H0 and Gravitational Lensing

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Two giants of the evolving universe

Edwin Powell Hubble (1889-1953)

Lemaitre

Hubble’s law: $d = H_0 t$

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ON THE POSSIBILITY OF DETERMINING HUBBLE’S PARAMETER AND THE MASSES OF GALAXIES FROM THE GRAVITATIONAL LENS EFFECT*

Sjur Refsdal
(Communicated by H. Bondi)
(Received 1964 January 27)

Summary

The gravitational lens effect is applied to a supernova lying far behind and close to the line of sight through a distant galaxy. The light from the supernova may follow two different paths to the observer, and the difference Δt in the time of light travel for these two paths can amount to a couple of months or more, and may be measurable. It is shown that Hubble’s parameter and the mass of the galaxy can be expressed by Δt, the red-shifts of the supernova and the galaxy, the luminosities of the supernova “images” and the angle between them. The possibility of observing the phenomenon is discussed.
The time delay between two images $i$ and $j$ is given by

$$\Delta t_{i,j} = \frac{1}{H_0} (1 + z_d) D(z_d, z_s, \Omega_0, \Lambda_0) f$$

where the function $f$ depends on the different path length to each image taking account of both the geometric path length, due to the image positions, and gravitational time delay due to the lens potential.
The time delay between two images \(i\) and \(j\) is given by

\[
\Delta t_{i,j} = \frac{1}{H_0} (1 + z_d) D(z_d, z_s, \Omega_0, \Lambda_0) f
\]

\[
D = \frac{D_s D_d}{D_{ds}}
\]

The function \(f\) depends on the different path length to each image taking account of both the geometric path length, due to the image positions, and gravitational time delay due to the lens potential.
Gravitational Lensing

**OPPORTUNITY**

Variable & multiply-imaged sources:
- quasars, gravitational waves, SNe
  → the ‘Refsdal Experiment’

H0LiCOW multiply-imaged quasars:
Sherry Suyu’s talk after lunch

- HE 0435-1223B1608+656 RXJ1131-1231
- HE 1104-1805 WFI2033-4723 HE 0435-1223

November 10 Falling Walls- H0 Controversy 2018
Gravitational Lensing

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UNCERTAINTY
Is there a fundamental limit to the accuracy with which measurements can be made?
Can we determine $H_0$ to 1%?

- *Scientific knowledge is a body of statements of varying degrees of certainty, some most unsure, some nearly sure, but none absolutely certain.*  
  Richard Feynmann
Sources - Supernovae

Angular scale of the source matters

\[ \theta_S \sim \theta_E = \sqrt{\frac{4GM}{c^2}} \frac{D_{ds}}{D_d D_s} \]

The remnant of a type 1a supernova that exploded in the year 1006. (NASA/CXC/et al)
Sources - quasars

C. Kindl, Diploma Thesis, U Heidelberg (IWR) 1995

Urry & Padovani 2002
**Sources – gravitational waves**

Gravitational lensing:

\[ h \propto \sqrt{\mu} \]

**M_{chirp} and h (strain):**

\[ D_l = \frac{M_{chirp}^5(z)}{\int h(t) f} \]

\[ M_{chirp} = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}} \]

Gravitational lensing

\[ h \propto \sqrt{\mu} \]
DEFLECTION OF LIGHT RAYS CROSSING THE UNIVERSE

IMAGES DES GALAXIES LOINTAINES DEFORMÉES PAR LA MATIÈRE NOIRE DE L'UNIVERS

SIMULATION COURTESY NIC GROUP, E. COLOMB, IAP

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Scale of Cosmological Homogeneity?

![Graph showing distribution of Scale (Mpc) vs. $R_H$ for different redshift ranges.]

- $0.1 < z < 0.3$
- $0.3 < z < 0.5$
- $0.5 < z < 0.7$
- $0.7 < z < 0.9$

Scrimgeour+2012
Background metric – numerically

Deviation of local Hubble parameter (l) and density (r)

Macpherson+ 2018
Measurements of $H_0$ locally

![Histogram of $H_D - H_{\text{all}}$ (km s$^{-1}$ Mpc$^{-1}$)](image)

- **Riess et al.** 1σ
- **All spheres**
- **Planck 1σ**
Background metric - distances

Angular diameter

Luminosity

Empty beam demagnification
Biased lines-of-sight

Wilson+ 2017

McCully+2016
Biased lines-of-sight (2)

The magnification PDF for small sources

Wilson+ 2017

Killdear+2011

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Cosmological Averaging

- Weinberg (1976): Flux is conserved $\langle \mu \rangle = 1$ averaged over sources
- Seitz, Schneider&Ehlers(1994): Focussing theorem $\rightarrow$ one image is always brighter with a lens (but ‘empty beams’ need to be taken into account)
- Kibble&Liu (2005): distinguish between averaging over sources and averaging over directions (relevant for CMB) arguing that $\langle \mu^{-1} \rangle = 1$
- Kaiser&Peacock(2016): support Weinberg and Kibble&Liu, but argue that a nonlinear function of $\mu$ such as $D \propto \mu^{-1}$ will be biased by $\sim \langle \kappa^2 \rangle$
- Ellis&Durrer(2018): consider there are still open questions, $\rightarrow$ not convinced the analysis is yet correct
Fractional correction to the distance $\Delta$

Umeh+2014
Redshift drift—or—the Sandage-Loeb Effect

\[ \frac{dz}{dt_{\text{obs}}} (t_0) = (1 + z)H_0 - H(z) \]

\[ c\delta(z) \sim 1 \frac{cm}{sec/yr} \]

Figure 2. The solid (dotted) lines and left-hand axis (right-hand axis) show the redshift drift \( \dot{z} \) as a function of redshift for various combinations of \( \Omega_M \) and \( \Omega_A \) as indicated. The dashed line shows \( \dot{z} \) for the case of dark energy having a constant \( w_X = -2/3 \) (and \( \Omega_M, \Omega_X = 0.3, 0.7 \)).
Measuring the SL Effect

**SKA**

- **Fiducial \( \Lambda \)CDM**
- \( \Omega_m = 0.28 \)
- \( w_0 = -0.95 \)
- SKA2 (5 bins) data
- SKA2 (10 bins) data

**E-ELT**

- Fiducial \( \Lambda \)CDM
- \( \Omega_m = 0.28 \)
- \( w_0 = -0.95 \)
- E-ELT data
Conclusions

• Measuring $H_0$ – path to the 1% experiment
• Scale of homogeneity is \( \sim 100 \) Mpc
• General relativistic cosmological models with structure formation framework
• Issues in measuring $H_0$ to better than 1% - theoretical

• SL test – hard but worth it