

Non-Hermitian and Topological Photonics

836. WE-Heraeus-Seminar

**15 Jun - 20 Jun 2025
at the Physikzentrum Bad Honnef/Germany**

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Introduction

The Wilhelm und Else Heraeus-Stiftung is a private foundation that supports research and education in science with an emphasis on physics. It is recognized as Germany's most important private institution funding physics. Some of the activities of the foundation are carried out in close cooperation with the German Physical Society (Deutsche Physikalische Gesellschaft). For detailed information see <https://www.we-heraeus-stiftung.de>

Aims and scope of the 836. WE-Heraeus-Seminar:

In recent years new concepts for controlling the flow of light have emerged. Perhaps the most important among these concepts are non-Hermitian optics (seeking to utilize gain and loss to control light properties) and topological photonics that rely on global invariant features of the systems. While the former grew out of interest in the mathematical notion of parity-time reversal symmetry and its associated exotic effects such as the presence of exceptional points, the latter was inspired by the Nobel prize work on topological material in condensed matter physics. Very recently, the marriage between these two concepts has been proposed and shown to have far reaching consequences both at the fundamental and application levels.

The aim of this seminar is to bring together a highly diverse group of experts in experimental and theoretical non-Hermitian topological photonics and microwave systems as well as experts in open quantum systems to present their recent results and discuss the important open problems in the field; and in doing so provide a roadmap for further development in the area of non-Hermitian topological quantum systems. Potential participants range from world-leading scientists in different fields of non-Hermitian optics and topological photonics to young scientists who are interested in these interdisciplinary research fields. We expect every participant to contribute to this seminar by either giving a talk or by presenting a poster.

Scientific Organizers:

Prof. Dr. Jan Wiersig	Otto-von-Guericke-Universität Magdeburg, Germany E-Mail: jan.wiersig@ovgu.de
Prof. Ramy El-Ganainy	Saint Louis University, USA E-Mail: ramy.elganainy@slu.edu
Prof. Sahin Ozdemir	Saint Louis University, USA E-Mail: sahin.ozdemir@slu.edu

Introduction

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Registration:

Martina Albert (WE Heraeus Foundation)
at the Physikzentrum, reception office
Sunday (17:00 h – 21:00 h) and Monday
morning

Program

Program

Sunday, June 15, 2025

17:00 – 21:00 Registration

18:00 *BUFFET SUPPER and informal get-together*

Program

Monday, June 16, 2025

07:30	<i>BREAKFAST</i>	
08:30 – 08:45	Jan Wiersig Ramy El-Ganainy Sahin Ozdemir	Opening and Welcome
08:45 – 09:30	Demetrios Christodoulides	Non-Hermitian systems and optical thermodynamics
09:30 – 10:15	Henning Schomerus	Unified framework for non-Hermitian response across all spectral scenarios
10:15 – 10:45	<i>COFFEE BREAK</i>	
10:45 – 11:30	Yogesh Patil	Knots, non-abelian braids, and geometric phases in non-Hermitian systems
11:30 – 11:45	Video contribution	About the Wilhelm und Else Heraeus Foundation
11:45 – 12:00	Conference Photo	
12:00	<i>LUNCH</i>	
14:00 – 14:30	Kaiwen Ji	Phase singularity in a complex Rice-Mele nanolaser array
14:30 – 15:15	Flore Kunst	Exceptional points, symmetries and similarities

Program

Monday, June 16, 2025

15:15 – 16:00 *COFFEE BREAK*

16:00 – 16:30 Cem Yuce **Nonlinear skin and subskin modes**

16:30 – 17:00 Jacob Barnett **Symmetry breaking in Non-Hermitian
bipartite lattice models**

17:00 – 18:00 **Discussion**

18:00 *DINNER*

20:00 – 21:00 **Discussion**

Program

Tuesday, June 17, 2025

07:30	<i>BREAKFAST</i>	
08:45 – 09:30	Stefan Rotter	Coherent perfect absorption and emission of light in non-Hermitian and time-varying media
09:30 – 10:15	Anja Metelmann	'Nonreciprocity in action'
10:15 – 10:45	<i>COFFEE BREAK</i>	
10:45 – 11:30	Clara Wanjura	Non-Hermitian topology and directional amplification
11:30 – 12:15	Said Rodriguez	Signatures of criticality and thermodynamic irreversibility in optical bistability
12:30	<i>LUNCH</i>	
14:00 – 15:15	Poster	
15:15 – 16:00	<i>COFFEE BREAK</i>	
16:00 – 18:00	Poster	
18:00	<i>HERAEUS DINNER at the Physikzentrum</i> <i>(cold and warm buffet, with complimentary drinks)</i>	
20:00 – 21:00	Discussion	

Program

Wednesday, June 18, 2025

07:30	<i>BREAKFAST</i>	
08:45 – 09:30	Konstantinos Makris	Exponential sensitivity based on pseudospectra scaling
09:30 – 10:15	Yogesh N. Joglekar	Non-Hermitian dynamics in bosonic systems: topological and quantum realizations
10:15 – 10:45	<i>COFFEE BREAK</i>	
10:45 – 11:30	Armando Perez Leija	Non-Hermitian quantum photonic filters
11:30 – 12:15	Sebastian Klemmt	Topological edge and corner modes in polariton lattices
12:30	<i>LUNCH</i>	
14:00 – 18:00	Excursion	
18:00	<i>DINNER</i>	
20:00 – 21:00	Poster / Discussion	

Program

Thursday, June 19, 2025

07:30	<i>BREAKFAST</i>	
09:00 – 09:45	Mercedeh Khajavikhan	Selective filtering of photonic quantum entanglement via anti-parity-time symmetry
09:45 – 10:30	Li Ge	Novel resonances and exceptional points in non-Hermitian optical systems
10:30 – 11:00	<i>COFFEE BREAK</i>	
11:00 – 11:30	Rui Su	Topological exciton polaritons in perovskite microcavities
11:30 – 12:15	Ahmed Dorrah	Structured light from the ground up: multilayer meta-optics and topological evolution in optical vortices
12:30	<i>LUNCH</i>	
14:00 – 14:30	Courtney Fleming	Modelling the onset of selective Non-Hermitian skin effects
14:30 – 15:15	Alejandro Giacomotti	A photonic crystal Rice-Mele nanolaser array: from the generation of chiral light to photonic computing applications
15:15 – 16:00	<i>COFFEE BREAK</i>	
16:00 – 16:30	Nicholas Lambert	Using controllable microwave resonators and non-Hermitian dynamics for coherent control of magnon-polaritons
16:30 – 17:00	Nicola Mayer	Enantiosensitive exceptional points in open chiral systems
17:00 – 18:00	Discussion	
18:00	<i>DINNER</i>	
20:00 – 21:00	Poster award	

Program

Friday, June 20, 2025

07:30	<i>BREAKFAST</i>	
08:45 – 09:30	Sahin Ozdemir	Non-Hermiticity as a Resource in Photonics
09:30 – 10:15	Kurt Busch	Multiphoton dynamics in tight-binding lattices
10:15 – 10:45	<i>COFFEE BREAK</i>	
10:45 – 11:15	Julius Gohsrich	A rigorous framework for topological plasmonics
11:15 – 11:45	Daniel Grom	Graph theory for the spectrum of perturbed higher-order exceptional points
12:00	<i>LUNCH</i>	

End of seminar and departure

Abstracts of Lectures

(in alphabetical order)

Symmetry Breaking in Non-Hermitian Bipartite Lattice Models

Jacob Barnett

Basque Center for Applied Mathematics, Bilbao, Spain

Non-Hermitian systems with Parity-Time (PT) symmetry exhibit a striking spectral bifurcation between two regimes: an unbroken phase characterized by entirely real spectra, and a broken phase where non-real eigenvalues emerge in complex-conjugate pairs—behavior not possible in Hermitian settings. However, given a fixed notion of P and T , PT-symmetric Hamiltonians are only a special case of the broader class of pseudo-Hermitian Hamiltonians that also exhibit this phase structure. Realizing generic pseudo-Hermitian systems experimentally remains challenging due to their algebraic complexity, often requiring long-range or fine-tuned couplings. In this talk, I introduce a physically motivated subclass of pseudo-Hermitian Hamiltonians local to bipartite lattices. Although these systems are not manifestly PT-symmetric, I derive closed-form expressions for their spectra and identify the symmetry-breaking point analytically. The clear geometric formulation of these models makes them particularly amenable to implementation in experimental platforms, including photonic and optical lattice systems, offering a new route for engineering tunable non-Hermitian phase transitions.

Title:

Multiphoton dynamics in tight-binding lattices

Author:

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Abstract:

Indistinguishable and/or entangled photons propagating in waveguide arrays (WAs) represent a promising platform whose utility ranges from research on fundamental aspects of quantum mechanics all the way to applications in quantum sensing and quantum information processing. In this talk, an overview of a number of recent developments of multiphoton physics in WAs will be provided. When such systems are combined with ideas from topological photonics and non-Hermitian photonics such as synthetic dimensions and exceptional points, several interesting device applications can be developed.

For instance, by judiciously combining multi-photon states with the idea of synthetic dimensions in WAs yields the notion of a synthetic atom and Fock graphs [1] and, in turn, this provides entirely novel perspectives on the dynamics of such multi-photon states that include applications in discrete fractional Fourier transforms [2] and high-order exceptional points for quantum sensing. [3,4]. In addition, novel theoretical approaches need to be developed.

References:

- [1] Photonics Research **8**, 1161 (2020)
- [2] Journal of the Optical Society of America B **35**, 1985 (2018)
- [3] Photonics Research **7**, 862 (2019)
- [4] Laser & Photonics Reviews **16**, 2100707 (2022)

Non-Hermitian Symmetries & Dynamics in Optics & Photonics

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In recent years, there has been a surge of activity in the broad areas of non-Hermitian and topological physics. Although many key concepts—such as parity-time symmetry—originated in the realm of mathematical physics, they began to flourish in earnest only after being introduced into the framework of optics and photonics nearly two decades ago. By now, these concepts have permeated numerous other subfields across physics and technology, ranging from nuclear and quantum systems to optical, microwave, electronic, and mechanical platforms. In this talk, we will present a broad overview of these developments, with a particular focus on systems that exhibit additional symmetries, such as topological characteristics. Other emerging research directions aimed at exploring thermalization dynamics in nonlinear, highly multimode non-Hermitian systems will also be discussed.

References

- [1] K. G. Makris, R. El-Ganainy, D. N. Christodoulides, and Z. H. Musslimani, *Phys. Rev. Lett.* **100**, 103904 (2008).
- [2] R. El-Ganainy, K. G. Makris, M. Khajavikhan, Z. H. Musslimani, S. Rotter, and D. N. Christodoulides, *Nat. Phys.* **14**, 11–19 (2018).
- [3] H. Hodaei, A. U. Hassan, S. Wittek, H. Garcia-Gracia, R. El-Ganainy, D. N. Christodoulides, and M. Khajavikhan, *Nature* **548**, 187–191 (2017).
- [4] C. E. Rüter, K. G. Makris, R. El-Ganainy, D. N. Christodoulides, M. Segev, and D. Kip, *Nat. Phys.* **6**, 192 (2010).
- [5] M. P. Hokmabadi, A. Schumer, D. N. Christodoulides, and M. Khajavikhan, *Nature* **576**, 70–74 (2019).
- [6] F. O. Wu, A. U. Hassan, and D. N. Christodoulides, *Nat. Photonics* **13**(11), 776–782 (2019).
- [7] G. Pyrialakos, H. Ren, P. Jung, M. Khajavikhan, and D. Christodoulides, *Phys. Rev. Lett.* **128**, 213901 (2022).

Structured Light from the Ground Up: Multilayer Meta-Optics and Topological Evolution in Optical Vortices

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Shaping light's spatial and topological structure is central to many applications in photonics and quantum optics. This talk covers two synergistic developments in structured light; one rooted in multilayer metasurface design, the other in the topological evolution of vortex beams. In the first part, we introduce a platform for multilayer, free-standing meta-optics that overcomes the fundamental limitations of single-layer metasurfaces [1]. Vertical integration of TiO₂ nanofins, which are fabricated via a dual-step lithography process using orthogonal e-beam resist/developer pairs, enables multilayer metasurfaces suspended in air. This design freedom allows broadband dispersion engineering, generalized polarization transformations, and asymmetric transmission. Such capabilities are difficult to achieve with single-layer metasurfaces. To demonstrate the platform, we realized polarization-multiplexed blazed gratings and an orbital angular momentum (OAM) plate generating vortex beams with ± 2 topological charge, with 80% measured diffraction efficiencies. Our structures impart pure geometric phase on linearly polarized light by inducing a cyclic path on the Poincaré sphere—determined by the relative orientation between stacked nanofins. This contrasts with traditional Pancharatnam-Berry metasurfaces, which operate on circular polarization. The method is adaptable across various materials and wavelengths, offering scalability and compactness, and advancing beam steering, holography, and vertical photonic integration.

The second part of the talk introduces optical rotatum, a recently reported class of structured light where a vortex beam exhibits a quadratic chirp in its OAM along the propagation axis—unlike conventional vortex beams which maintain a fixed OAM value [2]. This evolution is driven by an azimuthal variation in the spatial frequency of the beam, causing different points along the beam's azimuthal direction to experience distinct phase shifts. As the beam propagates, this results in a twisting of the phasefront, where the helical nature of the beam continuously deforms. By carefully designing the spatial frequency gradient in the beam, the OAM can evolve in a controlled manner, such as following a linear or quadratic dependence along the optical path. Remarkably, optical rotatum gives rise to a beam structure that follows a logarithmic spiral—a geometric form commonly observed in natural systems such as seashells and galaxies. This evolving OAM profile also suggests a new class of optical forces and torques, with possible implications for micromanipulation, light-driven transport, and emerging fields like valleytronics and spintronics. The results expand the landscape of singular optics and open new paths for synthetic photonic dimensions, non-Hermitian systems, and the design of topologically rich light-matter interactions—bridging device-level innovation with fundamental physics.

[1] Ahmed H. Dorrah et al. *Nature Communications* **16**, 3126 (2025).

[2] Ahmed H. Dorrah et al. *Science Advances* **11**, eadr9092 (2025).

Modelling the Onset of Selective Non-Hermitian Skin Effects

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In standard condensed matter physics, bulk-boundary correspondence within a lattice gives rise to topologically protected edge states (skin modes) and a class of robust bulk states (bulk modes) with no strong dependence on boundary conditions. This behavior comes from the combined properties of translational invariance and periodic boundary conditions and can be described in accordance with Bloch's theorem.

Recent studies in non-Hermitian physics, however, have shown that this framework fails to capture the behavior of open systems (where dissipative effects are relevant) using open boundary conditions. In this context, standard bulk–boundary correspondence breaks down and the bulk eigenstates are localized near boundaries by what's known as the non-Hermitian skin effect. This boundary sensitivity is well represented by the Hatano–Nelson (HN) model, a non-Hermitian tight-binding model which allows for asymmetric couplings between adjacent lattice sites. Here, we present an extension of the HN model for the non-Hermitian skin effect known as the Selective Non-Hermitian Skin Effect in which the parameter α is introduced as a position-dependent scaling factor of coupling strength. We consider two cases. In the first, this scaling comes from an exponential function of α , while the second is a linear function of α . We will describe the spectrum of our system and see how the ratio of skin modes/bulk modes depends on both our scaling factor α and the parameter β that comes from the HN model.

We find expectation values for each mode and define skin versus bulk modes relative to the threshold d , which represents skin depth. As we sweep α in the vicinity of $\alpha = 1$, we see how a system of purely skin modes (which resembles the effect described by the HN model) transforms into a system of predominantly bulk modes. We therefore find that bulk-boundary correspondence in a non-Hermitian lattice is strongly affected by the introduction of a position-dependent scaling factor α .

References

- [1] N. Okuma and M. Sato *Condens. Matter Phys.* 14, 83 (2023).
- [2] L. Xiao, et.al. *Nat. Physics.* 16, 761 (2020).
- [3] N. Hatano and DR. Nelson *Phys. Rev. Lett.* 77, 570 (1996).
- [4] L. Ge, *Innov. Discov.* 1, 4 (2024).

Novel resonances and exceptional points in non-Hermitian optical systems

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²*Graduate Center, City University of New York, New York, United States*

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Abstract: Exceptional points (EPs) are a unique non-Hermitian phenomenon, where not only do the eigenvalues of an operator (the scattering matrix, the Hamiltonian, etc.) become degenerate but also the corresponding eigenstates become identical. In this talk, I will first discuss EPs with previously unexpected optical dispersion relations, i.e., at the center of a 2D Dirac cone [1]. In such a case, an imaginary gauge transformation also establishes the equivalence between certain non-Hermitian systems of two different types, namely, those with optical gain/loss and those featuring asymmetric couplings. In the presence of a non-Hermitian gauge field, we identified a new mechanism that enables strong modal interactions and single-mode lasing in laser arrays [2]. Combining it with spin-orbital coupling, I will also discuss the underlying principle that led to the precise control of a micro-ring laser system emitting into a four-dimensional Hilbert space [3]. Lastly, I will talk about an active photonic resonance [4] that does not have an origin in a passive resonance, which ushered in new possibilities in the designing of laser cavities and other resonance structures. If time allows, I will touch on our recent work on robust non-Hermitian zero modes without a global symmetry [5] and the selective non-Hermitian skin effect [6].

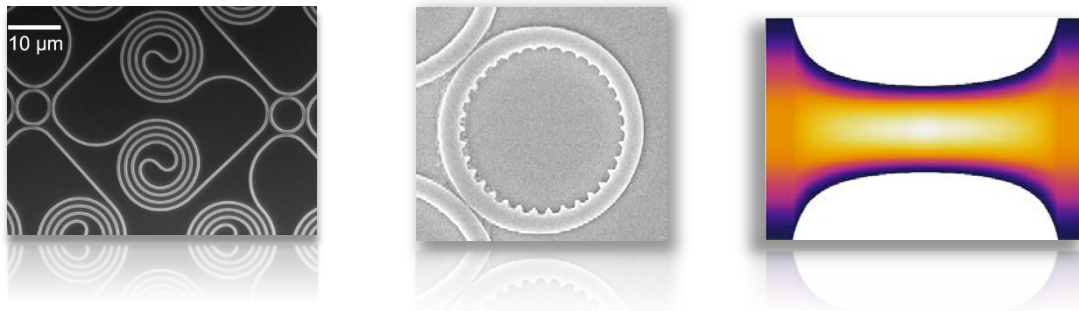


Fig. 1 (From left to right) A laser array with a non-Hermitian gauge field [2]; a ring resonator from a non-Hermitian photomolecule laser with spin-orbital coupling; a cavity hosting an active resonance [4];

References

1. Rivero, Feng, and Ge, Phys. Rev. Lett. **129**, 243901 (2022).
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3. Zhang et al., Nature **612**, 246 (2022).
4. Rivero et al., Phys. Rev. Lett. **126**, 163901 (2021).
5. Rivero et al., Phys. Rev. Lett. **131**, 223801 (2023).
6. Ge, Li, Innov. Discov. **1**, 4 (2024).

A photonic crystal Rice-Mele nanolaser array: from the generation of chiral light to photonic computing applications

Kaiwen JI, Giulio TIRABASSI, Melissa HEDIR, Qi ZHONG, Cristina MASOLLER, Li GE, Ramy EL GANAIFY and Alejandro M. YACOMOTTI

In this talk, I will present our recent results on active photonic crystal cavity arrays arranged in a closed loop of four evanescently coupled nanolasers—referred to here as a photonic “quartet.” In the presence of gain/loss modulation, sublattice frequency detuning, and coupling imbalance, this system can be mapped onto a Rice-Mele model with complex detunings. Under these conditions, the quartet exhibits second-order exceptional points, warranted by non-Hermitian chiral symmetry, which enable several promising photonic functionalities.

First, I will discuss the experimental realization of a chiral exceptional point where a phase singularity emerges. This enables the extreme miniaturization of photonic sources generating non-zero orbital angular momentum (OAM).

Second, I will show how the nanolaser quartet can function as a hidden complex layer for efficient photonic computing. This configuration demonstrates robust classification performance even under challenging conditions—such as distinguishing highly compressed handwritten digits with overlapping feature boundaries. These findings suggest that symmetry and/or topologically protected modes in nanolaser arrays can leverage robust optical connections to tackle complex problems without the need of scaling up the number of neurons.

A rigorous framework for topological plasmonics

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(Dated: May 25, 2025)

Plasmonics is the study of the interaction between light and free electrons at metal-dielectric interfaces at the nanoscale. A versatile platform in this field are systems of coupled metallic nanoresonators (MNRs), which are for example used to construct metasurfaces for sensing applications. Topological phenomena can provide novel functionalities to the already rich physics of such structures. To analyze and design topological properties of these systems, a non-Hermitian treatment is necessary to correctly account for Ohmic losses.

In my talk, I present a rigorous framework to analyze the topology of coupled MNRs. Having all relevant properties of arbitrarily shaped constituent MNRs, the properties of the coupled MNR system are encoded in the solutions of a non-Hermitian and nonlinear eigenvalue problem. Employing our method, we analyze a plasmonic analogue of the Su-Schrieffer-Heeger (SSH) model, and compare our results with the existing literature as well as with full-wave numerics. Our approach allows different approximation schemes, resulting in computational efficiency and scalability compared to full-wave numerics, making it a powerful tool for analyzing topological plasmonic structures and shaping our understanding of these. Beyond that, the presented methods allow for the optimization and design of topological plasmonic nanostructures with potential applications in sensing, microscopy and optical communication.

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Graph theory for the spectrum of perturbed higher-order exceptional points

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Exceptional points (EPs) are singularities in non-Hermitian systems where two or more eigenvalues and their corresponding eigenvectors coalesce. These points are associated with non-trivial spectral topology and lead to unique physical phenomena such as enhanced sensing possibilities. Higher-order EPs, where more than two eigenvalues coalesce, exhibit even richer behavior, but their spectral response to perturbations remains challenging to predict.

In this work, we present a novel graph-theoretical framework to analyze the eigenvalue spectra of systems hosting higher-order EPs [1]. By representing a non-Hermitian systems, especially the intrinsic couplings, as a graph, we establish a systematic approach to predict and classify the spectral unfolding. This method offers new insights into the topology of eigenvalue trajectories and their structural dependencies on system parameters.

Our results provide an alternative perspective to conventional analytical and numerical approaches, enabling an intuitive way to study perturbed higher-order EPs. To illustrate our findings, we use coupled microrings as example systems, demonstrating how graph-theoretical methods can effectively capture and predict spectral features in realistic settings. This framework has potential implications for engineered non-Hermitian photonic and quantum systems where precise control and understanding of spectral properties is essential.

References

1. D. Grom, J. Kullig, M. Röntgen, and J. Wiersig, arXiv:2409.13434 (2024)

Phase Singularity in a Complex Rice-Mele Nanolaser Array

Kaiwen Ji¹, Melissa Hedir¹, Ramy El-Ganainy², Li Ge³, Alejandro M. Yacomotti¹

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Thanks to the increasing flexibility of parameter tuning in photonic designs, micro and nanophotonic coupled cavity arrays —such as coupled micropillars, microrings and photonic crystal nanocavities— are ideal platforms to investigate topological photonics [1, 2]. Symmetry and/or topologically protected modes may arise in such lattices, which prove robust against some kind of disorder. Importantly, III-V semiconductor materials enable non-zero imaginary parts of the electrical susceptibility, leveraging active —e.g. lasing— regimes, and also nonlinear and non-Hermitian operation in the form of gain/loss spatial modulations.

In the context of topological lasers [2], devices able to generate vortex beams from modes with nonzero orbital angular momentum (OAM) have attracted considerable attention. In Ref. [3] a six-coupled micropillar laser in a ring with uniform couplings —or photonic “benzene” molecule— has been utilized to generate non-zero OAM laser beams. In this case, the pump is uniformly distributed in the system, therefore the ring can be mapped to a Hermitian system in which clockwise (CW) and counter clockwise (CCW) modes are degenerated. This can be seen as consequence of the eigenmodes of a symmetric real matrix: these can always be rescaled to be purely real functions, i.e. standing waves. Hence, chiral modes $e^{im\pi}$, where m is an integer, are linear superpositions of pairwise degenerated standing waves. In Ref. [3] mode selection is realized using pump beams with circularly polarized light (nonzero spin). Noticeably, pure chiral modes are no longer possible for arbitrary Hermitian couplings because degeneracies become lifted in general. In such a case, (non-Hermitian) gain/loss distributions can restore OAM. In this work, we show that $e^{im\pi}$ chiral modes and phase singularities can be generated at the exceptional points (EPs) of a four coupled nanolaser ring with alternating couplings and sublattice pump imbalance, which can be seen as a non-Hermitian version of the Rice-Mele model, where the on-site potentials are originally real. The gain/loss configurations naturally enable mode selection without the need of introducing other mechanisms as circular polarization of the pump beam.

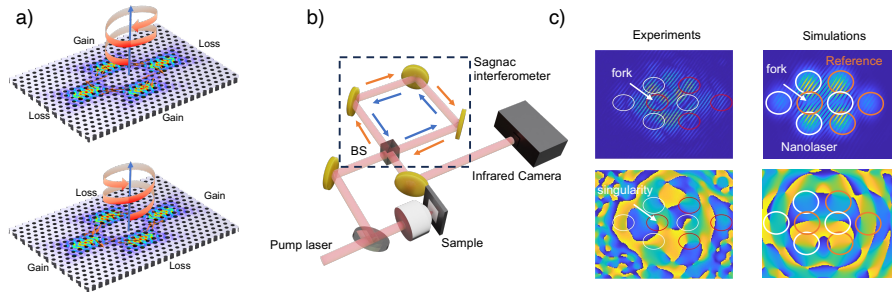


Fig. 1 (a) Schematic of the chiral mode operating at the EP in a complex Rice-Mele photonic crystal coupled nanolaser array, with arrows indicating the Poynting vector, under nonuniform (sublattice) pump distribution. (b) Experimental setup for the near-field interference measurements. (c) Near-field interference pattern near the EP (left). The circles mark the positions of the cavities, and a fork-shaped feature appears at the center of the structure. The lower panel shows the corresponding phase distribution, revealing a clear phase singularity. The right panel presents the corresponding simulation results.

As shown in Fig. 1(a), two chiral modes with opposite OAM are generated at the EPs by inverting the sublattice pump imbalance. A Sagnac interferometer [Fig. 1(b)] enables robust near-field interference measurements with spatially shifted patterns to determine the phase distribution of these chiral modes. When operated close to the EP, a fork-shaped feature appears at the center of the structure [Fig. 1(c), left panel]. Furthermore, off-diagonal Fourier filtering [4] confirms provides direct evidence of the formation of the phase singularity. This work presents a flexible approach to achieving nonzero OAM via non-Hermiticity in a coupled semiconductor nanolaser array, potentially benefiting various applications in fields such as optical communications.

References

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Non-Hermitian dynamics in bosonic systems: topological and quantum realizations

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Open systems governed by a non-Hermitian Hamiltonian, called Parity-Time (PT) symmetric systems, have been deeply investigated over the past decade. Although most of their realizations are with waves, in recent years, these ideas have been extended to the quantum domain. With a photonic Floquet topological insulator and number-resolved phonon dynamics as examples, I will present salient properties of such systems in the classical and quantum domains respectively. (Work done in collaboration with Alexander Szameit's group (University of Rostock, Germany), Anthony Laing's group (University of Bristol, UK), and David Allcock's group (University of Oregon, US)).

Selective filtering of photonic quantum entanglement via anti-parity-time symmetry

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Abstract

Entanglement is key resource for quantum computing, sensing, and communication, however it is highly susceptible to decoherence. To address this, quantum optics has explored filtering techniques like photon ancillas and Rydberg atom blockade to restore entangled states. Here, we introduce a an entirely new approach to entanglement retrieval exploiting non-Hermitian systems. By employing an anti-parity-time two-state guiding configuration, we demonstrate efficient extraction of entanglement from any input state. This filter is implemented on a lossless waveguide network using Lanczos transformations, consistent with Wigner-Weisskopf theory. This scheme achieves near-unity fidelity under single- and two-photon excitation and is scalable to higher photon levels while remaining robust against decoherence during propagation. Our work offers new insights into using non-Hermitian symmetries to address central challenges in quantum technologies.

Topological edge and corner modes in polariton lattices

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Topological Photonics is an emerging and novel field of research, adapting concepts from condensed matter physics to photonic systems adding new degrees of freedom. After the first demonstrations of topological photonic insulators [1,2], the field has moved on to study and exploit the inherent non-hermiticity of photonic systems and the interplay with their topological nature. In my talk, I will attempt to give an overview about the quickly emerging field of topological photonics. In this context, I will discuss topological lasers as a prime example of using topological concepts potentially for new technologies in the broad context of synthetic (photonic) matter. Examples will be given from novel photonic lattice devices resulting from the coupling of individual vertical III-V semiconductor microresonators. Here, the so-called exciton-polaritons – hybrid states of light and matter – can emerge in the strong coupling regime. The specific geometry as well as the hybrid light-matter nature allow for ways to break time-reversal symmetry and implement topologically non-trivial systems. We were able to experimentally demonstrate the first exciton-polariton topological insulator, manifesting in chiral, topologically protected edge modes [3]. In order to study topological effects in combination with optical non-linearities, so-called topological lasers have been envisaged and realized. We have presented the first experimental demonstration of a topological insulator vertical cavity laser array [4], using the crystalline topological insulator model (see Fig. 1). Following this works, I will discuss recent advances towards electrical operation and lasing from a topological defect [5]. In addition, so-called corner modes, fully localized higher-order topological defects in two-dimensional lattices in breathing Kagome and 2D-SSH lattices are discussed, with a particular focus on the robustness against (deterministic) fabrication imperfections. I will discuss recent advances in using polarization degrees of freedom in the context of artificial gauge fields and the spin quantum hall effect of light. Finally, I will discuss the potential of the experimental platform to enable precisely tunable gain and loss, realizing exceptional points and opening the field for non-Hermitian polaritonics.

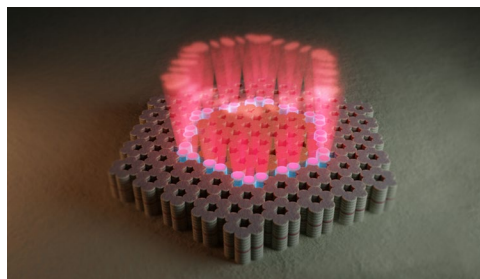


Fig. 1: Schematic drawing of a topological vertical-cavity surface-emitting laser array.

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Exceptional points, symmetries and similarities

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Exceptional points (EPs) are truly non-Hermitian (NH) degeneracies at which eigenvalues and eigenvectors coalesce. Exceptional points of order n (EP n s) emerge if $2(n - 1)$ real constraints are imposed [1], such that EP2s generically appear in two dimensions and are thus extremely ubiquitous in NH systems. In my talk, I will show that local symmetries generally reduce the number of constraints to find EP n s from $2n - 2$ to $n - 1$ or n in some cases [1]. We will see that these symmetries are captured by three different generalized similarities, which are at the root of the emergence of higher-order EPs in lower dimensions [2]. I will furthermore show in several concrete two-dimensional examples that the appearance of higher-order EPs comes hand-in-hand with lower-order EP structures as well as Fermi structures in the real and imaginary energy spectrum [3]. As it is not easy in general to construct models featuring higher-order EPs, I will present two methods to do this, which lead to the emergence of EPs of any desired order [1, 4]. I will end by illustrating how these results can be used to identify the existence of EPs in experiment.

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Using controllable microwave resonators and non-Hermitian dynamics for coherent control of magnon-polaritons

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Systems comprising two or more coupled oscillators display a rich variety of phenomena. There has been particular interest in the coupling of modes supported by different physical systems – so-called hybrid systems. In particular, the coupling of quantum systems with microwave and optical cavities has proved fruitful, allowing new types of excitations to be created. For example, embedding magnetic materials in a microwave cavity results in excitations that are combinations of magnons and photons, termed magnon-polaritons. Magnons are candidate systems for quantum information processing due to long excitation lifetimes and wide ranging frequency tunability [1, 2]. However, coherent control of magnonic systems is challenging due to the difficulties of creating rapidly changing magnetic fields to tune the frequencies of the magnons.

Here, using microwave resonators for which we have control over the centre frequency and loss/gain of microwave modes, we demonstrate that the exceptional point allows coherent control of magnon-polaritons. In particular, we show that encircling the EP allows excitations to be switched between modes, and traversing the EP permits deterministic preparation of a superposition of states. We demonstrate that a trajectory encircling an exceptional point allows transfer of energy between normal modes. Circling must occur with the correct sense for switching to occur, and the sense required is dependent on the target mode. We also study the effect of traversing through the exceptional point. Here, the effect of the generalised eigensystem on the evolution of the system must be considered, and we show that the time reversal symmetry of trajectories through the EP leads to robust preparation of the system in an equal superposition of magnon polariton modes. These results offer a pathway to novel coherent control of quantum hybrid systems.

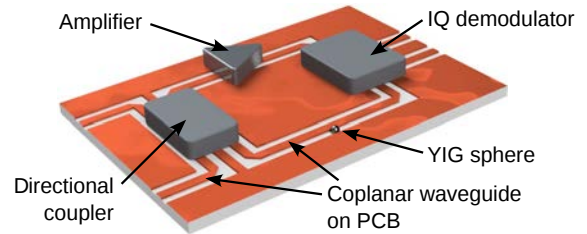


FIG. 1: Cartoon of tunable microwave resonator with embedded magnetic element.

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Exponential Sensitivity based on pseudospectra scaling

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One of the important features of non-Hermitian Hamiltonians H is the existence of a unique type of singularities, the so-called exceptional points (EPs) [1]. When a system operates near an EP, it exhibits ultra sensitive behavior due to the eigenstate non-orthogonality, a property that has led to the novel microresonators and gyroscopes, among other applications. An alternative way to achieve ultrasensitive behavior in lattices, lies on asymmetric couplings, where the non-Hermitian skin effect [2] occurs. It has been demonstrated that the spectra of asymmetric lattices display exponential dependence on the lattice size, when the boundary conditions are varied, and this sensitivity has been linked to the lattice's topological properties [3]. In this work, we apply the pseudospectra theory [4-7] and show that not only the eigenspectra, but also the power growth exhibit exponential variations with system's size, and that this effect is due to the extreme lattice non-normality [8]. In particular, we have presented a scaling analysis of pseudospectra in exponential sensitive lattices, and determine the conditions under which, the lattice exhibits exponential sensitivity or not. Given that recent experiments in non-Hermitian photonics have realized such asymmetric optical lattices [2], our results could enable the efficient design of ultrasensitive devices without relying on higher-order exceptional points or topology.

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Enantiosensitive exceptional points in open chiral systems

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The presence of exceptional points (EPs) in non-Hermitian (NH) physics leads to a plethora of remarkable phenomena, such as topological population transfer between the eigenstates or the enhanced sensitivity of NH systems to external perturbations [1]. Here, we apply these EP-based non-Hermitian phenomena to the problem of detection of molecular chirality. In particular, we show that the position of EPs in open chiral systems describing the interaction of light with chiral molecules depends on their handedness.

First, we show that selective excitation of chiral enantiomers can be achieved by coupling a chiral molecule to the continuum using a three-color field. Tuning the field parameters, we achieve highly efficient topological population transfer between the adiabatic states of only one of the molecular enantiomers, while its mirror twin is left in the initial state. We then demonstrate that enantiosensitive EPs in photodissociating chiral molecules can amplify weak chiral effects induced by the magnetic component of chiral light fields, enabling control over the amount of dissociated chiral molecules in a racemic mixture in an enantioselective fashion. Finally, we devise a chiral sensor based on a twisted non-Hermitian optical fiber interacting via evanescent modes with a solution of chiral molecules. By engineering the gain/loss profile of the optical fiber, we show that the fiber operates in the PT-symmetric or PT-broken regime depending on the handedness of the enantiomeric solution of chiral molecules. Our work is a first step in an exciting new direction at the crossroad between the development of new enantiosensitive techniques and non-Hermitian physics. Here we show that non-Hermiticity can provide much-needed enhancement of chiral phenomena, while chiral interactions open new avenues for the exploitation of EP-induced effects.

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'Nonreciprocity in Action'

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The concept of dissipation engineering has enriched the methods available for state preparation, dissipative quantum computing and quantum information processing. Combining such engineered dissipative processes with coherent dynamics allows for new effects to emerge. For example, we found that any factorisable (coherent) Hamiltonian interaction can be rendered nonreciprocal if balanced with the corresponding dissipative interaction. This powerful concept can be exploited to engineer nonreciprocal devices for quantum information processing, computation and communication protocols, e.g., to achieve control over the direction of propagation of photonic signals. However, although nonreciprocal concepts are realizable in quantum architectures, nonreciprocity itself holds up to the classical level and is not inherently quantum; the fundamental aspects of nonreciprocity in the quantum regime have yet to be fully investigated. In this talk we will address these aspects and discuss routes of how nonreciprocal concepts can find application beyond quantum information processing. Moreover, we will take a closer look on how to actually operate an embedded nonreciprocal amplifier on chip, and discuss the resulting efficiency and backaction for dispersive qubit-read-out.

Knots, non-abelian braids, and geometric phases in non-Hermitian systems

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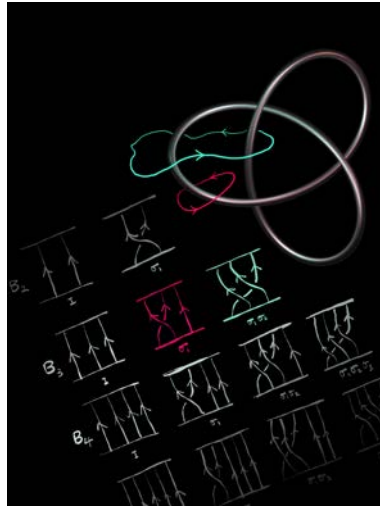
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For non-Hermitian systems of coupled oscillators, the spectrum of resonance frequencies is complex. If the system's parameters are varied along a path in parameter space and returned to their initial point (i.e., the parameters are varied around a closed 'loop'), the spectrum may return to itself in a topologically non-trivial manner, a phenomenon known as spectral flow. For an arbitrary- N -coupled-oscillator system, we find that the spectral flow is determined by how a loop encircles degeneracies. We also find that parameter loops generically produce braids of eigenfrequencies. These braids form a non-Abelian group (for $N > 2$), reflecting the non-trivial geometry of the space of degeneracies (e.g., knots) [1, 2].

In the context of dynamically tuning non-Hermitian systems along parameter loops, we find that non-Hermiticity changes the concept of Berry phase (geometric phase) from being a real number to being a complex number. Its imaginary part signifies a geometric gain or loss. This opens new avenues for controlling the flow of energy in oscillators and allows for a novel, unconventional class of amplifiers involving a 'slow' modulation of the system parameters.

Furthermore, we find that the Berry phase accumulated when a parameter loop is traversed within a subspace of degeneracies (i.e., of the so-called 'exceptional points') is determined by the homotopy class of the loop. It is real and discretized in units of $2\pi/n$, where n is the degeneracy order of the subspace [3].

I will describe our cavity optomechanical experiments demonstrating these phenomena. It should be emphasized, though, that these phenomena are generic to any physical domain and to any system whose dynamics is captured by the ubiquitous equation $\partial_t \psi = -iH\psi$ with non-Hermitian H .



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Title: Non-Hermitian Quantum Photonic Filters

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Abstract:

Dissipative quantum systems typically relax towards mixed steady states, losing their initial purity and quantum coherence. However, a remarkable counterpoint to this trend lies in the ability to precisely engineer dissipation to act as a selective filter for specific quantum states within these mixed ensembles. In this presentation we will explore the physics and practical implementation of dissipative photonic systems designed for the filtering of quantum light states. Strikingly, this method offers a robust and versatile linear filter for quantum state selection through dissipation, providing a powerful tool for manipulating and preparing certain quantum states of light.

Coherent perfect absorption and emission of light in non-Hermitian and time-varying media

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In my talk, I will speak about our recent work on coherent perfect absorption (CPA) [1,2] and emission [3] of light in tailor-made cavities. I will highlight, in particular, the possibility to engineer spatial and spectral degeneracies for broadband absorption of arbitrary wavefronts at an exceptional point as well as the topological aspects associated with perfect emission of thermal radiation. In the second part of my presentation, I will present new insights on the CPA effect in time-varying media, which can be well described by a pseudo-unitary Floquet scattering matrix [4].

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Unified Framework for Non-Hermitian Response Across All Spectral Scenarios

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Non-Hermitian systems exhibit remarkable response effects tied to distinct spectral scenarios, most notably exceptional points of different order. Existing frameworks treat these cases separately based on their specific spectral decomposition, which complicates the analysis in their vicinity, and limits the practical applicability due to the ill-conditioned nature of these decompositions. Furthermore, much less attention has been paid to the more complicated case of degeneracies with higher geometric multiplicity.

I present a general response theory that applies uniformly across all these spectral scenarios. By developing a universal expansion of the spectral quantization condition and Green's function, both based solely on directly computable Hamiltonian data, I describe how one can capture the qualitative response differences while ensuring smooth parameter dependence.

To highlight the applicability of the formalism, I describe how it allows to formulate (1) precise conditions for spectral degeneracies of higher geometric multiplicity, and (2) a continuous hierarchy of spectral response strengths that characterize all degeneracies quantitatively. Combining both aspects, it follows that scenarios with higher geometric multiplicity give rise to unique super-Lorentzian response, for which the spectral and physical response strength become two separate quantities.

Joint work with Subhajyoti Bid

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Topological exciton polaritons in perovskite microcavities

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Microcavity exciton polaritons are hybrid bosonic quasiparticles with combined advantages from semiconductor excitons and cavity photons. They hold significant promises for the next generation of optoelectronic devices, such as lasers, circuits and quantum simulators, which have witnessed prosperous developments in the past decades. Particularly, with the recent advances of lattice potential trapping and topological concepts, the extension of topological lattices into polariton systems have furnished novel functionalities to their optoelectronic applications towards low energy consumption and robust operations against perturbations. In this talk, I will introduce our recent progress in achieving topological exciton polaritons with lead halide perovskites in both Hermitian and non-Hermitian regimes, including valley Hall polaritons, topological disclinations and non-Hermitian exciton polariton skin effect at room temperature.

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Non-Hermitian topology and directional amplification

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Topology has been a major research theme in condensed matter physics and is associated with a number of remarkable phenomena such as robust edge states. A prominent example is the quantum Hall effect, in which the topological invariant is directly observable through the Hall resistance. More recently, topology started to be investigated in systems experiencing gain and loss sparking the field of non-Hermitian topology. However, for a long time, a clear observable signature of non-Hermitian topology had been lacking.

In this talk, I will show that non-trivial, non-Hermitian topology is in one-to-one correspondence with the phenomenon of directional amplification [1-3] in one-dimensional bosonic systems, e.g., cavity arrays. Directional amplification allows to selectively amplify signals depending on their propagation direction and has attracted much attention as key resource for applications, such as quantum information processing. Remarkably, in non-trivial topological phases, the end-to-end gain grows exponentially with the number of sites. Furthermore, this effect is robust against disorder [2] with the amount of tolerated disorder given by the separation between the complex spectrum and the origin.

In collaboration with the group of Ewold Verhagen at AMOLF, Amsterdam, we experimentally demonstrate the connection between non-Hermitian topology and directional amplification in a cavity optomechanical system [4] by realising a bosonic version of the Kitaev-Majorana chain proposed in [5]. Furthermore, we show in the experiment that a similar system proposed in [6] can be utilised as a sensor with a sensitivity that grows exponentially with system size [4].

Our work opens up new routes for the design of multimode robust directional amplifiers and sensors based on non-Hermitian topology that can be integrated in scalable platforms such as superconducting circuits, cavity optomechanical systems, plasmonic waveguides and nanocavity arrays.

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Nonlinear Skin and Subskin Modes

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Non-Hermitian systems, characterized by asymmetric couplings and complex spectra, exhibit novel localization phenomena that challenge conventional understandings of localization. In this talk, we explore both subskin and skin modes in the presence of nonlinear interactions, with particular emphasis on the mechanisms underlying their emergence.

Unlike traditional skin modes, which accumulate at system boundaries, subskin modes localize beneath the boundary. These modes do not emerge generically but require a specific relationship between system size and coupling parameters. However, the inclusion of nonlinearity relaxes these constraints, enabling the robust formation of subskin modes without fine-tuned parameters. Moreover, long-range hopping may lead to the localization of subskin modes deep within the lattice, sharply distinguishing them from both skin modes and conventional subsurface waves.

The second part of the talk addresses the nonlinear non-Hermitian skin effect using a fixed-point approach. Unlike the linear regime, nonlinear interactions induce power-dependent, degenerate modes and introduce power-energy discontinuities. These nonlinear skin modes exhibit spectral properties markedly distinct from their linear counterparts under open boundary conditions. Importantly, the introduction of a coupling impurity gives rise to localized dark and anti-dark solitons, as well as a novel class of modes that are neither conventional skin states nor scale-free localized modes.

Taken together, these findings deepen our understanding of boundary-localized phenomena in nonlinear non-Hermitian systems and demonstrate how nonlinearity not only deforms but enables new localization behaviors.

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Posters

Posters

Ceyhun Bulutay	A phase-controlled sensing paradigm in non-Hermitian closed-loop optomechanical system
Niladri Chakraborty	Topology and entanglement in Non-Hermitian systems
Roos De Boer	Slowed relaxation in an optical cavity with memory
Dwaipayan Debnath	Classical characterisation of a parity-time (PT) symmetry breaking of a photonic waveguide system by intensity measurement
Marcel Eichelmann	Implications of exceptional points on photonic auto- and cross-correlations for coupled microresonators containing semiconductor quantum dots
Chitres Guria	Measuring the complex geometric phase in a non-Hermitian system
Steven Kim	Universal description of the parametric instability
Lukas König	Braided phases
Julius Kullig	Computation of spectral response strength of exceptional points in optical microcavities
Maxine McCarthy	Non-Hermitian phenomena in directed graphs: topology with arbitrary connectivity
Carlo Panu	Topological edge states and exceptional points
Jann Erik Sowart	Theoretical framework for quantum light in Non-Hermitian waveguide systems
Igor Tsukerman	Analytical and computational tools for 2D and 3D topological photonics

Abstracts of Posters

(in alphabetical order)

A phase-controlled sensing paradigm in non-Hermitian closed-loop optomechanical system

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Abstract

We theoretically study optomechanical sensing in a non-Hermitian ternary coupled system composed of an optical cavity and two mechanical resonators. The couplings among the cavity and mechanical resonators form a closed-contour interaction accompanied by a Peierls phase. We demonstrate a method for controlling sensitivity through precise manipulation of the closed-loop phase in the system. The sensitivity in measurement can be enhanced for specific values of the global phase. We also analyze the system's performance by accounting for correlations between optical output quadratures by choosing homodyne angle other than 0° and 90° . Another control parameter is the mechanical coupling constant chosen close to the so-called exceptional point. Overall, the realistic and optimal parameter set enables us to enhance sensitivity.

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Topology and Entanglement in Non-Hermitian Systems

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Non-Hermitian topology has been connected to fascinating phenomena such as directional amplification - the selective amplification of signals in only one direction. While so far, most studies have revolved around classical effects, current experimental capabilities are reaching a level of control that will enable the exploration of quantum phenomena in non-Hermitian driven-dissipative systems. In this poster, quantum phenomena such as the generation of end-to-end entanglement induced by non-Hermitian topology in driven-dissipative bosonic quantum systems will be discussed, highlighting its potential relevance for quantum information processing. This work is relevant for a range of experimental platforms including cavity optomechanics, superconducting circuits, and BEC lattices.

Slowed relaxation in an optical cavity with memory

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The relaxation time to a steady state is a fundamental property of every dynamical system. For statistical systems relaxing to thermal equilibrium, this time scale is known as the thermalization time. In recent years, it was observed that certain many-body systems can relax to a ‘prethermal’ state that is transient but nevertheless extremely long-lived. The lifetime of this state can exceed the internal relaxation times of the system by many orders of magnitude. Very recently, we observed signatures of this phenomenon for the first time in an optical system. We study a single-mode laser-driven thermo-optical cavity where the coupling between light and temperature effectively gives memory to the optical response, making the dynamics non-Markovian in the presence of noise. Experimentally, this is achieved by using an oil-filled microcavity. Interestingly, while the photons in the cavity have a picosecond lifetime, and the temperature relaxes within a few microseconds, under a perturbation of the state of the intra-cavity field, we observe the emergence of a transient state lasting many milliseconds. Only afterwards, the intra-cavity field settles into its true steady state. The observed transient state seems to have many of the features associated with prethermalization, but now realized in a strongly out-of-equilibrium system. Besides this slowed relaxation, we also found non-Hermitian degeneracies in the fluctuation spectrum of our system. The combination of slow dynamics and the rich eigenvalue structure of our system makes it an interesting, experimentally realizable platform for further research.

Classical characterisation of a parity-time (PT) symmetry breaking of a photonic waveguide system by intensity measurement

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PT-symmetric and non-Hermitian photonic waveguide systems offer some unique ways to manipulate light, which is not possible in traditional Hermitian systems. For example, they offer unidirectional light transport, enhanced sensitivity at exceptional points, and non-reciprocal light propagation. That is why characterizing PT-symmetric and non-Hermitian photonic systems is important to understand and harness these novel properties. In this regard, simple intensity measurements to characterize these systems are practical and easy to access. In this project, we are trying to characterize a PT-symmetric two-waveguide system classically by simple intensity measurement. By this intensity profile, we aim to detect exceptional points and identify phase transitions between PT-symmetric and PT-broken regimes.

Implications of exceptional points on photonic auto- and cross-correlations for coupled microresonators containing semiconductor quantum dots

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Exotic features such as exceptional points (EPs) as non-Hermitian degeneracies have attracted much interest in various fields including open classical wave and optical systems. In this realm widely utilized is the coupled-mode theory to generate an effective coupled-mode Hamiltonian to study the effects of EPs with respect to the eigenenergies. But those are not the only quantities of interest in photonic systems. Additionally, auto- and cross-correlations constitute interesting generic quantities. To this end we study two evanescently coupled microdisks – a so-called photonic molecule. In the simplest case both of the microresonators contain one optical mode each. Additionally, one electrically pumped semiconductor quantum dot is located in one of the resonators. Starting with the appropriate Hamiltonian in rotating-wave approximation, we utilize the generalized Ehrenfest theorem combined with the expectation value based cluster expansion method to derive the equations of motion for relevant dynamical system variables. From this resulting system of nonlinear, coupled differential equations of motion, we are able to extract a dynamical matrix that is used to gain further insights into photonic auto- and cross-correlations at the EP and beyond.

Measuring the complex geometric phase in a non-Hermitian system

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Geometric phase is a fundamental feature of oscillatory dynamics and is found ubiquitously in all areas of physics. It is defined when (a) the system's parameters are varied smoothly around a closed path (a loop), and (b) at the end of this variation, the system's state has changed by an overall phase factor. Under these conditions, the phase factor includes a contribution that is solely determined by the shape of the loop. This contribution is known as the geometric phase.

Non-Hermitian systems of coupled oscillators differ dramatically from their Hermitian counterparts. Notable examples include the topology of their spectra, their response to perturbations, and their performance in sensing and control applications.

The interplay between non-Hermiticity and geometric phase, can induce qualitative changes in both. Perhaps, the most fundamental difference is that the Hermitian geometric phase is a real number, while the non-Hermitian geometric phase is complex-valued [1]. This difference has an important practical consequence, as the imaginary part of the geometric phase affects the oscillations' amplitude (in contrast with the usual real part, which only affects their phase).

In this work, we measure the complex geometric phase in the dynamics of two coupled harmonic oscillators in an optomechanical platform. Using the dynamical back-action effect [2], we parametrically tune the system (in real time) - around arbitrary paths and demonstrate a number of uniquely non-Hermitian features of the geometric phase. These include the (gauge-invariant) geometric gain for open paths (i.e., non-loops) in parameter space [3], and the presence of the adiabatic limit for only a unique eigenmode [4]. In addition, we demonstrate that the geometric gain obtained from this type of control can sustain and amplify the motion of an intrinsically damped oscillator, for arbitrarily large times.

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Universal description of the parametric instability

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When the driving strength overcomes the dissipation, an oscillator that is parametrically driven at twice its resonance frequency undergoes a transition where self-sustained oscillations are enabled. Quantum fluctuations wash out this abrupt transition and enable the emission of photons already below the classical threshold. The threshold, indicates a dissipative phase transition where a critical slowing down of the dynamics occur. In this work, we derive a universal Liouvillian that describes the relevant slow, long-time dynamics for the degenerate and non-degenerate parametric oscillator. At the critical point, it is vital to take both fluctuations and nonlinearities into account. We apply the Liouvillian to study the statistics of the emitted radiation. We find that, e.g., the cumulants of the photon current obey a power-law scaling as a function of the nonlinearity and that the Fano factor shows a maximum close to the critical point.

Braided Phases

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We show that two-dimensional non-Hermitian systems have robust features that we can understand as topological phases. These phases are protected by the braiding of eigenvalues, which is non-Abelian and contrasts the Hermitian (typically Abelian) eigenvector topology. This implies a new class of phases, which we call gapless as they feature irremovable nodal points. We show explicitly that these points have no partner charge: even if all nodal points in a Brillouin zone are fused, the result is a topologically stable monopole, which breaks the usual notion of Fermion doubling. To transition out of such a phase, a nodal point has to move around the Brillouin zone torus. We show that under certain symmetries this leads to non-Abelian charge inversion, where, in the course of the phase transition, the nodal point transitions as well. Finally, we present experiments that observe telltale signatures of these phases in terms of bulk Fermi arcs.

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Computation of spectral response strength of exceptional points in optical microcavities

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Exceptional points are spectral degeneracies in open systems where not only the eigenvalues but also the corresponding eigenvectors coalesce. The fundamental reason behind the research interest in exceptional points stems from the enhanced system's sensitivity to external perturbations. Specifically, when a system at an exceptional point of order N is perturbed, the involved eigenvalues diverge generically with a characteristic N -root topology. The resulting eigenvalue splitting depends on the specific kind of perturbation and on a quantity—the spectral response strength—which can be assigned independently of the perturbation to a particular exceptional point. The response strength is therefore a quantitative measure for the system's sensitivity to external perturbations and allows to compare different exceptional points of the same order.

So far, the theory for the spectral response strength is restricted to non-Hermitian Hamiltonians on finite-dimensional Hilbert spaces which can occur naturally as an effective description, e.g. within coupled-mode theory. On the other hand, realistic systems described by a wave equation give rise to an underlying infinite-dimensional Hilbert space. In this work we present a scheme to calculate the spectral response strength directly from the numerical results of wave simulations. To do so, the relation between the spectral response strength and the Petermann factor of the eigenstates near the exceptional point is exploited. For a validation of our approach, we consider three different photonic systems: a microring dimer, waveguide-coupled microrings, and a weakly deformed microdisk. For all three cases our approach shows an excellent agreement to results provided by an effective Hamiltonian based on coupled-mode theory.

Non-Hermitian phenomena in directed graphs: topology with arbitrary connectivity

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Most approaches to topological physics in non-Hermitian systems are under the assumption of periodicity, however, any weighted directed graph may be interpreted as a discrete non-Hermitian Hamiltonian. Enforcing periodicity may obscure the generality of many topological phenomena, or even make some phenomena hard to discern as topological in origin. A comprehensive understanding of topological effects therefore must also be extended to this translationally variant case. We present certain results for extending topological classification of non-Hermitian systems to directed graphs, without imposing translational invariance. Our work indicates that topological indices may be defined in terms of graph invariants and underlying connectivity, which we use to predict boundary phenomena that hold for translationally varying systems.

Topological Edge States and Exceptional Points

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We analyse the topological properties of non-Hermitian systems, and apply these insights to realistic physical models. We start from a Hermitian two-dimensional lattice model characterized by a non-trivial Chern number: the Qi-Wu-Zhang model [1]. We then explore a non-Hermitian extension of the model to examine how key topological features, such as edge states and Berry curvature, are altered. Introducing an off-diagonal non-Hermitian term gives rise to exceptional points (EPs) that inhibit the formation of separable bands [2]. This behaviour indicates that, although EPs are topologically significant in momentum space, their presence can obstruct the emergence of topologically protected edge states. Conversely, when the band structure remains separable, the Chern number remains well-defined, and the bulk-boundary correspondence proves robust against non-Hermitian perturbations, allowing topological states to persist. Furthermore, as illustrated in Fig. 1, these states exhibit resilience to the non-Hermitian skin effect [3].

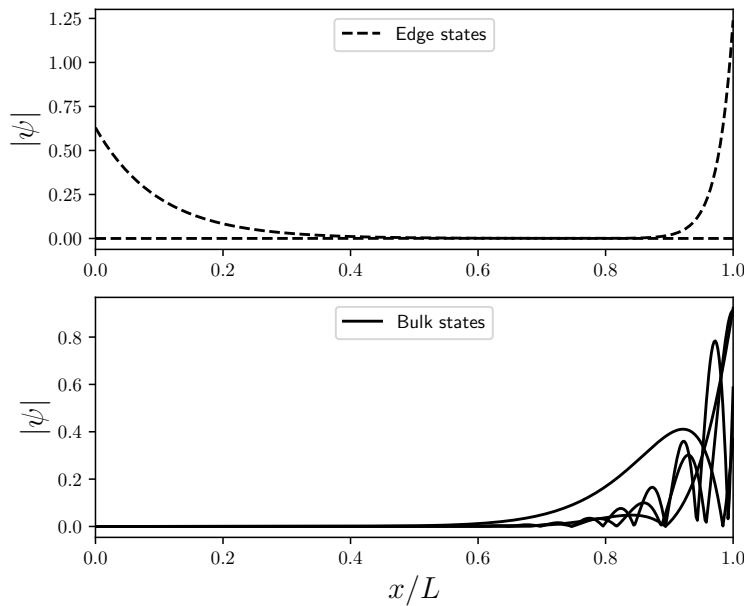


Figure 1: Norm of the right eigenstates of the non-Hermitian extension of the Qi-Wu-Zhang 2D lattice model [2]. Top panel: edge states under open boundary conditions. Bottom panel: bulk states illustrating the non-Hermitian skin effect.

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Theoretical Framework for Quantum Light in Non-Hermitian Waveguide Systems

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A theoretical framework for studying the dynamics of quantum light in dissipative integrated waveguiding systems is presented. The approach is based on Lindblad master equations which account for both, an effective non-Hermitian Hamiltonian describing non-unitary dynamics and quantum jump term. Via a suitable transformation (similar to a transformation to the interaction picture in Hermitian systems) a novel type of quantum jump super-operator is introduced and analytical solutions for a number of systems can be derived. The approach is validated by comparing with existing exact results which have been obtained with other methods. This includes results for two-photon Fock states in a waveguide coupler with one dissipative and one lossless waveguide and more complex photon states in symmetric dissipative waveguide couplers. Contrary to these other methods, the present approach can be extended in a number of ways and some first applications are presented in order to highlight the relevance of this approach for the dynamics of nonclassical states in integrated photonics within the broad context of non-Hermitian quantum systems.

Analytical and Computational Tools for 2D and 3D Topological Photonics

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The bulk-boundary correspondence principle (BBCP), which links the topological invariants of Bloch bands to the existence of interface modes, is the cornerstone of topological photonics. The BBCP is rigorously established for 1D systems [1–4], but proving it in 2D and 3D Maxwell’s electrodynamics is much more challenging [5–9]. We present theoretical and numerical evidence supporting BBCP and connect its validity to the positivity of electromagnetic energy density, a key physical constraint.

Most current studies rely on general-purpose commercial solvers, while we are striving to develop versatile computational methods and public-domain software for computing Bloch modes and analyzing interface states in periodic structures. On the computational side, our future work [10] will involve high-order Trefftz methods, advanced model reduction techniques [11–14], and rapidly-converging continued-fraction expansions implemented in the software package RectDisp [15–16].

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