Maximum Power
for
Two-Level Thermal Machines:
Optimality of Fast Cycles

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Thermal Machines

1) Heat engine

\[ Q_H \geq 0 \quad Q_C \leq 0 \quad W \geq 0 \]

2) Refrigerator

\[ Q_H \leq 0 \quad Q_C \geq 0 \quad W \leq 0 \]

3) Heater

\[ Q_H \geq 0 \quad Q_C \leq 0 \quad W \geq 0 \]

4) Thermal accelerator

\[ Q_H \geq 0 \quad Q_C \leq 0 \quad W \leq 0 \]
Thermal Machines: Characterization

Heat engine

Power: \[ P_E = \frac{W}{\tau} \]

Efficiency: \[ \eta = \frac{W}{Q_H} \leq \eta^{(c)} = 1 - \frac{T_C}{T_H} \]
Thermal Machines: Optimization

Maximize \textit{Efficiency:}
- Reversible Transformations
  - Infinitely slow
    - \( \eta = \eta^{(c)} \)
    - However, \( P = 0 \)

Maximize \textit{Power:}
- Finite-time thermodynamics
- Microscopic model
  - Universal strategy?
  - Efficiency at maximum power?
The Model

\[ \mathcal{H}_S(t) = \epsilon(t) \sigma^+ \sigma^- \]

\[ \frac{d}{dt} \hat{\rho} = -\frac{i}{\hbar} [\mathcal{H}_S, \hat{\rho}] + \mathcal{D}_H[\hat{\rho}] \]

\[ \frac{d}{dt} p(t) = -\sum_{\alpha} \lambda_{\alpha}(t) \Gamma_{\alpha} [\epsilon(t)] (p(t) - p_{eq}[\epsilon(t)]) \]

\[ \text{Tr} [\sigma^+ \sigma^- \hat{\rho}(t)] \text{ Bath Switch} \]

Dissipation Rate

Hot Bath

\[ p_{eq}^H = \frac{1}{1 + e^{\beta_H \epsilon}} \]
Power Maximization

Given a fixed physical system, i.e. fixed $\Gamma_C(\epsilon), \Gamma_H(\epsilon)$

Maximize over $\epsilon(t)$

Arbitrary input periodic control $\epsilon(t)$

Equ. of motion

Compute periodic state $p(t)$

Average Power: $P_E = -\frac{\int_0^\tau \dot{\epsilon}(t) p(t) dt}{\tau}$
Maximum Power: Infinitesimal Otto Cycle

Two Stroke Cycle:

- Contact with HOT: $\epsilon_H$
- Contact with COLD: $\epsilon_C$
- Repeat

\[ \tau_H + \tau_C \rightarrow 0 \]

\[ \frac{\tau_H}{\tau_C} = \sqrt{\frac{\Gamma_C}{\Gamma_H}} \]
Power of Optimal Cycle

\[ P_{[v]}^{(\text{max})} = \max_{\epsilon_H, \epsilon_C} \frac{\Gamma_H \Gamma_C}{(\sqrt{\Gamma_H} + \sqrt{\Gamma_C})^2} \tilde{\epsilon}_{[v]} \left[ p_{\text{eq}}^{(H)} - p_{\text{eq}}^{(C)} \right] \]

Where:

\[ \tilde{\epsilon}_{[E]} = \epsilon_H - \epsilon_C, \quad \tilde{\epsilon}_{[R]} = -\epsilon_C, \]
\[ \tilde{\epsilon}_{[A]} = +\epsilon_C, \quad \tilde{\epsilon}_{[H]} = \epsilon_C - \epsilon_H, \]
Optimality of fast cycles

Fast driving regime little attention, but:


...
Single level *Quantum Dot*:

Metallic Lead \( \Gamma^+_H(\epsilon) \) Tunnel Junction

Metallic Lead \( \Gamma^+_C(\epsilon) \) Tunnel Junction

States: occupation of QD \((n=0,1)\)

1. \( \Gamma^+ = k f(\beta \epsilon) \)

2. \( \Gamma^- = k [1 - f(\beta \epsilon)] \)

\[ \Gamma \equiv \Gamma^+ + \Gamma^- = k \]

Two-level *Atom in Dissipative EM Environment*:

\[ \Gamma = k \coth(\beta \epsilon/2) \]
Efficiency at Maximum Power (EMP)

Many upper bounds in literature:

- **Curzon-Ahlborn EMP:**
  \[ \eta_{CA} = 1 - \sqrt{1 - \eta_C} \]
  
  [C. Van den Broeck, Phys. Rev. Lett. 95, 190602 (2005)]

- **Schmiedl-Seifert:**
  \[ \eta_{SS} = \frac{\eta_C}{2 - \eta_C} \]
  
  [T. Schmiedl and U. Seifert, EPL 81, 20003 (2008)]

*Endoreversible Engines*
*Linear Irreversible Thermodynamics*
*Brownian Heat Engine*
*Universal in low dissipation regime*
Carnot Efficiency at Maximum Power

Low dissipation / slow driving: \[ \eta_{SS} = \frac{\eta_c}{2 - \eta_c} \]

Fast driving / far from equilibrium:

Feasibility

What happens for finite $dt$?
Conclusions

- Maximum Power in Two-Level Systems: Optimality of Fast Cycles
- “Duality” power - efficiency
- There is no fundamental upper bound for the EMP (Heat Engine and Refrigerator) thanks to the Fast Driving Regime.
- Applicable to Relevant Systems (Fermionic, Bosonic, etc).
- Feasibility of the Protocol.
Thanks for your attention

[P. A. Erdman et al., New J. Phys. 21, 103049 (2019)]
From asymptotic to infinitesimal

\[ \epsilon(t) \]

Choose the best subcycle

Quench

\[ p(t) \]
From asymptotic to infinitesimal

- \( p(t) \approx p \) is almost constant during a cycle
- \( \epsilon(t) \) switches between two extremal values \( \epsilon_C \) and \( \epsilon_H \)
- \( p(t) \approx p \) needs a very fast driving: the population does not have time to relax
Fast Driving Regime

- $t_b$: Bath correlation time timescale
- $t_s$: System timescale
- $t_r$: Relaxation timescale (induced by bath)
- $t_d$: Driving timescale

$t_b \ll t_s \ll t_d \ll t_r$

Markovian Approx. Secular Approx. Fast Driving