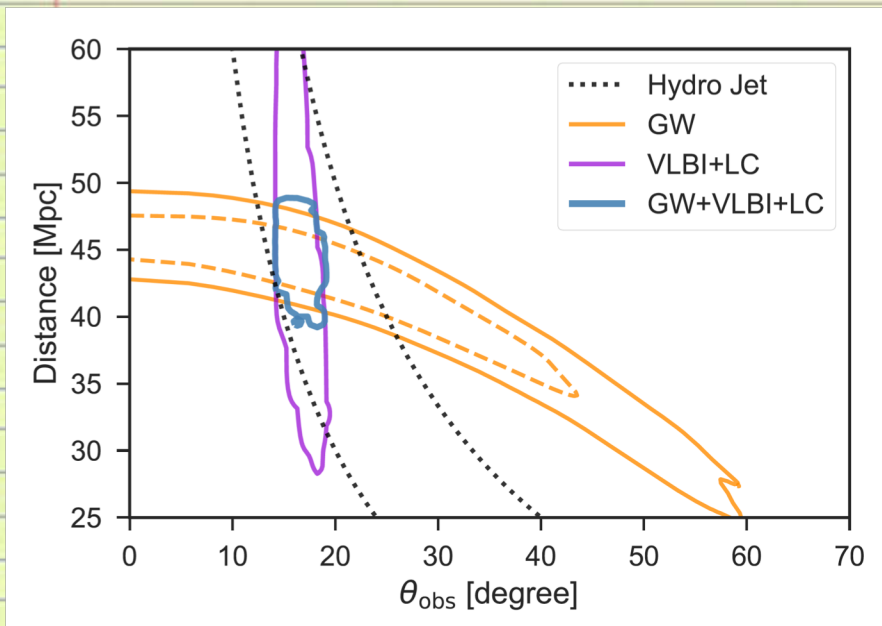
$$d_L = r_{\text{em}} (1+z) = d_A (1+z)^2$$

$$r_{\text{em}} = \int_{t_{\text{em}}}^{t_{\text{obs}}} \frac{cdt}{a(t)} = \frac{c}{H_0} \int_0^z \frac{dz'}{\sqrt{\Omega_m (1+z')^3 + \Omega_\Lambda}}$$

Given that a standard ruler (BAO) occupies an observed angle on the CMB sky: elevating H_0 requires lowering the dark matter density today (through decay to radiation or dark energy)

Effect of weak lensing on SNe is too small even out to $z \sim 0.5$ (Smith et al. 2013)
arXiv:1307.2566

(GW+EM signals from NS-NS merger)



Hotokezaka et al. (2018)
arXiv:1806.10596

Discrepancy = non-Standard Composition

$$\frac{H(t)}{H_0} = \left[\frac{\Omega_m}{a^3} + \Omega_\Lambda + \frac{\Omega_r}{a^4} \right]^{1/2}$$

CMB at $a \sim 0.001$ argues for a bigger ratio than measurements at $a \sim 1$ imply

Options:

- Additional radiation degrees of freedom (energy density), such as "**sterile neutrinos**". Hints from LSND, MiniBooNE oscillation experiments.
- **Decaying dark matter** (DM; e.g. evaporating primordial black holes), but the allowed range in the *DM mass fraction-lifetime* plane offers no solution (Poulin et al. arXiv:1606.02073).

$$p_{\text{vac}}/c^2 = -\rho_{\text{vac}}$$

- **Growing dark energy** (Morsell & Dhawan

Other Exotica

- **Temporary dark energy contribution at matter-radiation equality (Karwal & Kamionkowski 2016).** Fine tuning in amplitude, timing and duration is required.
- **Large scale density perturbation.** But since we are not likely to be at the center of a spherically-symmetric perturbed region, this would induce a too large quadrupole in the CMB through the Sachs-Wolfe effect.
- **Time dependence of Newton's constant.** But the required level (5%) is well above existing limits from lunar ranging or the CMB (<1%).



Alternatively, gravity might be slightly modified on cosmological scales...

- Caveat: the theory needs to accommodate all other tests of GR, including BBN, CMB anisotropies data and strong-field GW signals detected by LIGO. Motivation: other anomalies (such as the low CMB quadrupole).
- Fixing a single discrepant number with a new theory of gravity resembles to killing a fly with an atomic bomb. It is unwarranted by common sense.

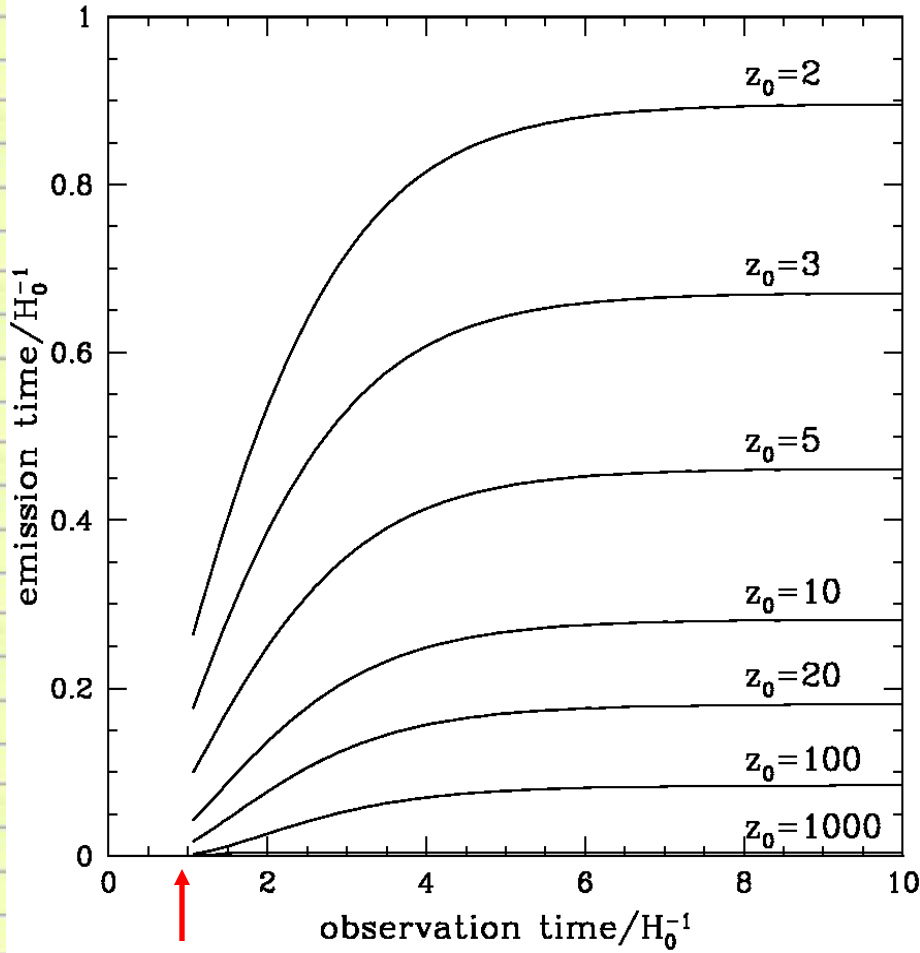
Or New Physics During Cosmological Recombination

- To increase the inferred values of H_0 from the CMB data, one needs to reduce the angular diameter distance or the sound horizon scale at cosmological recombination (given its measured angular size on the sky).
- Bearing BBN and CMB constraints in mind, this can be achieved by: increasing the energy density of the universe, making it expand faster, the sound horizon smaller, and H_0 larger. The extra constituent could be dark energy with an unusual equation of

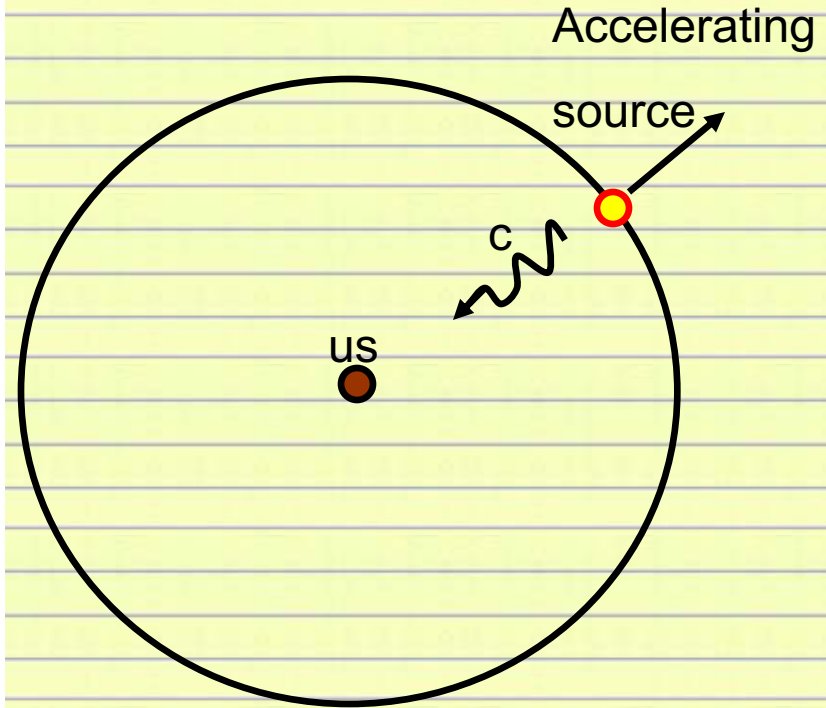
possibilities are not
compelling, we must
consider the possibility
of systematic
uncertainties in the

- Data without a compelling theoretical interpretation...resembles an orchestra without music.
- Weinberg's approach in "Gravitation and Cosmology (1972)": **average observational reports**. Led to $h=0.75$ in 1972. Not bad!
- Systematics are more likely in the SN Ia data set since it is based on astrophysical sources which are not

Future Implications for Extragalactic Astronomy



today

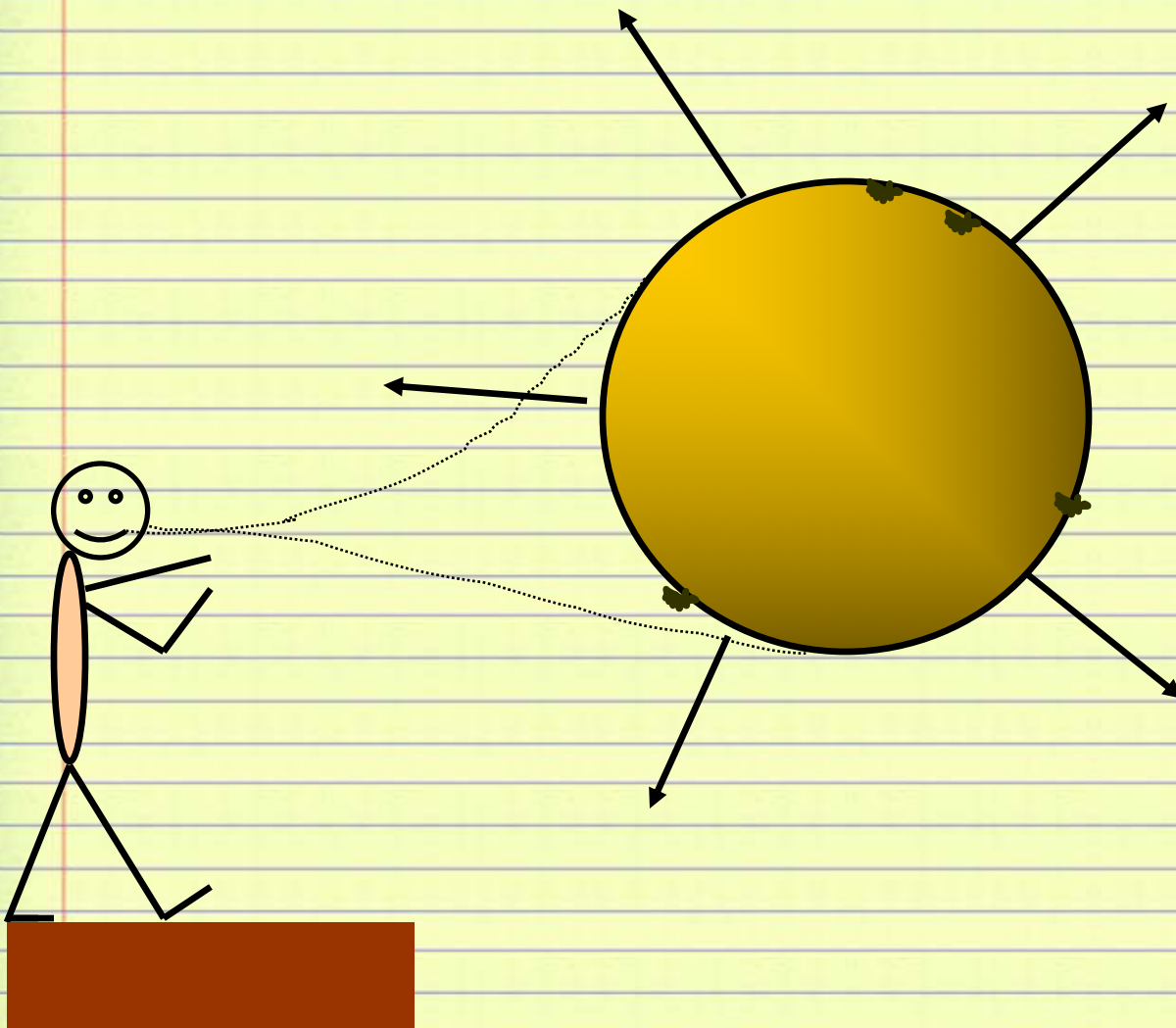


All galaxies beyond a redshift of $z=1.8$ are already outside our horizon (no cell phone communication to $z>1.8$!). (Loeb 2001)

Analogy

Ants = Photons

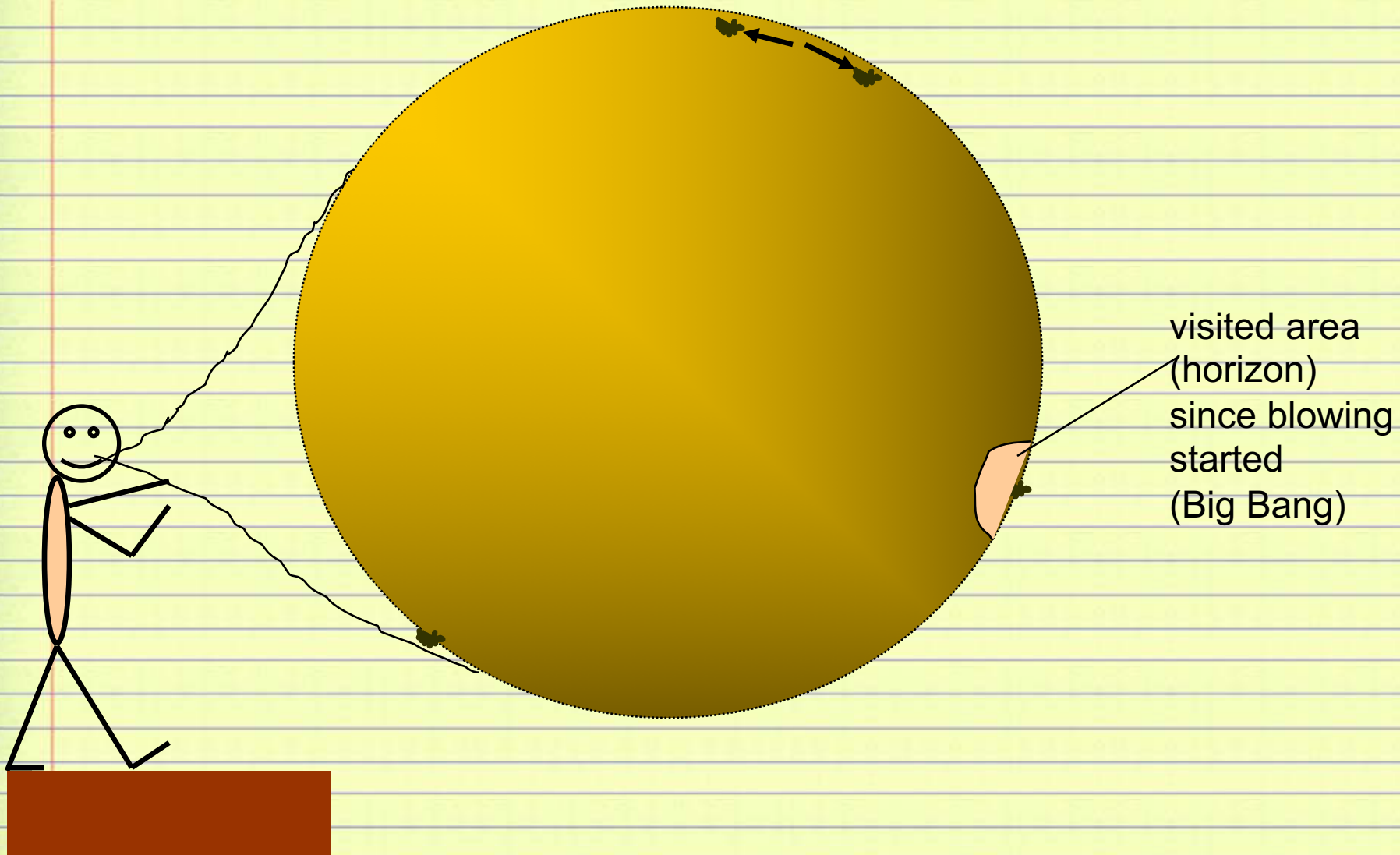
Balloon=Expanding Space



Analogy

Ants = Photons

Balloon=Expanding Space





Fourier Space

$$\delta_{\mathbf{k}} = \int d^3x \delta(\mathbf{x}) e^{i\mathbf{k}\cdot\mathbf{x}}$$

$$\delta(\mathbf{x}) = \int \frac{d^3k}{(2\pi)^3} \delta_{\mathbf{k}} e^{-i\mathbf{k}\cdot\mathbf{x}}$$

- Power spectrum: $P(\mathbf{k}) = \langle \delta_{\mathbf{k}} \delta_{\mathbf{k}'}^* \rangle = (2\pi)^3 \delta^D(\mathbf{k} - \mathbf{k}') P(\mathbf{k})$

$$(\delta M/M)^2 \propto k^3 P(k) \text{ for } k \sim 2\pi/\ell$$

primordial power-law spectrum $P(k) \propto k^{n_s}$ with $n_s \approx 1$

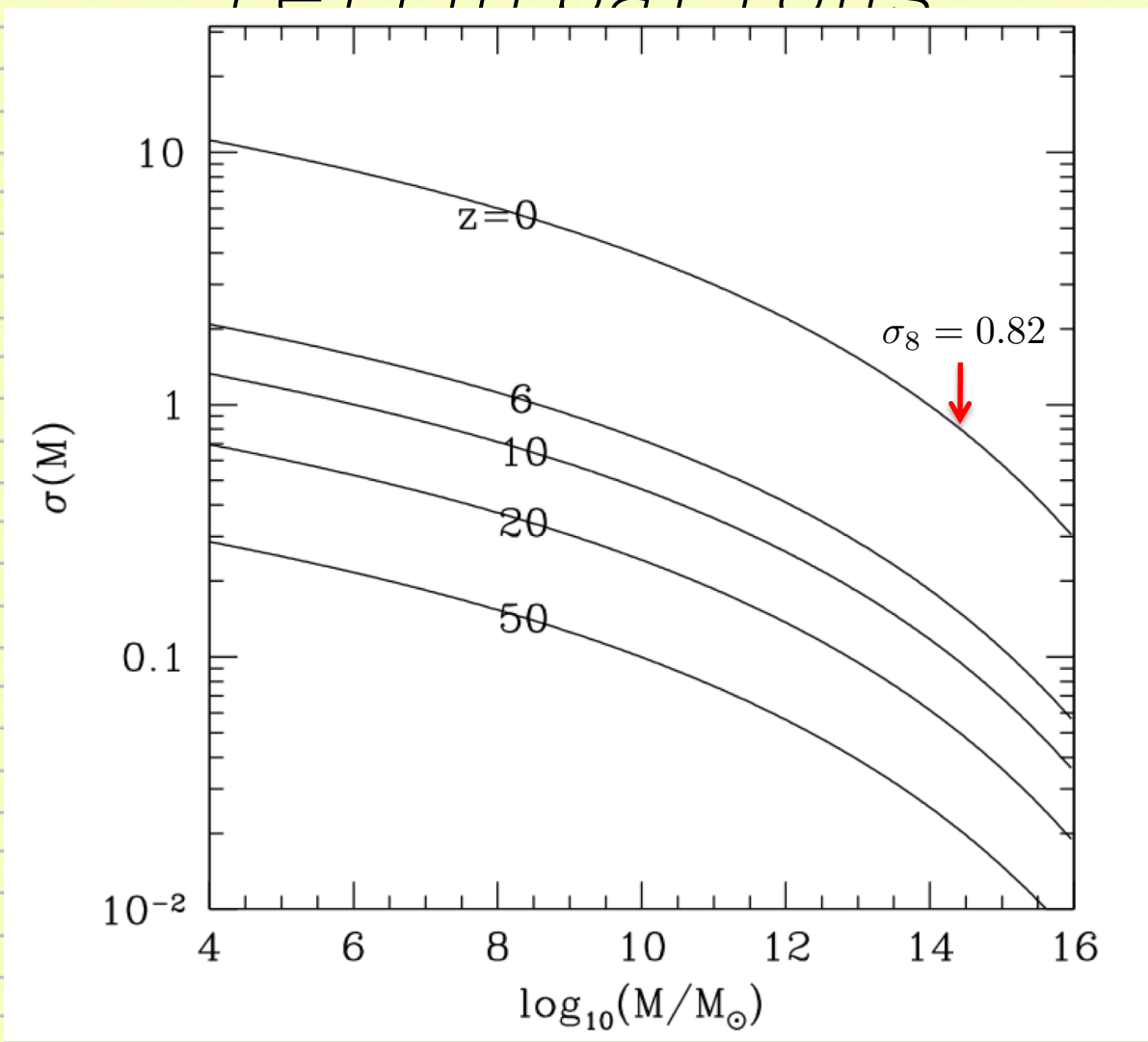
With cold dark matter, turnover at matter-radiation equality: $P(k) \propto k^{n_s-4}$

the gravitational potential, $\sim (G\delta M/\ell) \propto \ell^{(1-n_s)/2}$, is independent of scale if $n_s = 1$

(as expected from quantum fluctuations generated during a period of inflation with $H=const$ and all modes exiting the horizon with the same amplitude)

Amplitude of Density

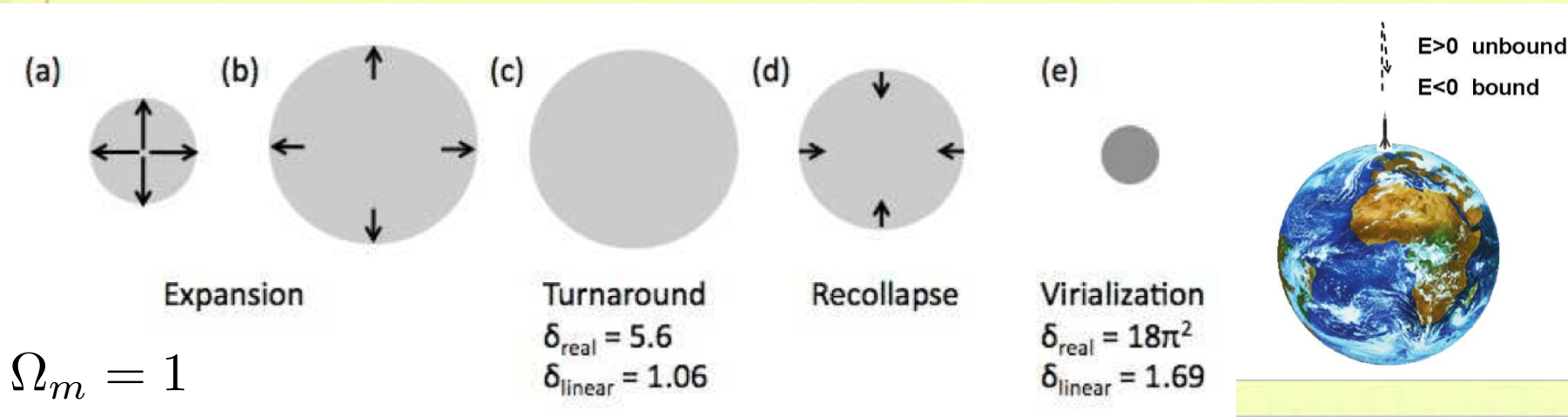
Perturbations



$$p(\delta)d\delta = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\delta^2/2\sigma^2} d\delta$$

Nonlinear Spherical Collapse

$$\frac{d^2 r}{dt^2} = H_0^2 \Omega_\Lambda r - \frac{GM}{r^2}$$



Stages of the spherical collapse model. At first, the overdensity (in gray) expands, though its excess gravity quickly slows that expansion below that of the Hubble flow. When the real fractional overdensity reaches $9\pi^2/16 \approx 5.6$ (corresponding to a *linearized* overdensity of 1.06), the expansion stops at *turnaround* and then begins to recollapse. When the overdensity reaches $18\pi^2 \approx 178$ (corresponding to a *linearized* overdensity of 1.69), the perturbation *virializes* as a collapsed dark matter halo.

Properties of Dark Matter

Halos

- Density contrast at virialization

$$\Delta_c = 18\pi^2 + 82d - 39d^2,$$

where $d \equiv \Omega_m^z - 1$ is evaluated at the collapse redshift, so that

$$\Omega_m^z = \frac{\Omega_m(1+z)^3}{\Omega_m(1+z)^3 + \Omega_\Lambda + \Omega_k(1+z)^2}$$

- Halo radius

$$r_{\text{vir}} = 1.5 \left[\frac{\Omega_m}{\Omega_m(z)} \frac{\Delta_c}{18\pi^2} \right]^{-1/3} \left(\frac{M}{10^8 M_\odot} \right)^{1/3} \left(\frac{1+z}{10} \right)^{-1} \text{ kpc}$$

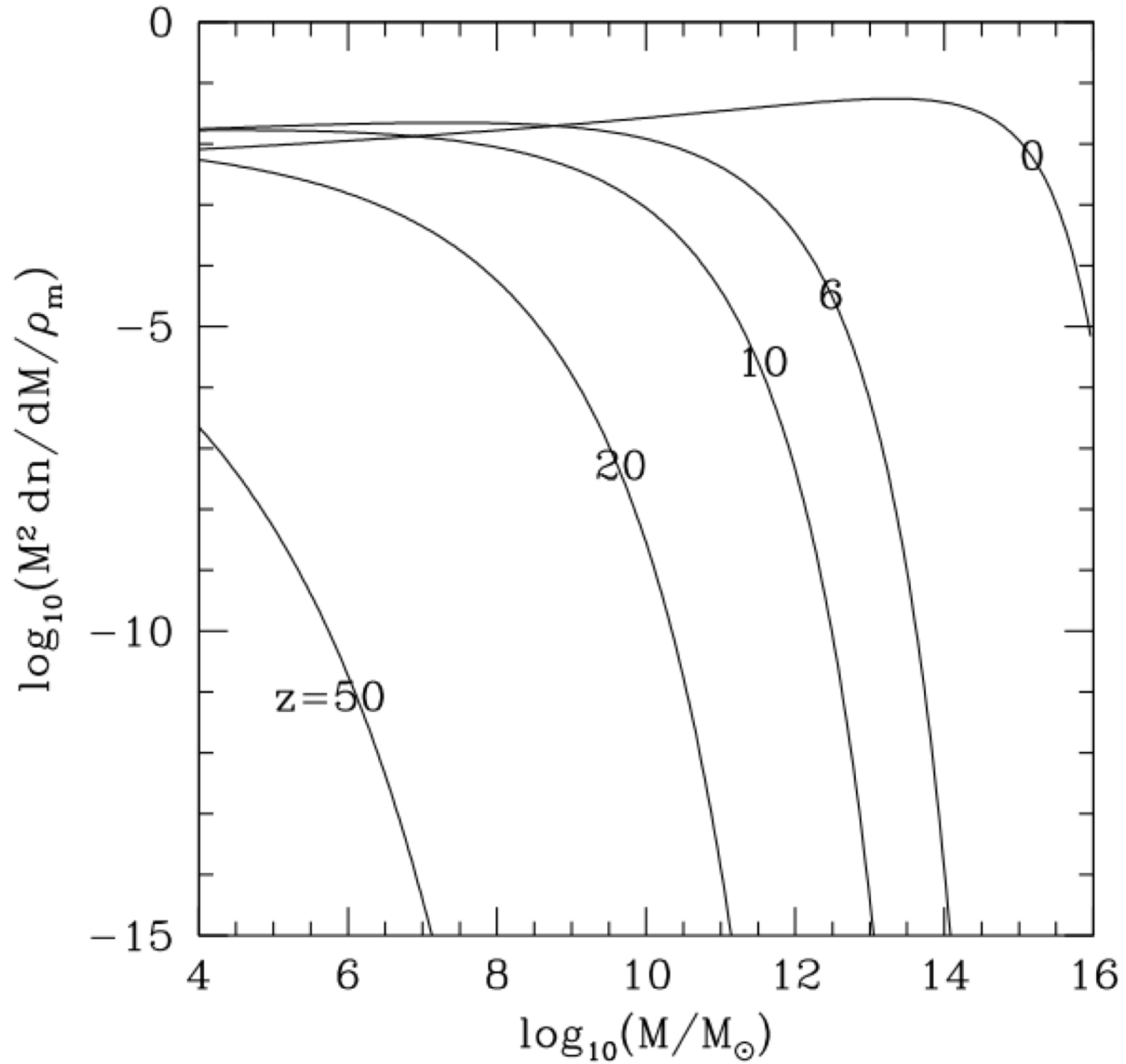
- Circular velocity

$$V_c = \left(\frac{GM}{r_{\text{vir}}} \right)^{1/2} = 17.0 \left[\frac{\Omega_m}{\Omega_m(z)} \frac{\Delta_c}{18\pi^2} \right]^{1/6} \left(\frac{M}{10^8 M_\odot} \right)^{1/3} \left(\frac{1+z}{10} \right)^{1/2} \text{ km s}^{-1}$$

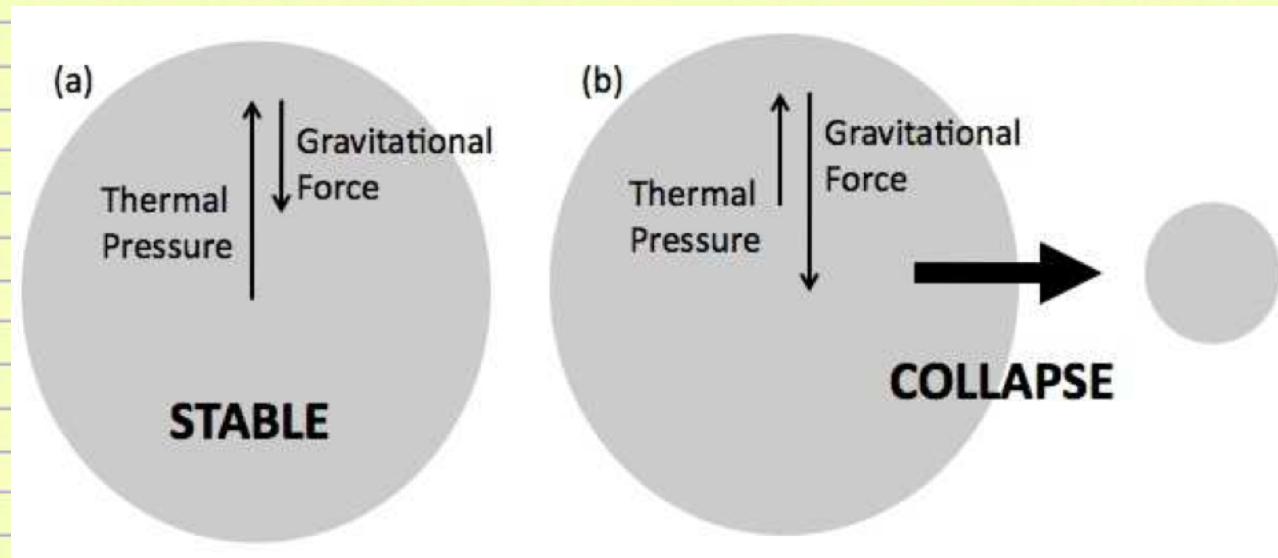
- Virial temperature

$$T_{\text{vir}} = \frac{\mu m_p V_c^2}{2k} = 1.04 \times 10^4 \left(\frac{\mu}{0.6} \right) \left[\frac{\Omega_m}{\Omega_m(z)} \frac{\Delta_c}{18\pi^2} \right]^{1/3} \left(\frac{M}{10^8 M_\odot} \right)^{2/3} \left(\frac{1+z}{10} \right) \text{ K}$$

Halo mass budget



Gas Pressure



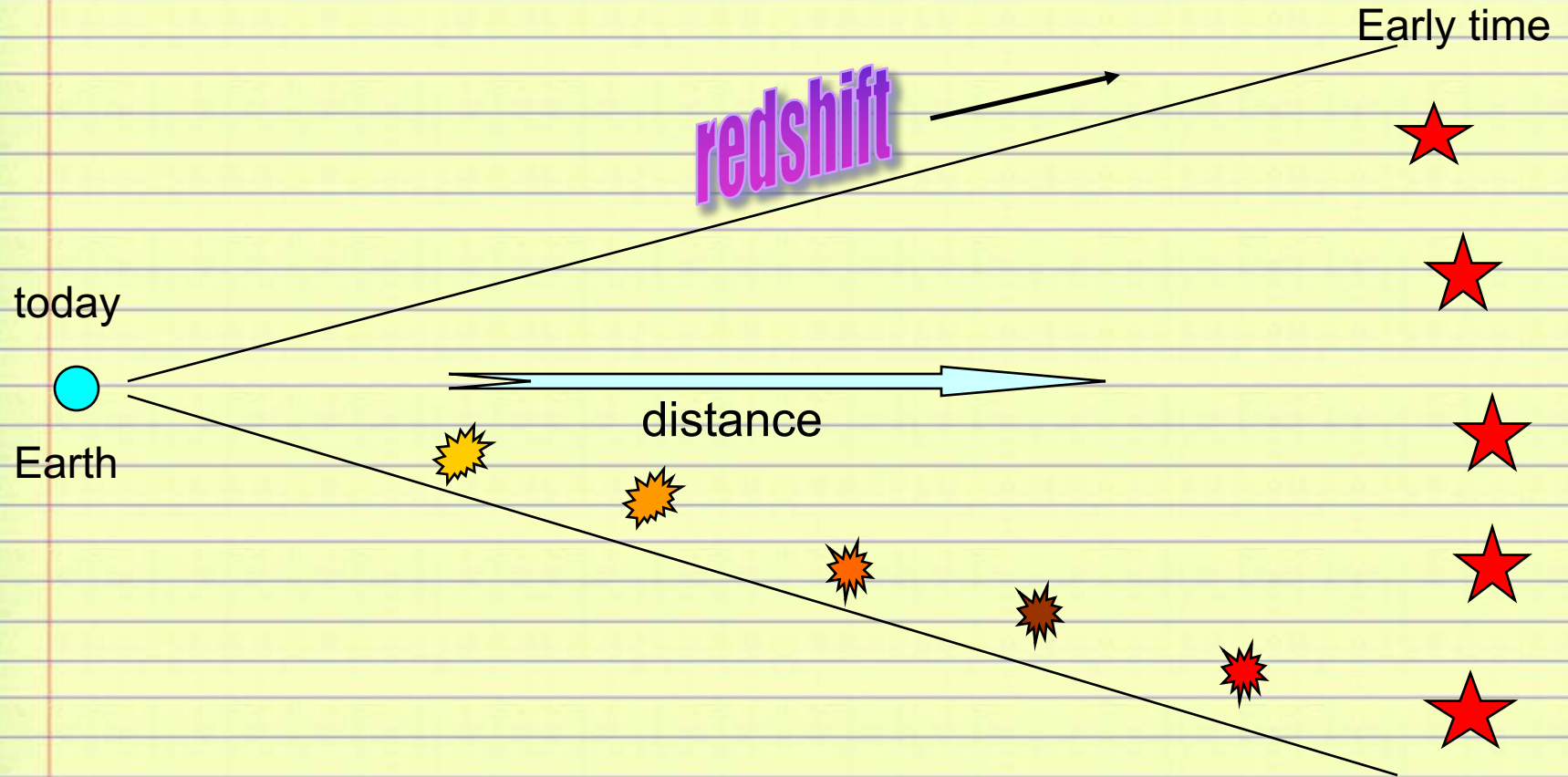
- Jeans mass

$$\frac{\lambda_J}{c_s} \sim \frac{1}{\sqrt{G\rho_m}}$$

$$M_J = 4.54 \times 10^3 \left(\frac{\Omega_m h^2}{0.15} \right)^{-1/2} \left(\frac{\Omega_b h^2}{0.022} \right)^{-3/5} \left(\frac{1+z}{10} \right)^{3/2} M_\odot$$

Grows to $\sim 10^9 M_\odot$. after intergalactic gas is photo-heated to $\sim 10^4 \text{K}$

Cosmic-Archeology



The more distant a source is, the more time it takes for its light to reach us. Hence, the light must have been emitted when the universe was younger. By looking at distant sources we can trace the history of the universe.

- Once the Universe ages by ~ 100 (a trillion years from now), the wavelength of the microwave background will exceed the scale of our horizon...
- At that time, all extragalactic atoms will be pushed out of the horizon and be unavailable for tracing the cosmic expansion...

