



THE HUBBLE CONSTANT CONTROVERSY

H₀ FROM GRAVITATIONAL WAVES

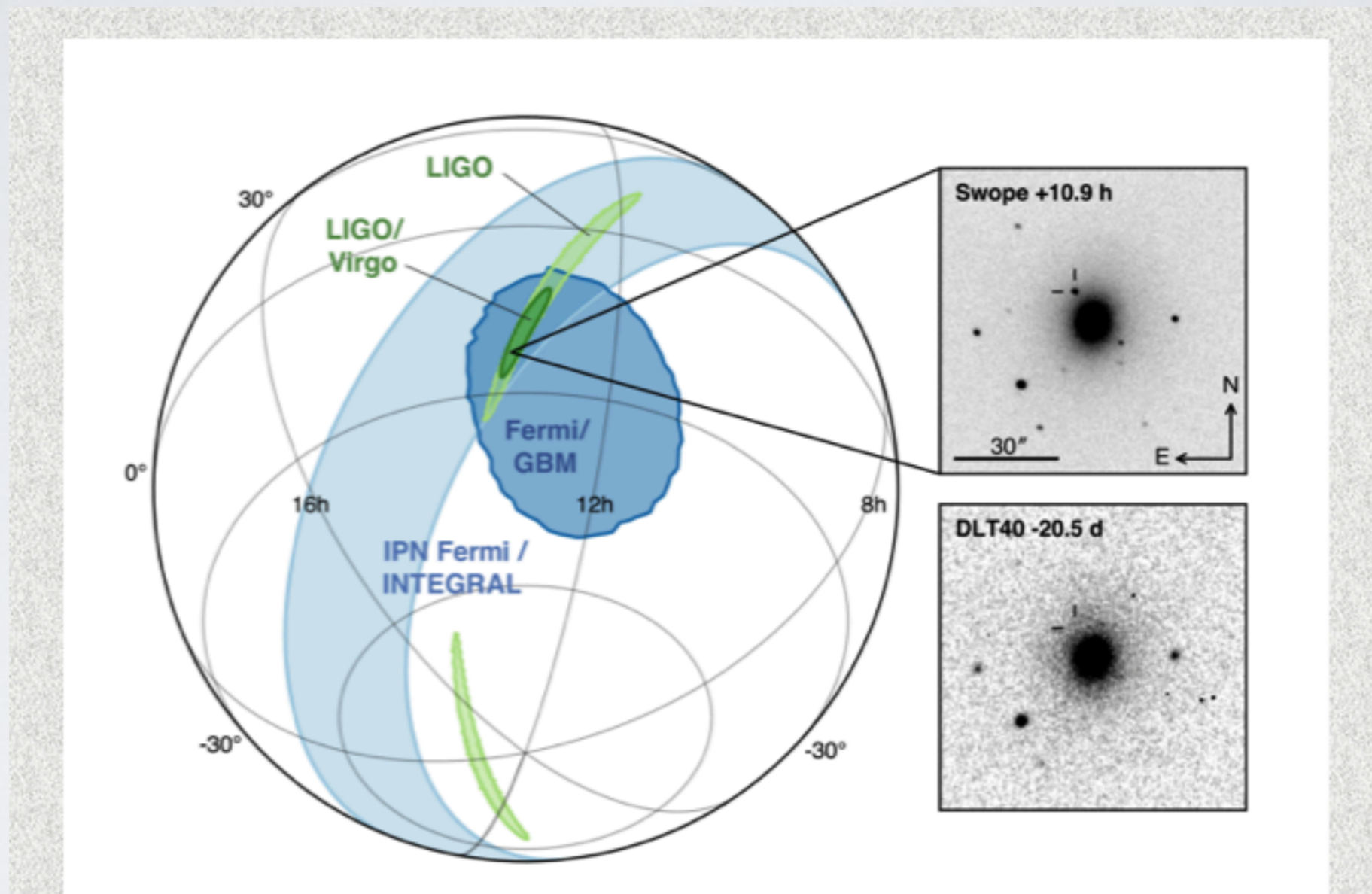
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GW170817



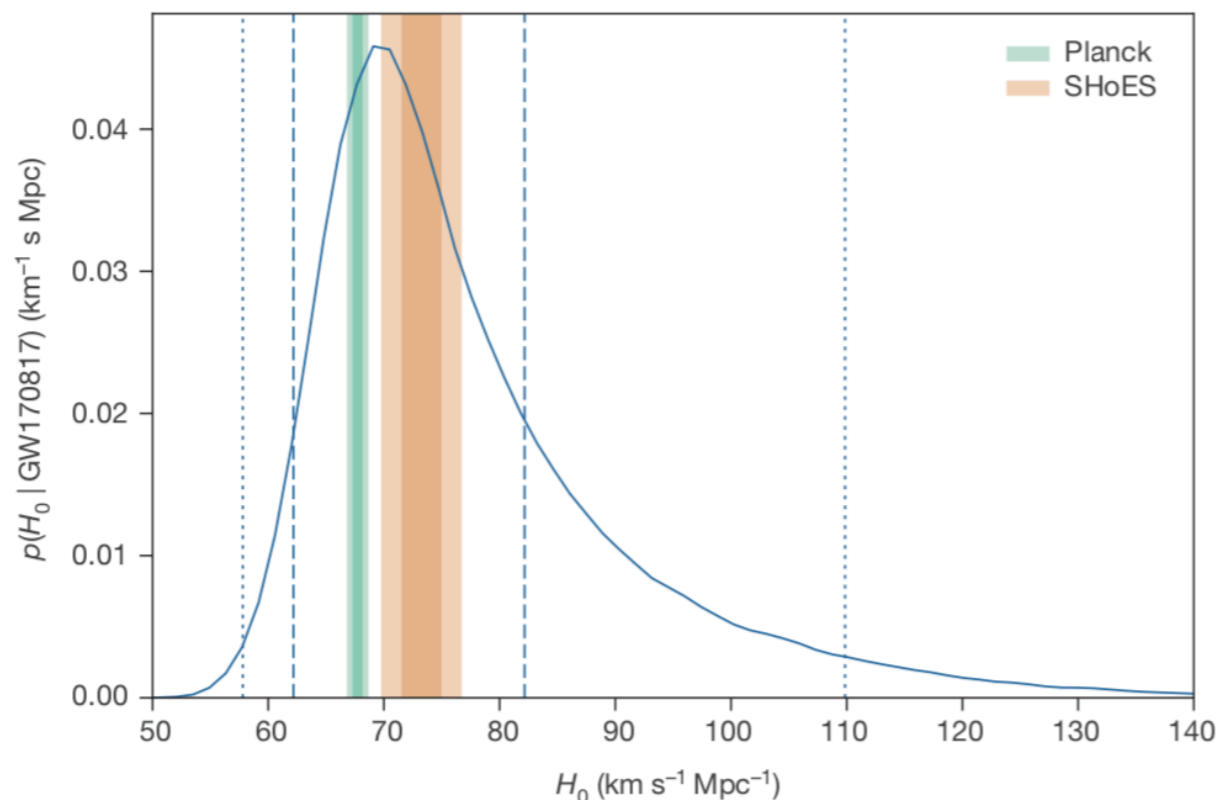
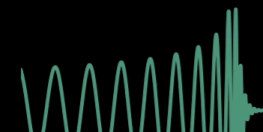


Figure 1 | GW170817 measurement of H_0 . The marginalized posterior density for H_0 , $p(H_0 | \text{GW170817})$, is shown by the blue curve. Constraints at 1σ (darker shading) and 2σ (lighter shading) from Planck²⁰ and SHoES²¹ are shown in green and orange, respectively. The maximum a posteriori value and minimal 68.3% credible interval from this posterior density function is $H_0 = 70.0^{+12.0}_{-8.0} \text{ km s}^{-1} \text{ Mpc}^{-1}$. The 68.3% (1σ) and 95.4% (2σ) minimal credible intervals are indicated by dashed and dotted lines, respectively.

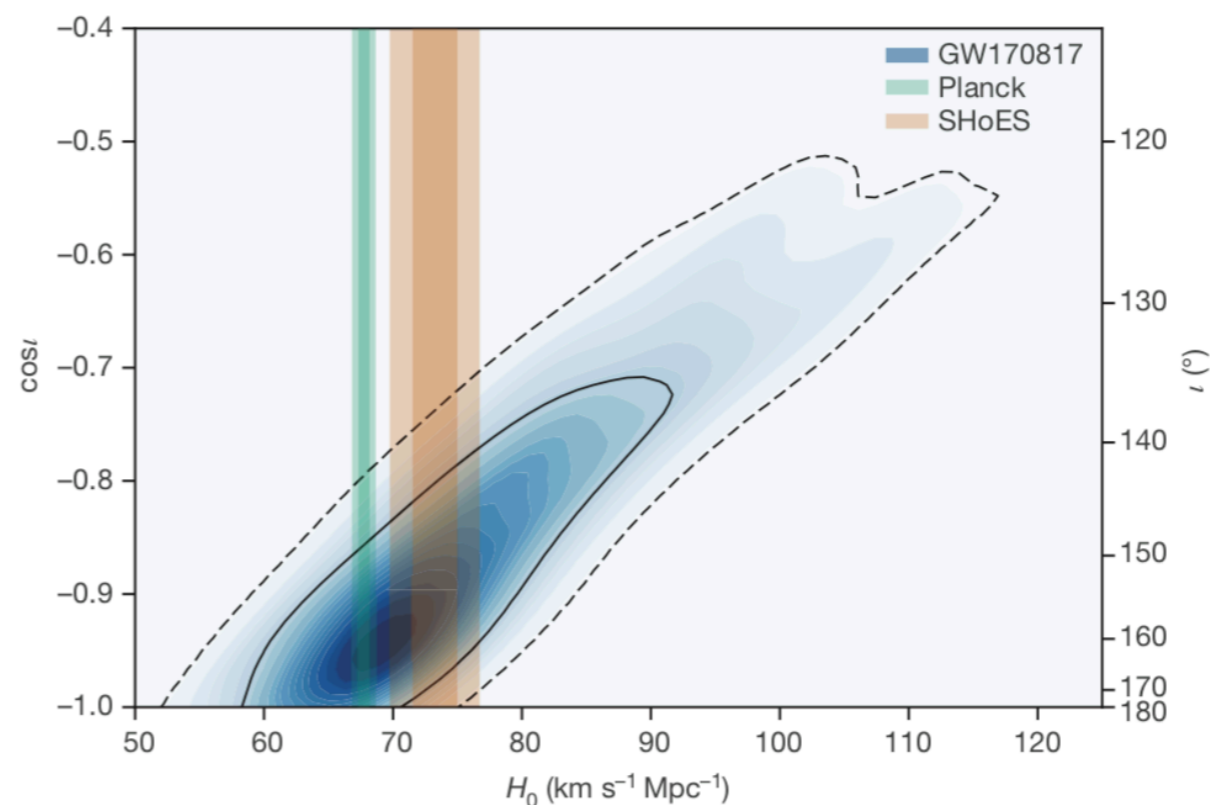


Figure 2 | Inference on H_0 and inclination. The posterior density of H_0 and $\cos i$ from the joint gravitational-wave–electromagnetic analysis are shown as blue contours. Shading levels are drawn at every 5% credible level, with the 68.3% (1σ ; solid) and 95.4% (2σ ; dashed) contours in black. Values of H_0 and 1σ and 2σ error bands are also displayed from Planck²⁰ and SHoES²¹. Inclination angles near 180° ($\cos i = -1$) indicate that the orbital angular momentum is antiparallel to the direction from the source to the detector.

- Counterpart was in NGC4993. $\mathcal{M} = 1.188 (+0.004, -0.002) M_\odot$, calibration $\sim 5\%$. But $d = 43.8 (+2.9, -6.9) \text{ Mpc}$, about 10% error. Due to inclination-distance degeneracy.



WHY THERE CAN BE STANDARD LUMINOSITIES

- GR is scale-free: the Schwarzschild solution depends only on M/r , so it scales perfectly with M . Unlike EM, no preferred mass scale.
- Luminosity is dimensionless if we set $c=G=1$! The fundamental luminosity scale is $L_0 := c^5/G = 10^{26} L_\odot = 3.6 \times 10^{59} \text{ erg/s}$. So any mass scale has to be cancelled by a distance or time. If all are the same (as for a BH) then L should be mass-independent.
- In the quadrupole-formula limit, a circular binary system with Newtonian gravitational potential φ radiates GW energy at a rate $\sim (\varphi/c^2)^5 L_0$. A black-hole binary therefore approaches L_0 .
- The first GW detection, GW150914, radiated $3 M_\odot c^2$ of energy in $\sim 200 \text{ ms}$, with a final peak $L \sim 3 \times 10^{56} \text{ erg/s} \sim 10^{-3} L_0$.
- Had the system been identical but with masses half as big, the peak luminosity would have been the same. The merger timescale would have been half as long, so the total radiated energy would have been half as much.



STANDARD SIRENS

- The binary orbit evolution problem was solved in the point-mass quadrupole GW approximation by Peters & Mathews in 1963 (PR 131,435) and Peters in 1964 (PR 136, B1224). Essentially they give h (the GW amplitude) in terms of the two masses m_1 and m_2 , the orbital separation a , and the distance to the source D : 4 parameters.
- Schutz in 1986 (Nature 323, 310) recast the Peters-Mathews equations in terms of *observables*. We can only observe 3 things: the amplitude h , the GW frequency f , and the chirp rate df/dt . So it does not look like one can infer r from the data. P-M did not go there.
- However, when one replaces a with the Newtonian orbit relation in terms of m_1 , m_2 , and Ω (the orbital angular velocity, which is half the GW phase angular velocity), one finds that the masses merge into just one mass, which we call the chirp mass \mathcal{M} , defined as

$$\mathcal{M} = (m_1 m_2)^{3/5} / (m_1 + m_2)^{1/5}$$

- This means that the three observables (h , Ω , $d\Omega/dt$) need to determine only three parameters: (\mathcal{M}, Ω, D) . The distance D can be measured for ANY binary!



HOW IT WORKS

- The full equations are (see Holz, Hughes, Schutz: Phys Today, Dec 2018

$$\frac{d\Omega}{dt} = \frac{96}{5} \left(\frac{GM}{c^3} \right)^{5/3} \Omega^{11/3}$$

which determines \mathcal{M} , and the two polarisations (ι is the inclination of the orbit)

$$h_+ = \frac{2c}{D} \left(\frac{GM}{c^3} \right)^{5/3} \Omega^{2/3} (1 + \cos^2 \iota) \cos [2\Phi(t)]$$
$$h_\times = \frac{4c}{D} \left(\frac{GM}{c^3} \right)^{5/3} \Omega^{2/3} \cos \iota \sin [2\Phi(t)] ,$$

- Measuring the two polarisations then determines D.
- At cosmological distances, D is the *luminosity distance* and $\mathcal{M}_{\text{obs}} = (1 + z)\mathcal{M}_{\text{intrinsic}}$.
- To infer h from the detector output one has to resolve some external variables as well: sky position (because detectors have anisotropic antenna patterns), orbit inclination (because the binary radiation pattern is also anisotropic). The inclination comes from the polarisation measurement. Three detectors can provide all the information, or two detectors plus a sky counterpart.



UNDER THE HOOD

- The measurement is in principle just one step: no distance ladder. And the binary model is very clean: no complex physics modelling. The only sources of uncertainty are measurement errors:
 - Too few detectors. One needs 3 GW detectors or a counterpart + 2 detectors. The first LIGO detections of BH mergers used only 2 detectors, so the errors in D were $\sim 50\%$.
 - Detector calibration. Errors in h translate directly into error in D . But calibration is difficult. LIGO and VIRGO claim conservatively that they do better than 10%. Actually it is probably closer to 5%. Pushing to 1% is foreseen. Technologies to go beyond 1% are currently under investigation.
 - Determining inclination. When a system is nearly face-on, so ι is nearly zero, then the distance is poorly determined: if h can be measured with relative uncertainty δ then D has an error of order $\delta^{1/2}$. This applies even for inclinations as much as 30° . Such systems are more common than one might expect because the radiation is stronger in direction perpendicular to the orbit, so they can be seen further away.
 - Weak lensing. Only matters on cosmological scale: 1% effect at $z = 0.5$. Relevant to LISA.
- Random errors for derived quantities (e.g. H_0 , w) average down over many measurements. The inclination error is random (must allow for Malmquist bias). Systematic errors in calibration will not average out. Controlling them is a major priority of the experimental teams. LISA will be self-calibrating, accurate to parts in 10^5 .



MEASURING H_0 WITH BNS

- We now expect all BNS to result in a kilonova, and we hope that the counterparts will be identified even if there is no gamma-burst.
- The random errors will decrease as $n^{-1/2}$, but more distant events will have lower SNR, higher errors.
- Chen, Fishbach, Holz (arXiv:1712.06331) predict a 2% measurement after 5 years of observing, starting February.
- But in my 1986 paper I devoted only one sentence to measuring H_0 using counterparts, because it was not clear that they would be easily identified. Instead, I asked how we could do it without identifications of the host galaxy. This is called the ‘statistical method’ and the LSC is preparing to use it on BBH mergers, which may be detected about once a week starting in O3.



STATISTICAL METHOD

- BBH mergers will be detected starting in February out to > 2 Gpc, and at a rate ≈ 2 per month. At this distance, one only needs to find the host galaxy cluster or group, not to its galaxy.
- Assuming the event is observed with three detectors, they give a sky localization and a GW distance, forming a 3D uncertainty cell for each event.
 - Take a catalog of redshifts of galaxy clusters and rich groups in that part of the sky (!). Select those that are within the GW distance uncertainty: use either the current H_0 or other distance measures, but with larger than claimed uncertainties to avoid bias. Each selected cluster gives a candidate value of H_0 formed from its redshift and the GW distance.
- Suppose a typical GW detection produces $N \gg 1$ candidate values of H_0 , only one of them (or possibly none) belongs to the correct host.
 - After n detections, construct a histogram of all nN candidate values. The values coming from the correct hosts begin to cluster in this histogram.
 - This clustering process initially converges *faster* than $n^{-1/2}$; simulations suggest closer to $n^{-3/4}$. Once the peak in the histogram becomes clear, then its width converges as $n^{-1/2}$.
- This will actually be implemented in a Bayesian way, but the process is the same. The LSC will be looking for EM partners who will help map the uncertainty cells to identify the clusters in them and measure their redshifts and richness. With ~ 50 detections (2 years) we may get to 2-4% errors, limited by calibration uncertainties.



PROSPECTS FOR NEAR FUTURE: O3

- When detectors come back online in February, we expect an improvement in range by perhaps a factor of 1.5, leading to detection rates ~ 3 times higher than last year.
- Aside from measuring H_0 , standard sirens are also distance calibrators: each BNS allows us to compare the GW distance measure with other measures. We are already doing this for NGC4993, where the Tully-Fisher distance is 40.7 Mpc, within the GW error range.
- In 2019-20 it is likely we will get a BNS at ~ 100 Mpc in a galaxy cluster. The chances are good that this cluster will have had a SNIa in the last 5-10 years. If that has been seen, we can compare the two distance measures directly. Over the next 10 years we may thereby be able to provide an alternative calibration of SNIa standard candles directly in terms of standard sirens.



LONGER TERM PROSPECTS

- Further improvements in existing detectors from 2020-25 are likely to give us hundreds of BNS, and perhaps thousands of BBH detections. We expect to push calibration to the 1% level. If we can't do better then that might limit the accuracy for H_0 .
- Standard sirens with counterparts will still be rare: volume event rate is 5% of SNIa rate. But the ensemble of standard siren distances could be used to calibrate the SNIa distance scale.
- LISA will launch after 2030, with SNR 10^4 - 10^5 for BBH at $z = 1$. It is self-calibrating so that source of systematics will be less troubling. But weak lensing could limit actual accuracy to 1%. Some chance of measuring Λ , perhaps even dark energy EOS. But the standard siren distances for distant BBH mergers will use known cosmological parameters to give redshifts, with SNR ~ 50 even at $z = 20$.
- Third-generation ground-based detectors, like ET in Europe and Cosmic Explorer in the USA, may be in operation after 2030. They will see essentially every BNS event in the universe, and all BBHs with components below $100 M_\odot$. They will presumably be calibrated to accuracy of 0.1%, so weak lensing may again limit accuracy. But uncertainties for H_0 approaching 0.1% are not impossible on this time-scale.

