Operator Theory and Polynomial Optimization in Quantum Information Theory

828. WE-Heraeus-Seminar

11 – 14 March 2025

at the Physikzentrum Bad Honnef, Germany

WILHELM UND ELSE HERAEUS-STIFTUNG

Introduction

The Wilhelm und Else Heraeus-Stiftung is a private foundation that supports research and education in science with an emphasis on physics. It is recognized as Germany's most important private institution funding physics. Some of the activities of the foundation are carried out in close cooperation with the German Physical Society (Deutsche Physikalische Gesellschaft). For detailed information see https://www.we-heraeus-stiftung.de

Aims and scope of the 828. WE-Heraeus-Seminar:

Quantum Information Theory is a rapidly growing field with far stemming foundational results (e.g., the Bell theorem) and applications (e.g. quantum cryptography and computing). While the complexity of technologically feasible protocols increases, the theoretical understanding of quantum devices capabilities is lagging behind.

Quantum correlations are central for the foundations of Quantum Information Theory, and for many of its applications. However, their characterisation is challenging. Existing mathematical and algorithmic approaches, based on operator theory and non-commutative polynomial optimization, are not adapted to some recent theoretical, experimental and technological developments in quantum physics, specifically for quantum network correlations. In recent years, this challenge has attracted researchers from quantum theory, operator theory, polynomial optimization and mathematical physics. While some progress has been made in each of these fields, a compartmented approach of each community cannot work.

The goal of our Operator Theory and Polynomial Optimization in Quantum Information Theory seminar is to bring together world leading experts and promising early-career researchers to unlock these mathematical barriers. The seminar will make the mathematical approaches from the operator theory and advanced tools from polynomial optimization available to quantum information researchers.

Scientific Organizers:

Prof. Dr. Mariami Gachechiladze	Technical University Darmstadt, Germany e-mail: mariami.gachechiladze@tu-darmstadt.de
Prof. Dr. Igor Klep	University of Ljubljana, Slovenia e-mail: igor.klep@fmf.uni-lj.si
Prof. Dr. Marc-Olivier Renou	Inria Paris-Saclay & CPHT, France e-mail: marc-olivier.renou@inria.fr

Introduction

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Registration:

Mojca Peklaj (WE Heraeus Foundation)
at the Physikzentrum, Reception Office
Tuesday (16:00 h - 21:00 h)

Program

Program

Tuesday, 11 March 2025

16:00 – 21:00	Registration	
18:00	*BUFFET SUPPER and informal get together*	
19:30 – 19:40	Scientific Organizers	**Opening remarks**
19:40 – 20:40	Antonio Acin	**Certified many-body physics**
20:40 – 21:40	**Discussions**	

Wednesday, 12 March 2025

08:00	*BREAKFAST*	
09:00 – 09:15	Stefan Jorda	**The Wilhelm and Else Heraeus Foundation**
09:15 – 10:15	Monique Laurent	**Approximations for polynomial optimization on the sphere and quantum de Finetti theorems**
10:15 – 10:45	Andreas Bluhm	**Computing noise robustness of incompatible quantum measurements**
10:45 – 11:15	*COFFEE BREAK*	
11:15 – 12:15	Victor Magron	**State polynomial optimization for nonlinear Bell inequalities**
12:15 – 12:45	Ranieri Vieira Nery	**Certification of entangled measurements**
12:45 – 14:00	*LUNCH / DISCUSSIONS*	

Program

Wednesday, 12 March 2025

14:00 – 15:00	Vern Paulsen (online)	**Synchronous Values of Games**
15:00 – 15:30	Costantino Budroni	**Optimization of Time-Ordered Processes in the Finite and Asymptotic Regimes**
15:30 – 16:00	*COFFEE BREAK*	
16:00 – 16:30	**Discussions**	
16:30 – 17:00	**Poster Flash Talks 1**	
17:00 – 19:00	**Poster Session 1**	
19:00	*DINNER*	

Program

Thursday, 13 March 2025

08:00	*BREAKFAST*	
09:00 – 10:00	Miguel Navascués	**Non-commutative optimization problems with differential constraints**
10:00 – 10:30	Victoria Wright	**Quantum field theory can be more contextual than non-relativistic quantum theory**
10:30 – 11:00	*COFFEE BREAK*	
11:00 – 12:00	Christopher Fewster	**Algebraic quantum field theory and operator algebras**
12:00 – 12:30	Abhishek Mishra	**Polynomial Optimization with partially commuting variables**
12:30 – 12:40	**Conference Photo**	
12:40 – 14:00	*LUNCH / DISCUSSIONS*	
14:00 – 15:00	William Slofstra (online)	**The mystery of linear system games over Z_p**
15:00 – 15:30	Hyejung Hailey Jee	**Practical semi-device-independent randomness generation based on energy constraint with probability estimation**
15:30 – 16:00	*COFFEE BREAK*	
16:00 – 16:30	**Discussions**	
16:30 – 17:00	**Poster Flash Talks 2**	
17:00 – 19:00	**Poster Session 2**	
19:00	*HERAEUS DINNER*	

Program

Friday, 14 March 2025

08:00	*BREAKFAST*	
09:00 – 10:00	Angelika Wiegele	**Solving Semdidefinite Programs using Alternating Direction Methods of Multipliers**
10:00 – 10:30	Victoria Sánchez Munoz	**Finite-size certification of randomness in semi-device-independent protocols based on photon-number constraints**
10:30 – 11:00	*COFFEE BREAK*	
11:00 – 12:00	David Gross	**De Finetti theorems for quantum and post-quantum polynomial optimization**
12:00 – 12:30	Máté Farkas	**Semidefinite representability of the set of quantum correlations in the simplest Bell scenario**
12:30 – 12:40	Scientific Organizers	**Closing remarks**
12:40	*LUNCH*	

End of the seminar and departure

For participants leaving on Saturday a self-service breakfast will be provided on Saturday morning.

Posters

Poster Session 1, Wednesday, 12 March, 17:00 h (CET)

Llorenç Balada Gaggioli	**Quantum optimal control via polynomial optimization**
Subhankar Bera	**Optimal demonstration of generalized quantum contextuality**
Bihalan Bhattacharya	**Qubit Schwarz maps with diagonal unitary and orthogonal symmetries**
Rutvij Bhavsar	**Rates for Device Independent Randomness Expansion Protocols Based on Two Input Two Output Bell Test**
Pawel Blasiak	**Identical particles as a genuine non-local resource**
Riccardo Castellano	**Parallel ergotropy: Maximum work extraction via parallel local unitary operations**
Kai-Siang Chen	**Bounding the minimal average communication cost of nonlocal correlations**
Nasra Daher Ahmed	**When can you trade causal order for locality?**
Arun Kumar Das	**An operational approach to classifying measurement incompatibility**
Jose De Ramon Rivera	**Deformations of the symmetric subspace of qubit chains**
Juan Diego Diaz Martinez	**A Quantum Mobius Strip?**
Prabhav Jain	**Communication Complexity Bounds using Information Causality**
Shashaank Khanna	**Which causal scenarios can support non-classical correlations?**

Poster Session 1, Wednesday, 12 March, 17:00 h (CET)

Gereon Kossmann	**Device independent bounds for the conditional von-Neumann entropy**
Robin Krebs	**Sums-of-squares techniques for classical XOR games**
Rufus Lawrence	**The Random Schrödinger Equation and Geometric Quantum Control**

Poster Session 2, Thursday, 13 March, 17:00 h (CET)

Zixuan Liu	Tsirelson bounds for quantum correlations with indefinite causal order
Rafael Macedo	Lieb-Robinson bounds and non-commutative polynomial optimization
Jan Mandrysch	Numerical quantum energy bounds in models with self-interaction
Nikolai Miklin	Enhancing robustness of self-testing of quantum resources
Davide Poderini	Interventional inequalities in minimal quantum causal networks
Albert Rico	Quantum advantage in q-dit communication
Jordi Romero Palleja	Multipartite Entanglement in the Symmetric Subspace
Rene Schwonnek	Finite-Dimensional Relaxations for Cone Programs on C∗-Algebras: A Unified Perspective
Sigurd Storgaard	Parallel repetition and gap-annihilation
Lucas Tendick	Distributed computing and its links to quantum information theory and the NPA hierarchy
Isadora Veeren	Characterizing high-dimensional quantum contextuality
Matthijs Vernooij	Lifting assumptions for robust self-testing
Giuseppe Viola	Quantum strategies for rendezvous and domination tasks on graphs with mobile agents

Poster Session 2, Thursday, 13 March, 17:00 h (CET)

Santiago Zamora	Partial interventions in the triangle quantum network
Lin Htoo Zaw	Tsirelson's Precession Protocol: A Theory-independent Bound Saturated by Quantum Mechanics, and Other Generalisations
Yuming Zhao	Robust self-testing for games with stable algebras

Abstracts of Lectures

(in alphabetical order)

Certified Many-body Physics

A. Acin

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When studying many-body systems, two approaches have been considered so far: analytical derivations and variational methods. The first provide exact results, as they do not involve any approximations, but generally scale exponentially with the number of particles, while the second scale much better but only provide estimates with no theoretical guarantees. Polynomial optimisation methods offer an alternative approach in which, contrary to variational methods, certified results, usually in the form of bounds, can be derived with much better scalability than exact methods. We illustrate this new approach in two paradigmatic many-body problems: the estimation of expectation values in ground states of Hamiltonian operators and in steady states of quantum open systems.

Computing noise robustness of incompatible quantum measurements

A. Bluhm[1], E. Evert[2], I. Klep[3], V. Magron[4], I. Nechita[5]

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We present here a connection between two very different problems: the compatibility of measurements in quantum information theory and the inclusion problem for free spectrahedra, which are objects appearing in convex optimization. In particular, the compatibility of a tuple of dichotomic quantum measurements is equivalent to the inclusion of the matrix diamond (see [1]), which is a matricial relaxation of the l_1-unit ball, inside a free spectrahedron defined by the measurements in question. A similar correspondence holds for quantum measurements with more outcomes and a different free spectrahedron, which we call the matrix jewel (see [2]).

Moreover, the minimal amount of noise necessary to render any tuple of measurements (in fixed dimension and with a fixed number of outcomes) compatible corresponds to the inclusion constant of the respective free spectrahedron (i.e., the matrix diamond or the matrix jewel). In order to compute the inclusion constants of interest and thereby the noise robustness of measurement incompatibility, we put forward hierarchies of semidefinite programs and demonstrate their usefulness with some examples. Finally, we will use the structure of the different types of extreme points of the free spectrahedra involved to further study the resulting optimization problems.

References

[1] A. Bluhm and I. Nechita, Journal of Mathematical Physics **59,** 112202 (2018).

[2] A. Bluhm and I. Nechita, SIAM Journal on Applied Algebra and Geometry **4,** 255–296 (2020).

Optimization of Time-Ordered Processes in the Finite and Asymptotic Regimes

Mirjam Weilenmann[1] , Costantino Budroni[2] , and Miguel Navascués[3]

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[2] University of Pisa, Italy,
[3] Institute for Quantum Optics and Quantum Information, Vienna, Austria

Many problems in quantum information theory can be formulated as optimizations over the sequential outcomes of dynamical systems subject to unpredictable external influences. Such problems include many-body entanglement detection through adaptive measurements, computing the maximum average score of a preparation game over a continuous set of target states and limiting the behavior of a (quantum) finite-state automaton. In this work, we introduce tractable relaxations of this class of optimization problems by combining ideas from polynomial optimization and dynamic programming. To illustrate their performance, we use them to: (a) compute the probability that a finite-state automaton outputs a given sequence of bits; (b) develop a new many-body entanglement detection protocol; (c) let the computer invent an adaptive protocol for magic state detection. As we further show, the maximum score of a sequential problem in the limit of infinitely many time steps is in general incomputable. Nonetheless, we provide general heuristics to bound this quantity and show that they provide useful estimates in relevant scenarios.

References

Weilenmann, Budroni, Navascués, PRX Quantum **5**, 020351 (2024)

Semidefinite representability of the set of quantum correlations in the simplest Bell scenario

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The simplest Bell scenario (two parties with two inputs and outputs) is relatively well-understood mathematically, but some fundamental questions still remain. The complexity of characterising this set in terms of polynomial and semidefinite optimisation is not clear. Without understanding the simplest case, it is really difficult to estimate the complexity of more complex scenarios such as those with more parties, inputs and outputs, or network Bell scenarios.

In this talk, I will present recent progress on the semidefinite representability of the set of quantum correlations in the simplest Bell scenario. We use a generic result from polynomial optimisation by Nie, stating that Lassere's semidefinite programming hierarchy—augmented by enforcing the Karush–Kuhn–Tucker conditions—collapses at a finite level for optimisation problems satisfying certain conditions. Optimising Bell inequalities in the simplest scenario satisfies these conditions, and thus in principle can be solved by semidefinite programming. We then use a connection between optimising Bell inequalities and linear functionals over symmetric product states, and use a result by Harrow, Natarajan and Wu (applying Nie's conditions) to prove that there is a universal (albeit extremely large) level of Lassere's hierarchy that is sufficient for every Bell inequality.

Despite these promising fundamental results, it remains unclear whether the set of quantum correlations in the simplest Bell scenario is semidefinite representable, that is, whether the membership problem can be decided with a finite semidefinite programme. To gain further understanding on this problem, the geometric properties of the set might prove useful. Furthermore, it is unclear whether the most-used semidefinite programming hierarchy, the Navascués–Pironio–Acín hierarchy, collapses on a finite level for optimising Bell inequalities. I will present a family of Bell inequalities potentially pointing towards a negative answer to this.

Algebraic quantum field theory and operator algebras

Christopher J. Fewster

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University of York, York, UK

Algebraic Quantum Field Theory (AQFT) is one of two axiomatic programmes for QFT that emerged in the 1950s, in response to the problem of making QFT mathematically precise. While Wightman's programme [1] maintains an emphasis on quantum fields, AQFT [2], developed initially by Haag, Kastler, Araki and others, takes the more radical step on focussing on local observables, with the idea that fields can emerge as natural ways of labelling some of the observables. To each suitable spacetime region, an AQFT assigns a *-algebra of associated observables, which are often either C*-algebras or von Neumann algebras. The subject has been in close contact with the theory of operator algebras since its early days.

This introductory lecture will aim to introduce the ideas of AQFT, emphasising links to operator algebras. As time permits I will also discuss recent topics such as measurement theory for AQFT [3] and the type of local von Neumann algebras appropriate in symmetric spacetimes, when a quantum reference frame is used to determine the location of experiments [4,5].

[1] R.F. Streater and A.S. Wightman, PCT spin and statistics, and all that,
Princeton Landmarks in Physics, Princeton University Press, Princeton, NJ, 2000

[2] R. Haag, Local quantum physics: Fields, particles, algebras, second ed.,
Texts and Monographs in Physics, Springer-Verlag, Berlin, 1996

[3] C.J. Fewster and R. Verch, Quantum fields and local measurements,
Comm. Math. Phys. 378 (2020) 851-889

[4] V. Chandrasekaran, R. Longo, G. Penington, and E. Witten,
An algebra of observables for de Sitter space. JHEP 2023(2) 1-56

[5] C.J. Fewster, D.W. Janssen, L.D. Loveridge, K. Rejzner and J. Waldron,
Quantum Reference Frames, Measurement Schemes and the Type of Local Algebras in Quantum Field Theory, Comm. Math. Phys. 406 (2025) 19:1-87

De Finetti theorems for quantum and post-quantum polynomial optimization

Laurens Ligthart[1], Mariami Gachechiladze[2], Martin Plavala[3], David Gross[1]

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Independent identically distributed variables are permutation-invariant. In the limit of many copies, one can often prove a converse: Permutation symmetry implies independence. In reference to the corresponding result in classical probability, such converses are often called *de Finetti theorems*. Here, we report on a number of applications of de Finetti theorems to design convergent convex hierarchies for polynomial optimization.

The first application is the *quantum network correlation* problem, where the task is to decide whether a set of correlations is compatible with a quantum model that satisfies "network" or "causal" constraints. In some instances, the problem reduces to the one of optimizing over the state space of a universal C^*-algebra given by generators and relations, subject to polynomial constraints in the state. We have developed a convergent SDP hierarchy for this task, based on a de Finetti theorem for general C^* tensor products and the *quantum inflation technique*.

More recently, we have generalized the method to polynomial optimization problems in *general probabilistic theories*. We consider nonnegative matrix rank as a sample application.

Depending on the progress of a project by the time of the meeting, I may show first formalizations of such results in the *Lean* proof verification system.

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[1] L.T. Ligthart, M. Gachechiladze, D. Gross, A convergent inflation hierarchy for quantum causal structures, *Comm. Math. Phys.* **401**, (2023), 2673-2714.

[2] L.T. Ligthart, D. Gross, The inflation hierarchy and the polarization hierarchy are complete for the quantum bilocal scenario, *J. Math. Phys.* **64**, (2023), 072201.

[3] M. Plávala, L. T. Ligthart, D. Gross, The polarization hierarchy for polynomial optimization over convex bodies, with applications to nonnegative matrix rank, arXiv:2406.09506.

Practical semi-device-independent randomness generation based on energy constraint with probability estimation

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The semi-device-independent (SDI) approach offers a good middle ground for quantum randomness generation (QRNG), where we can keep the high security advantage of the device-independent (DI) approach whilst avoiding the need of experimentally demanding implementations, by imposing extra assumptions on physical devices. In particular, a prepare-and-measure protocol developed in [1] provides a practical framework for SDI-QRNG against quantum adversaries with classical side information (i.e., classically correlated with the devices) based on simple constraints on the state preparation, such as energy, photon number or overlap with vacuum. Such constraints are easier to be monitored experimentally than previously suggested ones for SDI-QRNG like the dimension constraint. A full security proof for finite and non-IID (independent and identically distributed) samples has been proposed in [2] but suffers from a high number of rounds being required to verify enough entropy, which makes it difficult to integrate with necessary post-processing such as randomness extraction.

In this paper, we consider the simplest SDI-QRNG protocol in [1] and optimise its practicality by applying the probability estimation (PE) technique developed in [3]. The PE framework is known for its outstanding finite-data efficiency and thus allows us to significantly reduce the required number of rounds for the protocol. The performance of PE depends on a good outer polytope-approximation for the set of quantum correlations in the given setting, and we constructed such approximations for some parameter k – the larger the parameter k is, the tighter the approximation is. We show that we can reduce the minimum number of rounds required for some fixed desired entropy rate by orders of magnitude, which has a direct advantage of reducing the cost and time of implementations and allowing efficient integration with available randomness extraction algorithms.

References

[1] T. Van Himbeeck *et al.* Quantum **1,** 33 (2017)
[2] T. Van Himbeeck and S. Pironio, arXiv:1905.09117 (2019)
[3] Y. Zhang *et al.* Phy. Rev. A **98**, 040304 (2018), E. Knill *et al.* Phy. Rev. R. **2**, 033465 (2020)

Approximations for polynomial optimization on the sphere and quantum de Finetti theorems

Alexander Taveira Blomenhofer
University of Copenhagen

Monique Laurent
CWI Amsterdam, and Tilburg University

We revisit two approximation hierarchies for polynomial optimization on the unit sphere, whose convergence analysis for the r-th level bound was shown to be, respectively, in $O(1/r^2)$ by Fang and Fawzi (using the polynomial kernel method) and in $O(1/r)$ by Lovitz and Johnston (using a quantum de Finetti theorem of Christandl et al. for complex matrices with Bosonic symmetry).
We investigate links between these approaches, in particular, via duality of moments and sums of squares. In particular, we propose another proof for the analysis of the spectral bounds of Lovitz and Johnston, via a "banded" real de Finetti theorem, and we show that the spectral bounds cannot have a convergence rate better than $O(1/r^2)$. In addition, we show how to use the polynomial kernel method to obtain a de Finetti type result for real maximally symmetric matrices, improving an earlier result of Doherty and Wehner.

References

A. Taveira Blomenhofer and M. Laurent. Moment-sos and spectral hierarchies for polynomial optimization on the sphere and quantum de Finetti theorems. arXiv:2412.13191, 2024.

M. Christandl, R. Koenig, G. Mitchison, and R. Renner. One-and-a-half quantum de Finetti theorems. *Communications in Mathematical Physics*, 273(2):473-498, 2007.

A.C. Doherty and S. Wehner. Convergence of SDP hierarchies for polynomial optimization on the hypersphere. arXiv:1210.5048, 2013.

K. Fang and H. Fawzi. The sum-of-squares hierarchy on the sphere and applications in quantum information theory. *Mathematical Programming*, 190:331-360, 2021.

B. Lovitz and N. Johnston. A hierarchy of eigencomputations for polynomial optimization on the sphere. arXiv:2310.17827, 2023.

State polynomial optimization for nonlinear Bell inequalities

- Igor Klep (Department of Mathematics, Institute of Mathematics, Physics and Mechanics, Faculty of Mathematics and Physics, University of Ljubljana, Slovenia)
- Victor Magron (LAAS CNRS and Institute of Mathematics from Toulouse, France)
- Jurij Volčič (Department of Mathematics, Drexel University, Philadelphia, PA, USA)
- Jie Wang (Academy of Mathematics and Systems Science, Chinese Academy of Sciences, Beijing, China)

Abstract: This talk focuses on optimization over state polynomials, i.e., polynomials in noncommuting variables and formal states of their products.

An archimedean Positivstellensatz in the spirit of Putinar and Helton-McCullough is presented leading to a hierarchy of semidefinite relaxations converging monotonically to the optimum of a state polynomial subject to state constraints. This hierarchy can be seen as a state analog of the Lasserre hierarchy for optimization of polynomials, and the Navascués-Pironio-Acín scheme for optimization of noncommutative polynomials.

The motivation behind this theory arises from the study of correlations in quantum networks. Determining the maximal quantum violation of a polynomial Bell inequality for an arbitrary network is reformulated as a state polynomial optimization problem. Several examples of quadratic Bell inequalities in the bipartite scenario are analyzed.

To reduce the size of the semidefinite programs, sparsity, sign symmetry and conditional expectation of the observables' group structure are exploited.

This is based on the joint work [1] with Igor Klep, Jurij Volčič, and Jie Wang.

References:

[1] I. Klep, V. Magron, J. Volčič, and J. Wang. State polynomials: positivity, optimization and nonlinear Bell inequalities. *Mathematical Programming*, 207:645-691, 2023

Polynomial Optimization with partially commuting variables

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1 Abstract

Moment relaxations, or equivalently, sum of squares decompositions, have become a central tool to tackle polynomial optimization problems. These techniques have been adapted from commutative to non-commutative variables and even further to include trace and state polynomials. Their applications extend to a wide variety of research areas, including quantum information and many-body physics.
We propose computational tools tailored to exploit the tantamount structure of commutativity constraints in the polynomial optimization problems appearing in quantum information. We exploit canonical forms for monomials in partially commuting letters to present complete sets of replacement rules to identify polynomials under equality constraints. We employ the graph product of monoids to enlarge the set of equality constraints we can handle through normal forms. That also captures the subsysem picture of QM. We identify normal forms for equivalence of monomials under cyclicty,conjugation, commutations and frequently appearing equations in QM. The advantage in the computational cost is twofold: this both reduces the number of variables and constraints in the semidefinite programming relaxations.

References

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[2] I. Klep, V. Magron, and J. Volčič. Optimization over trace polynomials. In *Annales Henri Poincaré*, volume 23, pages 67–100. Springer, 2022.

[3] I. Klep, V. Magron, J. Volčič, and J. Wang. State polynomials: positivity, optimization and nonlinear bell inequalities. *arXiv preprint arXiv:2301.12513*, 2023.

[4] J. B. Lasserre. Global optimization with polynomials and the problem of moments. *SIAM Journal on optimization*, 11(3):796–817, 2001.

[5] T. Mora. An introduction to commutative and noncommutative gröbner bases. *Theoretical Computer Science*, 134(1):131–173, 1994.

[6] M. Navascués, S. Pironio, and A. Acín. A convergent hierarchy of semidefinite programs characterizing the set of quantum correlations. *New Journal of Physics*, 10(7):073013, 2008.

Non-commutative optimization problems with differential constraints

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Non-commutative polynomial optimization (NPO) problems seek to minimize the state average of a polynomial of some operator variables, subject to polynomial constraints, over all states and operators, as well as the Hilbert spaces where those might be defined. Many of these problems are known to admit a complete hierarchy of semidefinite programming (SDP) relaxations. NPO theory has found application in quantum information theory, quantum chemistry and statistical physics. In this work, we consider a variant of NPO problems where a subset of the operator variables satisfies a system of ordinary differential equations. We find that, under mild conditions of operator boundedness, for every such problem one can construct a standard NPO problem with the same solution. This allows us to define a complete hierarchy of SDPs to tackle the original differential problem. We apply this method to extrapolate quantum time series in a semi-device-independent way and sketch how one can use it to model Hamiltonian time evolution in many-body quantum systems.

References

Mateus Araújo, Andrew J. P. Garner and Miguel Navascués, arXiv:2408.02572.

SYNCHRONOUS VALUES OF GAMES
BAD HONNEF

VERN I. PAULSEN

A cooperative two-person game is called *synchronous* if the input(question) sets and output(answer) sets are the same for both players and among the rules of the game is the rule that whenever both players receive the same input, then they must reply with the same output. A good example of such a game is the *graph k-coloring game*, where the inputs are the vertices of a graph, the outputs are a set of k colors and the rules are that whenever the players receive a pair of vertices that are an edge, then they must reply with distinct colors and whenever they both receive the same vertex, they must reply with the same color.

A probabilistic strategy for such a game is represented by a conditional probability density, where p(a,b—x,y), is the probability that if the players receive the inputs (x,y), then they reply with outputs (a,b). Such a density is *synchronous* provided that p(a,b—x,x) =0 whenever $a \neq b$.

There has been a great deal of work on finding quantum-assisted values of games. In this talk we first motivate why we believe that for synchronous games, finding the quantum-assisted value when the densities are restricted to be synchronous is important and then present what we know about these values.

This talk is based on joint work with Bill Helton, Hamoon Mousavi, Sayed Nezhadi, Travis B. Russell, and Rupert Levene.

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Finite-size certification of randomness in semi-device-independent protocols based on photon-number constraints

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We study Quantum Random Number Generation (QRNG) in a semi-device-independent setting, where both the source and the measurements are untrusted. It is only assumed that the source emits single-mode optical systems whose photon-number distribution is constrained on average. In the practical case where the adversary is not entangled with the QRNG devices, the optimal asymptotic randomness and finite-size corrections can be analysed using trade-off functions, which relate the observed experimental correlations to the Shannon entropy [1]. In [1] it was shown how these trade-off functions can be determined via semidefinite programming for the simplest photon-number constrained scenario. Recently, this was extended in [2] to more general scenarios using non-commutative polynomial optimization.

In this work, we consider alternative trade-off functions based on Tsallis-like entropies that enhance randomness bounds in finite-size settings. We develop numerical schemes to compute these functions using outer approximations of the quantum correlations, determined using semidefinite programming relaxations for non-commutative polynomial optimization. These methods provide tighter randomness bounds and expand the toolkit for semi-device-independent QRNG protocols under realistic conditions.

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Certification of entangled measurements

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Certifying quantum properties in networks with uncharacterized devices is relevant both to elucidate foundational properties of quantum theory, and to enable applications that involve such properties. Much attention has been devoted to certification of entangled states as a source of correlations, and comprehensive tests for it have been developed in the literature. A counterpart of it—entangled measurements—has obtained only some limited results. In this work, we propose a hierarchy of moment matrices similar to that of Moroder et al. [1] that could allow for the test of entangled measurements in more general networks. We argue that there is a possible duality between tests for states and tests for measurements, in a way that tests for one could be mapped to the other.

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The mystery of linear system games over Z_p

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Finding the quantum value of a nonlocal game is an example of a noncommutative polynomial optimization problem. The MIP*=RE theorem tells us that it's undecidable to compute (or even approximate) the quantum value in general, but it's not hard for every instance of a nonlocal game. To study which instances are hard and which are easier, we can look at constraint system games. A pivotal class among constraint system games are the linear system games. We understand the theory of boolean linear system games fairly well, although we still don't know if it's undecidable to approximate the quantum value of such games (showing that it's undecidable would prove the existence of a non-hyperlinear group, a major open problem). However, over Z_p, there are many more basic problems that we still don't understand --- it's even been hard to find examples of such games which have perfect quantum strategies but no perfect classical strategies. In this talk, I'll describe work with Luming Zhang showing that games with perfect commuting operator strategies but no perfect classical strategies exist, and give an overview of the many remaining open problems for linear system games over Z_p.

Solving Semidefinite Programs using Alternating Direction Methods of Multipliers

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Semidefinite Programming (SDP) is an extension of Linear Programming (LP). A matrix-variable is optimized over the intersection of the cone of positive semidefinite matrices with an affine space. It turns out, that SDP can provide significantly stronger practical results than LP and that it can be applied in a lot of different areas, like combinatorial optimization, control theory, engineering, or polynomial optimization.

Given its wide range of applications, solving SDPs has become a widely studied topic. Interior point methods are the most popular algorithms for solving SDPs. However, for large-scale SDPs interior point methods are impractical due to either the large number of constraints or the size of the matrices.

In this talk, we will present alternative approaches for obtaining approximate solutions to SDPs in reasonable time and using affordable memory requirements. These methods are based on the augmented Lagrangian algorithm and are particularly effective when the set of variables can be naturally split into two (or more) groups, enabling efficient optimization over each group. When the underlying SDP lacks an interior, developing such algorithms requires the application of facial reduction techniques. We will illustrate these methods using examples of SDPs arsing from problems in combinatorial optimization.

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Quantum field theory can be more contextual than non-relativistic quantum theory

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Quantum theory allows for correlations between spacelike separated experiments that go beyond the set of local realist correlations of classical physics. However, since the resolution of Tsirelson's conjecture we know that the set of quantum commuting correlations---that is identified by the mathematical framework of algebraic quantum field theory (AQFT)---is strictly larger than the (closure of the) set of quantum spatial correlations---that are allowed in non-relativistic quantum theory (NRQT). An experiment observing correlations beyond the quantum spatial set would greatly impact our understanding of nature. Currently, these correlations are difficult to identify and it is not known whether any have a physical realisation. Both the theoretical and technological challenges to deciding these problems are great. In this presentation I will consider the sets of correlations identified by the AQFT and NRQT formalisms for a different type of experiment: prepare-and-measure experiments known as (generalised) contextuality scenarios. One may initially expect that the relative simplicity of a prepare-and-measure experiment would not allow us to probe the distinction between AQFT and NRQT. Nonetheless, using the non-local case [1] and a result about steering in C*-algebras [2,3], I will demonstrate that the set of potential AQFT correlations for contextuality scenarios is strictly larger than that of NRQFT. The simplicity of contextuality scenarios comes at the cost of requiring more assumptions about the experiment. However, this simplified setting may provide a more feasible testing ground for probing the difference in the predictions of AQFTs and NRQT both theoretically and experimentally.

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Abstracts of Posters

(in alphabetical order)

Quantum optimal control via polynomial optimization

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Quantum optimal control plays a crucial role in the development of quantum technologies. By optimizing the shape of a control pulse, we can prepare quantum states needed to initialize algorithms in a quantum computer and implement unitary operations on the system. However, most currently used optimization methods rely on gradient-based techniques, which are inherently non-convex and can lead to complex landscapes where they may get stuck in local minima. We propose QCPOP, a new approach that reformulates quantum optimal control as a polynomial optimization problem. This allows us to apply standard polynomial optimization methods to find global solutions more effectively.

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Optimal demonstration of generalized quantum contextuality

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The notion of general quantum contextuality encompasses preparation as well as measurement contextuality. Our methodology proposes a generalized noncontextual polytope that maintains constant dimension despite variations in measurements and outcomes, ensuring a consistent approach to noncontextual polytope construction. Our constructed polytope's facet inequalities, serve as necessary conditions for generalized noncontextuality, can be obtained computationally efficiently. We illustrate the efficacy of our methodology through several distinct contextuality scenarios involving up to six preparations and three measurements, obtaining the maximum quantum violations of our derived noncontextuality inequalities. Our investigation uncovers many novel non-trivial noncontextuality inequalities and reveals intriguing aspects and applications of quantum contextual correlations.

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Schwarz qubit maps with diagonal unitary and orthogonal symmetries

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I shall discuss Schwarz maps in this presentation and analyze the relationship between positivity, operator Schwarz inequality and complete positivity of a linear unital map. Next I shall discuss complete characterization of a class of unital qubit Schwarz maps with diagonal unitary and orthogonal symmetries. Such maps have already found a lot of applications in quantum information theory. M. D. Choi provided the first example of a Schwarz map which is not completely positive and it falls in this class. As a case study I shall also analyze the Schwarz property of Pauli maps.

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Rates for Device Independent Randomness Expansion Protocols Based on Two Input Two Output Bell Test

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We introduce a general family of Device Independent Randomness Expansion (DIRE) protocols [1] based on the simplest Bell scenario. This family allows recycling input randomness [2], permits choosing different input probability distributions, and considers a class of (multiple) Bell inequalities. We also provide a numerical technique to compute tight bounds on the randomness rates for our protocols, as existing techniques [3] are not tractable for our case. Using our techniques enables orders of magnitude improvements in the finite-round regime, from the same experimental setup, thus significantly enhancing the practicality of DIRE protocols.

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Identical particles as a genuine non-local resource

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All particles of the same type are indistinguishable, according to a fundamental quantum principle. This entails a description of many-particle states using symmetrised or anti-symmetrised wave functions, which turn out to be formally entangled. However, the measurement of individual particles is hampered by a mode description in the second-quantised theory that masks this entanglement. Is it nonetheless possible to use such states as a resource in Bell-type experiments? More specifically, which states of identical particles can demonstrate non-local correlations in passive linear optical setups that are considered a purely classical component of the experiment? Here, the problem is fully solved for multi-particle states with a definite number of identical particles. We show that *all* fermion states and *most* boson states provide a sufficient quantum resource to exhibit non-locality in classical optical setups. The only exception is a special class of boson states that are reducible to a single mode, which turns out to be locally simulable for any passive linear optical experiment. This finding highlights the connection between the basic concept of particle indistinguishability and Bell non-locality, which can be observed by classical means for almost every state of identical particles.

Reference

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Parallel ergotropy: Maximum work extraction via parallel local unitary operations

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(Dated: February 4, 2025)

Maximum quantum work extraction is generally defined in terms of the *ergotropy* functional, no matter how experimentally complicated is the implementation of the optimal unitary allowing for it, especially in the case of multipartite systems. In this framework, we consider a quantum battery made up of many interacting sub-systems and study the maximum extractable work via concurrent local unitary operations on each subsystem. We call the resulting functional *parallel ergotropy*. Focusing on the bipartite case, we first observe that parallel ergotropy outperforms work extraction via *egoistic* strategies, in which the first agent A extracts locally on its part the maximum available work and the second agent B, subsequently, extracts what is left on the other part. For the agents, this showcases the need of cooperating for an overall benefit. Secondly, from the informational point of view, we observe that the parallel capacity of a state can detect entanglement and compare it with the statistical entanglement witness that exploits fluctuations of stochastic work extraction. Additionally, we face the technical problem of computing parallel ergotropy. We derive analytical upper bounds for specific classes of states and Hamiltonians and provide recipes to obtain numerical upper bounds via semi-definite programming in the generic case. Finally, extending the concept of parallel ergotropy, we demonstrate that system's free-time evolution and application of local unitaries allow one to saturate the gap with the ergotropy of the whole system.

Bounding the minimal average communication cost of nonlocal correlations

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Bell nonlocality is a fundamental property that cannot be replicated by local hidden variable (LHV) models. However, allowing communication between parties enables LHV models to reproduce these correlations, making communication cost a nonlocality measure. In this work, we introduce more computationally feasible methods to lower bound the minimal average communication cost (MACC) for 2-partite Bell scenarios, including considering only the communication strategies with lesser communication cost and considering certain Bell inequality. More precisely, we show that for the former case, there is a hierarchy of lower bounds of MACC by considering different communication strategies with different levels of communication cost. Further, the exact MACC can be found without the full knowledge of all the communication strategies. On the other hand, for the latter one, we find out that our lower bound of MACC up to 1-bit based on a Bell inequality can achieve a higher value compared to the existing lower bound from the method by Pironio [1]. Additionally, we explore the relationship between nonlocality robustness (NR) and MACC and provide conjectured upper bound and lower bound of MACC based on NR. In the end, we provide the largest MACC lower bounds for quantum correlations we found in several Bell scenarios.

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When can you trade causal order for locality?

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We investigate the extent of the conceptual link between indefinite causal order (namely, process functions) and the phenomenon of quantum nonlocality without entanglement (QNLWE) that was uncovered in R. Kunjwal and Ä. Baumeler, Phys. Rev. Lett. 131, 120201 (2023). Specifically, we study the set of separable operations achievable via local operations and process functions (LOPFs). It is known that although some separable operations outside the set of local operations and classical communication (LOCC) can be achieved via LOPF, this is not the case for all of them. Our investigation therefore aims to uncover the conditions under which a separable operation falls in the set LOPF\LOCC, i.e., the conditions under which it is possible to trade causal order for locality.

An operational approach to classifying measurement incompatibility

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Measurement incompatibility has proved to be an important resource for information processing. In this work, we present an operational approach that leverages classical operations on the inputs and outputs of measurement devices to explore different layers of incompatibility among the measurements performed by the device. We study classifications of measurement incompatibility with respect to two types of classical operations, *viz.*, coarse-graining of measurement outcomes and disjoint-convex-mixing of different measurements. We derive analytical criteria for determining when a set of projective measurements is fully incompatible with respect to coarse-graining or disjoint-convex-mixing. Robustness against white noise for different layers of incompatibility for mutually unbiased bases is investigated. Furthermore, we study operational witnesses for incompatibility subject to these classical operations, using the input-output statistics of Bell-type experiments as well as for experiments in the prepare-and-measure scenario.

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Deformations of the symmetric subspace of qubit chains

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The symmetric subspace of multiqubit systems, that is the space of states invariant under permutations, is commonly encountered in applications in the context of quantum information and communication theory.

It is known that the symmetric subspace can be described in terms of irreducible representations of the group $SU(2)$, whose representation spaces form a basis of symmetric states, the so-called Dicke states. In this poster I will present deformations of the symmetric subspace as deformations of this group structure, which will be promoted to a quantum group $\mathcal{U}_q(\mathfrak{su}(2))$[1].

We will see that deformations of the symmetric subspace obtained in this manner correspond to local deformations of the inner product of each qubit, in such a way that departure from symmetry can be encoded in a position-dependent inner product.

Consequences and possible extensions of this result will also be discussed.

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A Quantum Mobius Strip?

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This study investigates the mathematical existence and dynamic behavior of a quantum Möbius strip within the framework of non-commutative geometry. By formulating a Hamiltonian as a perturbation of a model for a two-level atom driven by strong classical electric fields and interacting with a reservoir of double-mode bosons, we show that the classical Möbius strip's parametric equations are recovered from the quasi-coherent states of the system. Numerical simulations reveal that, only in an adiabatic regime characterized by low parametric velocities, a state initialized as a quasi-coherent state remains correlated (up to a phase factor) with its instantaneous configuration. Unexpectedly, the spin structure of these states remains invariant regardless of the trajectory's velocity, suggesting the presence of hidden symmetries not readily apparent from theory. These results possibly imply the presence of symmetries in the system that are not easily predicted by theory and motivate further investigation into symmetry-protected features in complex quantum systems.

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Communication Complexity Bounds using Information Causality

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In a distributed computing scenario, two parties (say Alice and Bob) aim to compute a given function with as minimum communication as possible. The communication cost or the complexity depends not only on the function itself but the shared resources to which both parties have access to such as public randomness or entangled Bell pairs. In this work we aim to study communication complexity in theories satisfying the information causality principle. The principle essentially states that the information potentially available to Bob about Alice's data cannot be higher than the amount of information Alice sends to Bob. We formulate an extension of the information causality principle which is valid for any distributed computation scenario and apply it to several well-known functions. We show a reduction for some of these problems to known functions and hence derive one-way communication complexity bounds in a theory independent manner. Finally, we prove that the information causality principle is stronger than the principle of non-trivial communication complexity.

Classifying Causal Structures: Ascertaining when Classical Correlations are Constrained by Inequalities

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The classical causal relations between a set of variables, some observed and some latent, can induce both equality constraints (typically conditional independencies) as well as inequality constraints (Instrumental and Bell inequalities being prototypical examples) on their compatible distribution over the observed variables. Enumerating a causal structure's implied inequality constraints is generally far more difficult than enumerating its equalities. Furthermore, only inequality constraints ever admit violation by quantum correlations. For both those reasons, it is important to classify causal scenarios into those which impose inequality constraints versus those which do not. Here we develop methods for detecting such scenarios by appealing to d-separation, e-separation, and incompatible supports. Many (perhaps all?) scenarios with exclusively equality constraints can be detected via a condition articulated by Henson, Lal and Pusey (HLP). Considering all scenarios with up to 4 observed variables, which number in the thousands, we are able to resolve all but three causal scenarios, providing evidence that the HLP condition is, in fact, exhaustive.

Device independent bounds for the conditional von-Neumann entropy

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Recent developments in quantum cryptography—especially in device-independent quantum key distribution (DIQKD) and randomness extraction—leverage the inherent unpredictability of quantum systems to protect communication from eavesdroppers. DIQKD protocols are based on the observation that correlations in measurement outcomes of entangled quantum systems cannot be explained by any local hidden variable theory, as evidenced by violations of Bell inequalities. Rather than relying on classical models, these correlations highlight the fundamentally non-local nature of quantum mechanics, thereby ensuring genuine randomness. A key advantage of DIQKD is its minimal reliance on the measurement devices themselves, requiring no assumptions about their inner workings.

Within this framework, accurately bounding entropy under non-asymptotic conditions emerges as a primary challenge. Noise and finite sample sizes in real-world experiments necessitate precise, non-asymptotic entropy bounds to quantify the randomness of measurement outcomes reliably. Such bounds are essential for assessing the security of device-independent protocols, which assume only the observed distribution and the validity of quantum theory.

In this work [1], we contribute to the field of device-independent randomness extraction by investigating how much usable randomness can be harvested from a given experiment. We propose a novel numerical method that provides provable lower bounds on the extractable randomness, relying solely on Alice and Bob's measurement outcomes and the premise that quantum theory governs the experiment. The relevance of such tools has grown with recent advancements in entropy accumulation theorems, which offer concrete solutions for randomness extraction in finite-size regimes by reducing it to calculating the conditional von Neumann entropy. Hence, after applying an entropy accumulation theorem, the remaining task is to establish provable lower bounds on this conditional von Neumann entropy for specific noise models and input-output data scenarios. By restricting the analysis to projective operators, our method achieves a high level of computational efficiency, making it suitable for practical implementations.

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Sums-of-squares techniques for classical XOR games

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Motivation Bipartite XOR games are a class of games important in quantum nonlocality, quantum cryptography and theoretical computer science. In these games, a verifier poses questions $x, y \in \mathcal{X} \times \mathcal{Y}$ drawn from a probability distribution $p(x, y)$ to two parties, Alice and Bob, respectively. Communication between parties is usually forbidden, or at least bounded. We consider the cases where communication is forbidden, but either classical or quantum correlations are available. Alice and Bob output one bit each $(a(x), b(y))$, which they send to the verifier. The verifier adds the two bits and compares the value $a \oplus b$ to a previously agreed-upon winning condition $f(x, y)$. If the clause is satisfied, then Alice and Bob win the game.

In contrast to other nonlocal games, the optimal quantum winning probability ω_q can be determined with a semidefinite program with $|\mathcal{X}|+|\mathcal{Y}|$ variables. No such simplification occurs in the classical case, which is NP-hard [1] to compute for general XOR games, with few analytical methods. This disparity is explained by the fact that every quantum XOR game can be associated to a non-commutative sum-of-squares (SOS) polynomial. In the classical case, we must rely on SOS hierarchies, which remain a valid tool since they are applicable to any polynomial optimization problem. We discuss simplifications facilitated by the specific polynomial structure of XOR games, which can be mapped to bilinear polynomials $\sum_{xy} B_{xy} s_x t_y$ on the hypercube defined by the equations $s_x^2 = 1$ $(\forall x)$, $t_y^2 = 1$ $(\forall y)$. We analyze the SOS decomposition of an XOR game by considering its SOS matrix as equivalent to an enlarged symmetric XOR game, which satisfies a simple geometric condition from [2]. This condition ensures that no quantum advantage exists in the enlarged game. On the other hand, when communication constraints are reintroduced in the larger game, it simplifies to the original game.

We discuss a possible connection between these observations and the values of games defined by the tensor product of bias matrices. Furthermore, we present group-theoretical methods for XOR games of the form $B = B_0^{\otimes n}$. These methods are capable of simultaneously utilizing symmetries of a single copy, and the overall permutation symmetry between copies, discussed here on the example of the CHSH game.

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The Random Schrödinger Equation and Geometric Quantum Control

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We study the random Schrödinger equation, with a noise term given by a random Hermitian matrix. We derive bounds on the error of the synthesised unitary in terms of bounds on the norm of the noise, and show that for certain noise processes these bounds are tight. We then show that for certain control-noise couplings, minimising the error is equivalent to finding a geodesic on $SU(2^n)$ with respect to a Riemannian metric encoding the coupling between the control pulse and the noise process, connecting our work to the work of Nielsen et al. [1-3] on the geometry of quantum computation.

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Tsirelson bounds for quantum correlations with indefinite causal order

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Quantum theory is in principle compatible with processes that violate causal inequalities, an analogue of Bell inequalities that constrain the correlations observed by a set of parties operating in a definite causal order. Since the introduction of causal inequalities, determining their maximum quantum violation, analogue to Tsirelson's bound for Bell inequalities, has remained an open problem. Here we provide a general method for bounding the violation of arbitrary causal inequalities, establishing limits on the correlations achievable by arbitrary local experiments and by arbitrary quantum processes with indefinite causal order. We prove that the maximum violation is generally smaller than the algebraic maximum, and determine Tsirelson-like bounds for a class of causal inequalities including some of the most paradigmatic examples. Surprisingly, we find that the algebraic maximum of causal inequalities in the simplest scenario can be achieved by a new type of processes that allow for information to flow in an indefinite temporal direction within the parties' laboratories.

Overall, our findings open up a search for physical principles determining the boundaries of the set of quantum correlations with indefinite causal order, and a search for applications of causal inequalities to quantum cryptography and other quantum technologies.

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Lieb-Robinson bounds and non-commutative polynomial optimization

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The interplay between locality of interactions and causality in quantum mechanics is subtle. It is well-established that subsystems defined by tensor products, whose interactions are governed by continuous-time Hamiltonian evolution, cannot exhibit perfectly sharp lightcones. Instead, their maximal causality violation is constrained by what are known as Lieb-Robinson bounds. In this work, we reformulate their computation as a relaxation of a differential non-commutative optimization problem, as recently introduced by Araujo et al [1]. Additionally, we investigate the connection between these bounds and "quantum mereology"[2], the study of how quantum systems decompose into tensor product structures [3]. Our results suggest a profound link between approximate causality and the locality of interactions, framed within the context of polynomial optimization problems.

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Numerical quantum energy bounds in models with self-interaction

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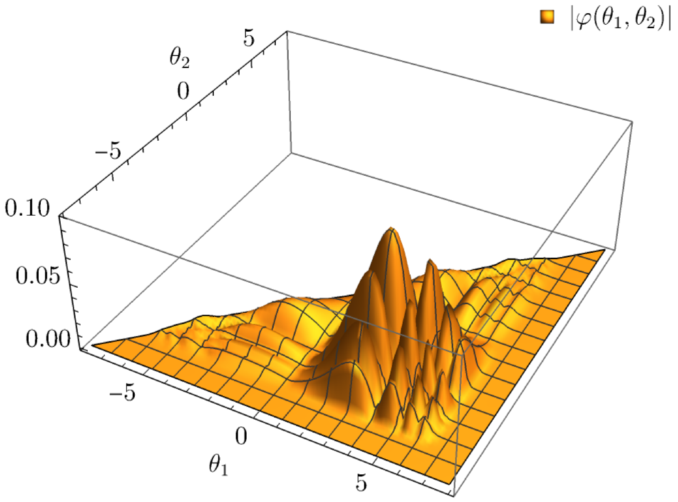
While quantum matter allows energy densities which are locally negative, i.e., below the ground state energy, its amount and duration obeys stringent constraints in physically reasonable models. In quantum field theory this can be captured by so-called quantum (weak) energy inequalities (QEIs), lower bounds of the smeared quantum-stress-energy tensor. QEIs could be proven in many free quantum field theory (QFT) models on both flat and curved spacetimes.

In interacting theories only few results exist, so that numerical investigations are favourable. We are here presenting numerical results on QEIs in interacting integrable QFT models in 1+1 dimension, in particular the O(N)-nonlinear-sigma and sinh-Gordon model at 1- and 2-particle level. In view of runtime complexity an extension of this method to higher-particle levels is highly limited and we advocate further research in this topic as a challenging research problem in operator-theoretic optimization.



Example of negative energy state at two-particle level

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Enhancing robustness of self-testing of quantum resources

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We introduce a novel framework for robust self-testing of various classes of quantum resources. Unlike previous approaches [1] that derive robustness bounds by explicitly constructing local isometries, our method optimizes over them, improving the self-testing statements. Specifically, the optimal robustness bounds are computed using a hierarchy of semidefinite programming relaxations [2]. We show that the method can be universally applied to different self-testing scenarios. We consider self-testing of steerable quantum assemblages, dimension-bounded self-testing of quantum states in a prepare-and-measure scenario, and one-sided device-independent self-testing of entangled quantum states. In all the instances mentioned above, our method improves upon previously reported bounds and, moreover, we show that the bounds obtained are optimal.

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Interventional inequalities in minimal quantum causal networks

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It is known that quantum correlations, the ones exhibited between two or more quantum systems, cannot in general be explained in classical terms. Previous research has shown that the causal modeling framework offers a powerful set of tools for analying nonclassicality, and extending this concept to general networks possibly involving independent sources of correlations and communication between parties [5]. This framework not only ensures robust security in information-processing tasks and cryptographic protocols but also serves as a versatile tool for certification tasks requiring minimal assumptions. Interventions are an essential tool in causal inference, which, unlike passive observations, involve locally changing the causal structure of an experiment, erasing all external influences that a given variable might have and putting it under the exclusive control of an observer [3]. In particular, through interventions, one can reveal the quantum behavior of a system that may appear classical at the observational level [4]. To illustrate this technique, I will present its application in the case of the Uncorrelated Confouders (UC) scenario, modeling a network with two independent sources, three observers and no inputs, with very weak causal assumptions. Remarkably, in the UC scenario, we were able to demonstrate the existence of a nonclassical quantum distribution, making this one of the simplest structure without external inputs where nonclassicality can be observed [1, 2]. The detection of this nonclassicality is possible using nonconvex inequalities, which can involve either observational only or a combination of interventional and observational data together. While the former case requires less measurements and assumptions, the noise robustness of the latter makes it much more likely to be used in experimental tests.

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Quantum advantage in q-$d$it communication

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The benefits of quantum communication over classical communication can be seen in terms of enhanced security and the scalability of the carrier system. In this contribution we focus on the latter, which is gathering increasing theoretical [1, 2] and experimental [1, 3, 4] attention.

We present a communication protocol between Alice and Bob where correlated events occur with zero probability, while anticorrelated events occur with maximal probability. We will demonstrate that this protocol can be realized using a quantum system of operational dimension $\mathcal{D}_Q = d$, whereas a classical implementation would require at least dimension $\mathcal{D}_C \geq 2d$. This research is hybrid in nature: bounds on the classical dimension $\mathcal{D}_C$ are established through polynomial optimization [5]; and the quantum dimension $\mathcal{D}_Q$ is determined by an explicit setup that is currently under experimental preparation with orbital angular momentum.

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Multipartite Entanglement in the Symmetric Subspace

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Our work focuses on the separability problem; i.e., determining whether a given composite state is entangled or not. This problem has attracted significant attention not only due to its intrinsic mathematical elegance but also because of its critical implications for potential quantum technologies. The separability problem is an extremely challenging problem which is known to be NP-Hard. The problem is only solved for bipartite cases in low dimensions; two qubits or one qubit-one qutrit, by means of the partial transposition criterion. In higher dimensions most criteria are necessary but not sufficient conditions rendering the problem of separability unsolved. Here we show that for symmetric states, separability is mathematically linked to the theory of copositive matrices.
We present a novel approach, dubbed pseudo-convex combination, which provides a constructive analytical procedure to solve the separability problem in the symmetric case of two qutrits and associates bound entangled states to exceptional copositive matrices.

Finite-Dimensional Relaxations for Cone Programs on C∗-Algebras: A Unified Perspective

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Many optimization problems in quantum information theory, such as determining achievable probability distributions in experiments or tackling ground state problems, can be framed as cone programs on C∗-algebras. These problems are inherently complex due to the infinite-dimensional nature of the associated algebras. We investigate a construction for finite-dimensional relaxations of such cone programs, providing outer bounds through positive maps and basic linear algebra techniques. This approach unifies well-known hierarchies, such as the NPA hierarchy, Lasserre's hierarchy, and symmetry reductions in semidefinite programming (SDP).

By leveraging the structure of positivity on C∗-algebras, this framework provides a systematic method for approximating solutions to complex optimization problems mathematically robust method for addressing optimization tasks, including those over infinite-dimensional spaces.

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PRODUCTS OF NONLOCAL GAMES AND PARALLEL REPETITION

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Abstract. Given a nonlocal game G, its n-fold parallel repetition is the game in which n copies of G are played simultaneously (i.e. in one round) and the provers win if and only if they win each of these copies.

The main question is how the value of the n-fold repeated game relates to the value of the base game, G. A celebrated result by Ran Raz states that classically, the value decays exponentially with n. In the quantum case, the best known decay rate for a general game (due to Yuen, '16) is polynomial. It is a long-standing open problem whether this can be promoted to exponential decay. An affirmative solution to this is expected since exponential decay has been shown for many classes of games.

We investigate the relation between the self-testing properties of a game and the behavior of its value under parallel repetition. In particular, we show that many games (both well-known ones and new ones) have the property that they are self-tests and with some number of repetitions, N, the quantum value coincides with either: (1) the classical value of the N-fold repeated game or (2) the quantum value of the base game to the power N (called perfect parallel repetition). Whenever (1) is the case we say that parallel repetition annihilates the qc-gap of the game. The values of the repeated games are obtained using the NPA hierarchy with symmetrization techniques.

An example is provided where the quantum value of the doubly repeated game exceeds both the classical value of the doubly repeated game and the quantum value of the base game squared. We will, however, also show that this specific game cannot possibly be a self-test.

This work naturally leaves open the question whether every self-testing game either obeys perfect parallel repetition or is gap-annihilated by parallel repetition.

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Distributed computing and its links to quantum information theory and the NPA hierarchy

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Distributed computing involves the use of multiple computers, interconnected through a network, to collaboratively solve a shared problem. In our model, these computers are organized according to the graph structure of the network, performing arbitrarily powerful local computations and exchanging messages through channels of unlimited capacity with their neighbors in a synchronized manner. A common task in distributed computing is, for instance, graph coloring. Here, each node outputs a color after T communication rounds such that no two neighboring nodes share the same color. Unlike centralized computing, the complexity of distributed problems arises from the nodes' lack of knowledge about the network topology and the need to coordinate with other nodes that are executing the exact same algorithm.

This raises the question: To what extent can quantum computers and the exchange of quantum information via entangled states reduce the time steps T required to solve a given task? Currently, this problem is approached indirectly. Upper bounds on T for quantum algorithms are determined by identifying effective algorithms, while lower bounds rely on no-signaling arguments. The best quantum algorithm is then compared to the best-known classical algorithm and the general no-signaling bound. Although this method is effective for specific problems, it often results in significant gaps for others.

In this work, we explore the potential applications of a to be developed extended version of the NPA hierarchy to distributed computing tasks. While the NPA hierarchy can already be extended to network nonlocality scenarios through the inflation technique, we discuss here what is conceptually needed to also incooperate the communication that takes place in a distributed computing algorithm. Furthermore, we discuss the role of symmetries to enforce that each computer follows the same algorithm. Finally, we give explicit examples of problems that could be tackled via this method.

Characterizing high-dimensional quantum contextuality

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As a phenomenon encompassing measurement incompatibility and Bell nonlocality, quantum contextuality is not only central to our understanding of quantum mechanics, but also an essential resource in many quantum information processing tasks. The dimension-dependent feature of quantum contextuality is known ever since its discovery, but there is still a lack of systematic methods for characterizing this fundamental feature. In this work, we propose a reliable method for certifying the high-dimensional advantages of quantum contextuality. Our work gives a complete characterization of the dimension-constrained quantum contextual behavior, and the nonconvex structure of this set can be fully explored. When applied, our method can be used for dimensionality certification of quantum information processing systems, and also for concentrating the quantum contextual behavior into lower-dimensional systems.

Lifting assumptions for robust self-testing

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In recent years, a lot of effort has been devoted to studying synchronous correlations, in part due to Tsirelson's problem. This has led to some very nice robust self-testing results for perfect non-local games. However, these results only apply to symmetric projective maximally entangled (PME) strategies, which we call PME-robust self-testing, as this is what is needed for synchronous correlations. In physical applications, only robust self-testing that holds for all strategies is useful. We show that a perfect non-local game that is a PME-robust self-test is a general robust self-test. This proof involves the spectral gap of the game operator, which turns out to be bounded away from zero by the PME-robustness of the self-test. We then apply this result to the quantum low degree test to obtain the most efficient robust self-testing result currently known for the n-qubit test.

Quantum strategies for rendezvous and domination tasks on graphs with mobile agents

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This work explores the application of quantum non-locality, a renowned and unique phenomenon acknowledged as a valuable resource. Focusing on a novel application, we demonstrate its quantum advantage for mobile agents engaged in specific distributed tasks without communication. The research addresses the significant challenge of rendezvous on graphs and introduces a new distributed task for mobile agents grounded in the graph domination problem. Through an investigation across various graph scenarios, we showcase the quantum advantage. Additionally, we scrutinize deterministic strategies, highlighting their comparatively lower efficiency compared to quantum strategies. The work concludes with a numerical analysis, providing further insights into our findings, through the use of convex optimization methods.

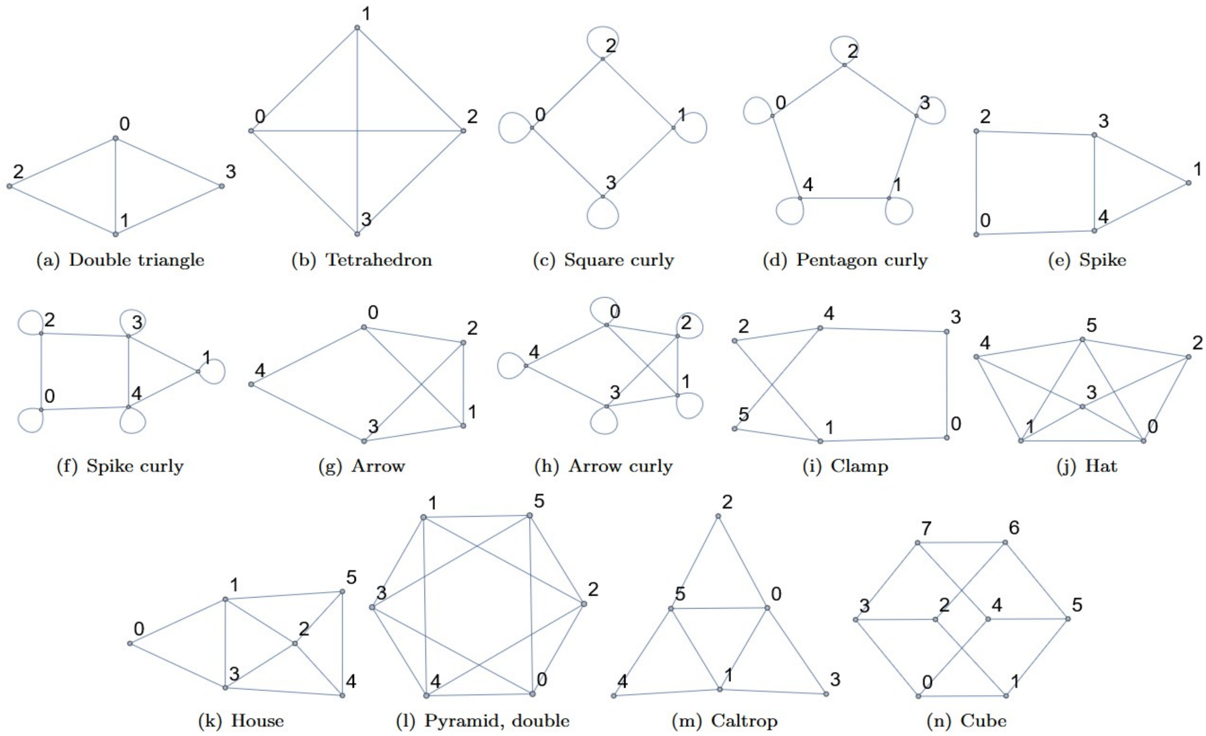

Figure: some of the graphs analysed in this work.

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Partial interventions in the triangle quantum network

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The study of quantum correlations in networks offers profound insights into both the foundational principles and practical applications of quantum mechanics. While tools like the NPA hierarchy have proven invaluable in the Bell scenario, their effectiveness is significantly limited in the context of nonconvex network structures. In this work, we introduce partial interventions, a novel causal modeling tool that enables the manipulation of latent nodes within quantum networks. Unlike classical interventions, which are restricted to observed nodes, partial interventions exploit the manipulability of quantum states to uncover properties that remain inaccessible to standard methods. We showcase the power of partial interventions through the triangle scenario, revealing previously hidden non-classical correlations and enhancing the detection of known incompatible quantum distributions. These results underscore the potential of causal inference methods as powerful tools for advancing the study of quantum correlations in complex quantum networks.

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Tsirelson's Precession Protocol: A Theory-independent Bound Saturated by Quantum Mechanics, and Other Generalisations

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The time evolution of uniformly-precessing variables $(X(t), Y(t))$—like the position and momentum of the harmonic oscillator, or components of a vector rotating with a fixed angular momentum—is the same in both classical and quantum theory: they evolve as $X(t) = \cos(2\pi t/T)X(0) + \sin(2\pi t/T)Y(0)$. In an overlooked preprint, Tsirelson found that the likelihood P_3 that a uniformly-precessing variable is positive at three equally-spaced times

$$P_3 := \frac{1}{3}\sum_{k=0}^{2}\left(\Pr\left[X(kT/3) > 0\right] + \frac{1}{2}\Pr\left[X(kT/3) = 0\right]\right) \tag{1}$$

satisfies $P_3 \leq 2/3$ for classical systems [1]. This "*Tsirelson inequality*" can be understood as a facet in a constrained conditional probability polytope [2], from which inequalities for generalisations of the original protocol can also be derived.

The Tsirelson inequality is violated by quantum systems: $P_3 \approx 0.709$ for the quantum harmonic oscillator and $P_3 = 0.75$ for spin systems [3]. Violations of Tsirelson inequalities have also been shown to certify Wigner negativity volume [3], non-Gaussian entanglement between coupled oscillators [4], and genuine multipartite entanglement of spin ensembles [5].

There remained several related open questions: *What is the maximum possible violation of the original protocol (or its generalisations) for a specific quantum system (or quantum theory in general)*? Our contributions answer one open question, and partially answer another.

Our first contribution is to derive a theory-independent bound for the original precession protocol for systems with a finitely many outcomes [6]. The derivation relies only on the linearity of the expectation value with respect to the observables. We then prove by construction that this general bound is saturated by quantum mechanics. This means that no general theory can outperform quantum theory in the precession protocol, and thus provides a simple test that can experimentally falsify general theories that do not also saturate this bound.

Our second contribution is to characterise all generalisations of the precession protocol with three probing times [7]. For the quantum harmonic oscillator, we prove that a violation for every such protocol is related to a violation of the original precession protocol, and provide improved upper bounds for the maximum possible quantum violation for harmonic oscillators.

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Robust self-testing for games with stable algebras

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A nonlocal game is a self-test if it has a unique optimal quantum strategy. In essence, self-testing allows us to infer the exact quantum state and measurements solely from the observed correlations. However, due to the presence of noise, empirical data typically leads to near-optimal values. The robustness of a self-test asks whether every near-optimal strategy is close to the ideal optimal one. For many self-tests, this robustness can also be formulated as a stability property of the underlying game algebra

In this talk, I will introduce an operator-algebraic formulation of robust self-testing in terms of states on C^*-algebras. Many nonlocal games of interest, including XOR games and synchronous games, have a ”nice” game algebra in the sense that optimal strategies correspond to tracial states on the game algebra. For these nonlocal games, I will show how robust self-testing is related to the uniqueness of tracial states on the game algebras. I will also present a generalized Gowers-Hatami theorem that is applicable to C*-algebras and discuss how it derives the quantitative robustness of many self-tests.