Half a Century of the Kuramoto Model: Explaining Emergent Order in Time and Space

860. WE-Heraeus-Seminar

28. June – 03 July 2026

at the Physikzentrum Bad Honnef, Germany

WILHELM UND ELSE
HERAEUS-STIFTUNG

Introduction

The Wilhelm und Else Heraeus-Stiftung is a private foundation that supports research and education in science with an emphasis on physics. It is recognized as Germany's most important private institution funding physics. Some of the activities of the foundation are carried out in close cooperation with the German Physical Society (Deutsche Physikalische Gesellschaft). For detailed information see https://www.we-heraeus-stiftung.de

Aims and scope of the 860. WE-Heraeus-Seminar:

Synchronization constitutes one of the most fundamental order forming processes observed across natural, engineered and social systems. A major open challenge is to quantitatively and qualitatively capture spatio-temporal, collective order-forming processes in networked and in large nonlinear systems. Half a century ago, Professor Yoshiki Kuramoto developed a solvable model of globally coupled heterogeneous phase oscillators, shifting the research paradigm to a new level and enabling a range of numerical and analytical works. Much theoretical and applied research adopted and extended the Kuramoto model to broader contexts. Current research focuses on uncovering mechanisms of emergent order forming phenomena in finite and infinite networks with different coupling topologies and different types of units (phase oscillators, high-dimensional oscillators, chaotic systems, excitable systems, rotators), in particular also beyond the weak and local coupling regimes. The proposed Heraeus event aims to review, discuss, and advance cutting-edge research concepts and approaches, bridge frameworks in nonlinear dynamics for coupled oscillators with those of statistical physics, and push for novel directions in theory and applications.

Scientific Organizers:

Dr. Simona Olmi	CNR, Italy E-mail: simona.olmi@cnr.it
Prof. Dr. Michael Rosenblum	Universität Potsdam E-mail: mros@uni-potsdam.de
Prof. Dr. Marc Timme	Technische Universität Dresden E-mail: marc.timme@tu-dresden.de
Dr. Edmilson Roque	MPI for the Physics of Complex Systems, Dresden E-mail : eroquedo@pks.mpg.de

Introduction

Administrative Organization:

Dr. Stefan Jorda
Elisabeth Nowotka

Wilhelm und Else Heraeus-Stiftung
Kurt-Blaum-Platz 1
63450 Hanau, Germany

Phone +49 6181 92325-12
Fax +49 6181 92325-15
E-mail nowotka@we-heraeus-stiftung.de
Internet: www.we-heraeus-stiftung.de

Venue:

Physikzentrum
Hauptstrasse 5
53604 Bad Honnef, Germany

Conference Phone +49 2224 9010-120

Phone +49 2224 9010-113 or -114 or -117
Fax +49 2224 9010-130
E-mail gomer@pbh.de
Internetwww.pbh.de

Taxi Phone +49 2224 2222

Registration:

Elisabeth Nowotka (WE Heraeus Foundation)
at the Physikzentrum, reception office
Sunday (17:00 h – 21:00 h) and Monday
morning

Program

Program

Sunday, 28 June 2026

17:00 – 21:00	Registration	
18:00	*BUFFET SUPPER and informal get-together*	

Monday, 29 June 2026

08:00	*BREAKFAST*	
09:00 – 09:30	Scientific organizers	**Opening**
09:30 – 10:00	Arkady Pikovsky	**Properties of finite Kuramoto-Sakaguchi ensembles: chaos, coherence, reliability**
10:00 – 10:30	Ulrike Feudel	**Global stability of complex networks of Kuramoto (-like) oscillators**
10:30 – 11:00	*COFFEE BREAK*	
11:00 – 11:30	Igor Belykh	**How time delay generates inertia and triadic coupling in Kuramoto networks**
11:30 – 12:00	Oleksandr Burylko	**Bifurcations and catastrophes in a frustrated spin system on a ring**
12:00	*LUNCH*	

Program

Monday, 29 June 2026

14:00 – 14:30	Hugues Chaté	**The Kuramoto-Vicsek model**
14:30 – 15:00	Carlo Laing	**Neural field models with delays**
15:00 – 15:30	*COFFEE BREAK*	
15:30 – 16:00	Mehrnaz Anvari	**Finite size effect in Kuramoto oscillators with second order dynamics**
16:00 – 16:30	Shamik Gupta	**Synchronization with annealed disorder and higher-harmonic interactions in arbitrary dimensions: When two dimensions are special**
16:30 – 17:00	**Poster flashes**	
17:00 – 18:00	**Poster session**	
18:00	*DINNER*	

Program

Tuesday, 30 June 2026

08:00	*BREAKFAST*	
09:00 – 09:30	Hildegard Meyer-Ortmanns	**The Kuramoto Model: From local to repressive to non-reciprocal couplings**
09:30 – 10:00	Albert Diaz-Guilera Conrad J. Pérez-Vicente	**Beyond synchronization: pattern formation in Kuramoto-type systems**
10:00 – 10:30	Antonio Politi	**Collective dynamics in the presence of long-range interactions**
10:30 – 11:00	*COFFEE BREAK*	
11:00 – 11:30	Katharina Krischer	**When amplitude counts: Collective behavior of globally coupled Stuart-Landau oscillators**
11:30 – 12:00	Iván León	**The emergence of chaos in the Kuramoto model with random interactions**
12:00 – 12:30	Stefano Iubini	**Internal reliability and antireliability in dynamical networks**
12:30	*LUNCH*	

Program

Tuesday, 30 June 2026

14:00 – 14:30	Bard Ermentrout	**Frequency plateaus revisited**
14:30 – 15:00	Bob Rink	**Reconstructing resonant interactions in oscillator networks from noisy time series**
15:00 – 15:30	*COFFEE BREAK*	
15:30 – 16:00	Hiroshi Kori	**Rethinking phase models: Data-driven pathways to synchronization analysis**
16:00 – 16:30	Amit Pando	**Synchronization of oscillator lattices with quenched disorder using coupled laser arrays**
16:30 – 17:00	**Poster flashes**	
17:00 – 18:00	**Poster session**	
18:00	*DINNER*	

Program

Wednesday, 01 July 2026

08:00	*BREAKFAST*	
09:00 – 09:30	Serhiy Yanchuk	**Singular basins in multiscale systems**
09:30 – 10:00	Oleh Omelchenko	**Mean-field approach to finite-size fluctuations in coupled oscillator systems**
10 :00 – 10:30	Diego Pazó	**Low-dimensional reductions and the analytic continuation problem**
10:30 – 11:00	*COFFEE BREAK*	
11:00 – 11:30	Hiroya Nakao	**Phase reduction approach to control and design of coupled oscillators networks**
11:30 – 12:00	Rok Cestnik	**Two-phase quadratic integrate-and-fire neurons: Exact low-dimensional description for ensembles of finite-voltage neurons**
12:00 – 12:30	Matthias Wolfrum	**Synchronization patterns with topologically protected incoherent spots**
12:30	*LUNCH*	
14:00	**Excursion**	
18:00	*HERAEUS DINNER* *(social event with cold & warm buffet with complimentary drinks)*	

Program

Thursday, 02 July 2026

08:00	*BREAKFAST*	
09:00 – 09:30	Erik Martens	**Lie meets network dynamics: Exact macroscopic descriptions**
09:30 – 10:00	Victor Buendía Ruiz-Azuaga	**Mesoscopic theory for coupled stochastic oscillators**
10:00 – 10:30	Christian Bick	**Higher-order phase interactions, phase reduction, and collective dynamics**
10:30 – 11:00	*COFFEE BREAK*	
11:00 – 11:30	Istvan Z. Kiss	**Collective dynamics of electrochemical oscillator networks described by higher order Kuramoto models**
11:30 – 12:00	Bastian Pietras	**Exact mean-field model for networks of spiking neurons with quadratic spike-frequency adaptation**
12:00 – 12:30	Niamh Fennelly	**A Novel Framework for Non-Local Hebbian Learning in Neural Fields**
12:30	*LUNCH*	

Program

Thursday, 02 July 2026

14:00 – 14:30	Xiaozhu Zhang	**Fluctuation-Induced Patterns in Power Networks — Localization, Resonances and Extreme Events**
14:30 – 15:00	Holger Kantz	**Transient times till synchronization in Reservoir Computing**
15:00 – 15:30	*COFFEE BREAK*	
15:30 – 16:00	Markus Bär	**Hydrodynamic synchronization of elastic cilia: Stability and dynamics of metachronal waves**
16:00 – 16:30	Tomer Hacohen	**Robustness to disorder using complex coupling in laser arrays**
16:30 – 18:00	**Discussion**	
19:00	*DINNER*	

Program

Friday, 02 July 2026

08:00	*BREAKFAST*	
09:00 – 09:30	Yuri Maistrenko	**Chaotic switching in Kuramoto model with inertia**
09:30 – 10:00	Pau Clusella Coberó	**From chimeras to extensive chaos in networks of heterogeneous Kuramoto oscillator populations**
10:00 – 10:30	Alessandro Torcini	**The π -transition : A tipping point for the Kuramoto model with triadic repulsive interaction**
10:30 – 11:00	*COFFEE BREAK*	
11:00 – 11:30	Sangita Dutta	**Double explosive synchronization transition in hypergraphs**
11:30 – 12:00	Ernest Montbrió	**Synchronization modes in bipartite oscillator networks**
12:00 – 12:20	Scientific organizers	**Closing remarks**
12:20	*LUNCH*	

End of the seminar and departure

Posters

Posters

Anish Acharya	**Controlling phases via Subsystem Resetting in many-body interacting systems**
Manuel Adams	**Characterizing brain-wide interactions in different states of consciousness: EEG analysis with ordinal patterns**
Iva Bačić	**Phase locking and multistability in the topological Kuramoto model on cell complexes**
Minwoo Bae	**Spectral dimension determines criticality in nonreciprocal phase oscillators**
Baptiste Blanc	**Metabolic gels: from chemical microreactors to polymeric actuators**
Martin Brešar	**Structural, functional, and effective connectivity in chimera states**
Timo Bröhl	**Emergence of a tipping subnetworks during a critical transitions in coupled oscillator networks**
Timoteo Carletti	**Synchronization of higher-dimensional Kuramoto oscillators on networks: from scalar to matrix-weighted couplings**
Maciej Chudak	**Effect of kinetics on non-equilibrium phases of a driven Potts model**
Gonzalo Contreras-Aso	**Order amidst a world of chaos: recent developments in cluster synchronization**
Thomas Geert De Jong	**Designing networks of coupled oscillators for computing**
Arpan Dey	**Collective drift and pinning in Kuramoto-type active rotator networks with mixed-sign local feedback**

Posters

Bengi Donmez	**Characterizing resonant solitary states as relative equilibria via symmetry reduction**
Pezhman Ebrahimzadeh	**Chaotic switchings in the minimal pendula network**
Juan Gancio	**Anticipating explosive synchronization in networks of Kuramoto oscillators**
Ricardo Gutierrez	**Nonequilibrium criticality in the dynamics of synchronization in one dimension**
Carter Hinsley	**Symbolic and statistical structure of chaotic neuronal bursting due to slow oscillating currents**
Anastasiia Hrabovets	**Gradient Kuramoto-type model for energy optimization in frustrated spin systems**
Pitambar Khanra	**Taming synchronization transitions: Tiered and double explosive transitions**
Nadia Kevine Kouonang	**Synchronization in a power network of wind turbines**
Jan Meibohm	**Collective behaviour of driven Potts models**
Riccardo Muolo	**A route to higher-order Kuramoto models through phase reduction theory**
Komofor Isidore Ngongiah	**Synchronization of mechanical arms driven by Josephson junction circuits: mimicking myriapod locomotion**
Conrad J. Pérez-Vicente Albert Diaz-Guilera	**Beyond synchronization: Pattern formation in kuramoto-type systems**
Max Potratzki	**Synchronization dynamics of Kuramoto oscillators on power grid models**

Posters

Yannick Schöhs	**Origin of frequency clusters and self-organized triplet locking in the kuramoto model with Inertia**
Wilfried Segnou	**Synchronization of nonlinearly coupled Stuart–Landau oscillators on networks**
Nirmala Jenifer Selvaraj	**A robust method for classification of chimera states**
Rommel Tchinda Djeudjo	**Chimera states on m-directed hypergraphs**
Zeynep Naz Tuna	**Synchronisation in polar active solids**
Cyriel van Velzen	**Extreme synchronization and hysteresis in phase oscillator networks**

Abstracts of Talks

(in alphabetical order)

Hydrodynamic synchronization of elastic cilia: Stability and dynamics of metachronal waves

Albert von Kenne and Markus Bär

Physikalisch-Technische Bundesanstalt, Berlin, Germany

We describe cilia that interact hydrodynamically as elastically bound microspheres on circular orbits. The inclination of these orbits with respect to a no-slip wall models the ciliary power and recovery stroke. This produces an anisotropy in the viscous flow, governed by steady Stokes equations [1]. By considering microsphere dynamics on the slow timescale of synchronization, we reduce the model to coupled phase oscillators, resulting in dynamics reminiscent of Kuramoto-Sakaguchi oscillators. We determine analytical metachronal wave solutions and their stability in a periodic chain setting. Near the surface, the flow typically stabilizes metachronal waves with long wavelengths that propagate in the direction of the power stroke and metachronal waves with short wavelengths that propagate perpendicular to the power stroke. However, the dynamics of metachronal waves differ fundamentally in open chains of phase oscillators. In this case, the elasticity of the model cilia controls the wave direction and selects a particular wave number. At high elasticity, waves traveling in the direction of the power stroke are stable, while at low elasticity, waves traveling in the opposite direction are stable. For intermediate elasticity, both wave directions coexist.

In addition, we present a model of elastic cilia coupled by unsteady flow in the bulk fluid. This model was obtained by using the unsteady Stokes equation to derive a modified phase oscillator model. In this model, vorticity diffusion qualitatively and quantitatively impacts cilia coordination. Notably, stable metachronal waves are found in the absence of surface effects for cilia chains larger than the viscous penetration depth. Lastly, we discuss the role of coupling symmetries in cilia synchronization and address the stability properties of metachronal waves in two-dimensional arrays using a phenomenological description of cilia coupling.

References:

[1] A. von Kenne, M. Bär and T. Niedermayer. Physical Review E 109 (5), 054407 (2024)

[2] A. von Kenne, S. Schmelter, H. Stark and M. Bär. Phys. Rev. Research 7, L012029 (2025).

How time delay generates inertia and triadic coupling in Kuramoto networks

Igor Belykh[1]

[1]Department of Mathematics and Statistics, Georgia State University, P.O. Box 4110, Atlanta, Georgia, 30302-410, USA

Time-delayed phase-oscillator networks model diverse biological and physical systems, yet standard first-order phase reductions cannot adequately capture their high-dimensional collective dynamics. In this talk, we will present a second-order reduction for a broad class of time-delayed Kuramoto networks, transforming the original delayed Kuramoto system of one-dimensional phase oscillators into a delay-free network of two-dimensional rotators [1]. The resulting model shows that coupling delay generates intrinsic dynamics and higher-order (triadic) interactions, and it accurately predicts the emergence of complex collective patterns such as splay, cyclops [2,3], and chimera states. The reduction further reveals a qualitative division of roles: time delay acts primarily as effective inertia for higher-dimensional dynamics, including splay states, whereas the induced triadic interactions are decisive for lower-dimensional patterns such as chimeras. The method applies to networks with arbitrary topology, higher-harmonic coupling, and intrinsic-frequency heterogeneity, yielding a compact, parameter-explicit reduced model. This universal reduced description of time-delayed oscillator networks opens the door to systematic prediction and analysis of nontrivial collective dynamics in delay-coupled systems.

References

[1] L.A. Smirnov, V. Munyaev, M.I. Bolotov, and I. Belykh, arXiv:2512.10806.
[2] M.I. Bolotov, V.O. Munyayev, L.A. Smirnov, G.V. Osipov, and I. Belykh, Physical Review E, 112 (5), L052202 (2025).
[3] V.O. Munyayev, M.I. Bolotov, L.A. Smirnov, G.V. Osipov, and I. Belykh, Physical Review Letters, 130, 107201 (2023).

Mesoscopic theory for coupled stochastic oscillators

Victor Buendía

Department of Computing Sciences, Bocconi University, 20136 Milano, Italy

The theory of synchronization in systems of coupled oscillators has found a plethora of applications outside of Physics. In Neuroscience, it has been used to understand neural dynamics from microscopic systems, such as quadratic integrate-and-fire neurons [1], to whole-brain models [2].

The stochasticity present in biological systems poses an important challenge for the study of coupled oscillators. First, ansatzs such as Ott-Antonsen are only exact for deterministic systems. Several authors have tried to tackle this problem. In 2018, Tyulkina and collaborators presented a "circular cumulant" approach, improving the Ott-Antonsen ansatz's results [3]. In 2021, an alternative method with pseudocumulants was suggested [4]. However, these descriptions do not include finite-size fluctuations of the order parameters. These are essential to model biological systems, as well as to understand criticality and synchronization phase transitions.

I have proposed a theory that addresses this limitation [5]. Starting from the Dean-Kawasaki equation, which describes the evolution of the fluctuating oscillator density, I derived Langevin equations for the system's Kuramoto-Daido order parameters. I applied the results to the stochastic version of the Kuramoto model, finding finite-size corrections even for small system sizes. Exploiting the structure of fluctuations I found a reduced version of the Ott-Antonsen ansatz, improving previous numerical results. Moreover, I computed probability densities of the Kuramoto order parameter, even close to the critical point, as well as the critical exponent γ, showing it is indeed $\gamma = \gamma' = 1$. I will also show recent applications for excitatory populations of theta-neurons, studying a stochastic extension of the Montbrió-Pazó-Roxin model.

References

[1] E. Montbrió, D. Pazó, and A. Roxin, Macroscopic Description for Networks of Spiking Neurons, Physical Review X **5**, 21028 (2015).

[2] A. Daffertshofer, R. Ton, M. L. Kringelbach, M. Woolrich, and G. Deco, Distinct Criticality of Phase and Amplitude Dynamics in the Resting Brain, Neuroimage **180**, 442 (2018).

[3] I. V. Tyulkina, D. S. Goldobin, L. S. Klimenko, and A. Pikovsky, Dynamics of Noisy Oscillator Populations beyond the Ott-Antonsen Ansatz, Physical Review Letters **120**, (2018).

[4] D. S. Goldobin, M. di Volo, and A. Torcini, Reduction Methodology for Fluctuation Driven Population Dynamics, Physical Review Letters **127**, 38301 (2021).

[5] V. Buendía, Mesoscopic Theory for Coupled Stochastic Oscillators, Physical Review Letters **134**, 197201 (2025).

Bifurcations and catastrophes in a frustrated spin system on a ring

O. Burylko[1,2], **A. Hrabovets**[1], **B. Zwicknagl**[2]

[1] Institut für Mathematik, Humboldt-Universität zu Berlin, Berlin, Germany
[2]Institute of Mathematics of the NAS of Ukraine, Kyiv, Ukraine

Our aim is to describe the minimization of the energy of a frustrated spin system using a Kuramoto-type model of coupled phase oscillators. The system is ring-like, and each oscillator has attractive couplings with its nearest neighbours, repulsive couplings with its second nearest neighbours, and no interaction with other oscillators. Such a system is gradient and therefore imposes significant restrictions on the types of possible trajectories. Despite this, it exhibits high multistability which depends both on the parameters of the system and on the oscillator number. We describe possible symmetries of the system that give rise to invariant cluster manifolds, as well as certain types of collective dynamics. The system has permanent synchronization, anti-phase regimes (pi-states) and rotating waves. The existence of these regimes does not depend on any system parameters but the stability of each solution does depend significantly on them. We have demonstrated that the stability of rotating waves depends on their rotation number and identified the conditions under which these modes exhibit multistability as a function of the parameters. Statements regarding critical transitions between global minima were also proven, which is consistent with Maxwell sets from the Catastrophe theory. It turns out that such transition jumps also have a certain hierarchical structure, which depends on the rotation numbers. We have shown that pi-states are almost always unstable in the frustrated system but they play a significant role in the formation of bifurcations. In addition to the aforementioned periodic equilibria, the system also possesses other isolated equilibria, as well as continuous manifolds of them. The symmetric structure of the system gives rise to degenerate and non-standard bifurcations, such as the circular pitchfork. We also attempt to understand the properties of such a system during a thermodynamic transition. The results obtained are consistent with previous studies on energy minimization in the frustrated spin systems, and also complement them.

Two-phase quadratic integrate-and-fire neurons:

Exact low-dimensional description for ensembles of finite-voltage neurons

Rok Cestnik[1]

[1]Centre for Mathematical Sciences, Lund University, Märkesbacken 4, Lund, Sweden

We introduce a two-phase quadratic integrate-and-fire (QIF) neuron whose membrane potential evolves according to two alternating Riccati equations within finite bounds. This simple extension removes the unphysical voltage divergence of the standard QIF model while producing realistic spike waveforms. Despite this modification, the system retains an exact low-dimensional description in the thermodynamic limit, governed by a single complex Riccati equation. Expressions for collective quantities such as the firing rate and mean voltage remain compact and analytically tractable. Because the formalism preserves the mathematical structure of the standard QIF ensemble, it inherits its many generalizations and can serve as a drop-in replacement in existing mean-field frameworks, providing a more biologically plausible yet still exactly solvable neuronal model.

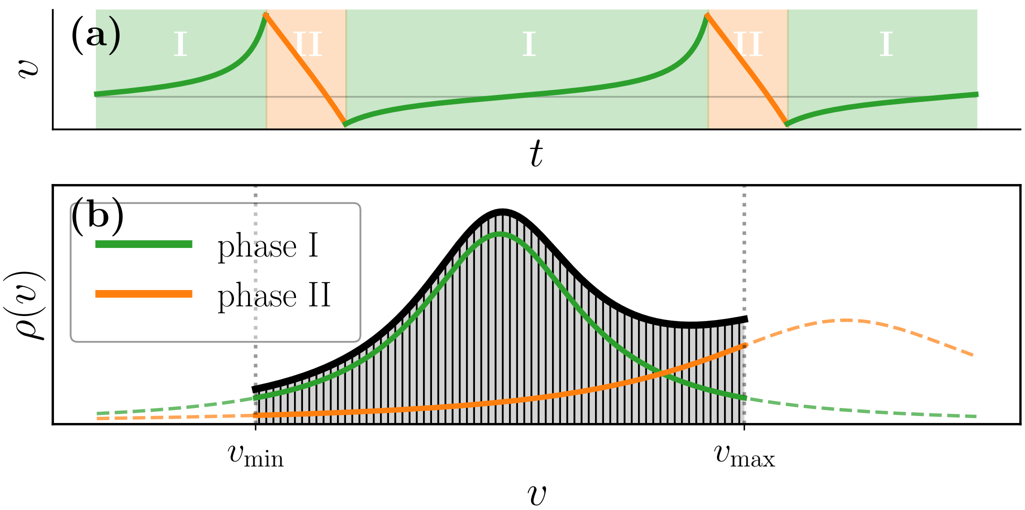


Figure 1: Schematic representation of a single two-phase QIF voltage time series in (a) and the ansatz distribution in (b). Contributions of the first phase are marked with green and of the second phase with orange. In (b) the full distribution $\rho(v)$ is depicted with black.

The Kuramoto-Vicsek model

Hugues Chaté

CEA – Saclay, France

I will summarize a host of results obtained over the last few years on two-dimensional Vicsek-style models where the self-propelled particles are endowed with an intrinsic chirality or frequency. This Kuramoto-Vicsek model, unsurprisingly, displays much richer collective behavior. In particular, one generically finds globally ordered condensates able to overcome frequency disorder even though the interactions at play are strictly local, resulting in one rare instance of synchronization in a two-dimensional system.

From chimeras to extensive chaos in networks of heterogeneous Kuramoto oscillator populations

P. Floriach[1], J. Garcia-Ojalvo[2] and P. Clusella[3]

[1]*Department of Mathematical Sciences, University of Copenhagen, Copenhagen, Denmark*
[2]*Department of Medicine and Life Sciences, Universitat Pompeu Fabra, Barcelona, Spain*
[3]*EPSEM, Department of Mathematics, Universitat Politècnica de Catalunya, Manresa, Spain*

Populations of coupled oscillators can exhibit a wide range of complex dynamical behavior, from complete synchronization to chimera and chaotic states. We can, thus, expect complex dynamics to arise in networks of such populations. Here, we analyze the dynamics of networks of populations of heterogeneous mean-field coupled Kuramoto–Sakaguchi oscillators and show that the instability that leads to chimera states in a simple two-population model also leads to extensive chaos in large networks of coupled populations.

Formally, the system consists of a complex network of oscillator populations whose mesoscopic behavior evolves according to the Ott–Antonsen equations. By considering identical parameters across populations, the system contains a manifold of homogeneous solutions where all populations behave identically. Stability analysis of these homogeneous states provided by the master stability function formalism shows that non-trivial dynamics might emerge on a wide region of the parameter space for arbitrary network topologies.

As examples, we first revisit the two-population case and provide a complete bifurcation diagram. Then, we investigate the emergent dynamics in large ring and Erdös–Rényi networks. In both cases, transverse instabilities lead to extensive space–time chaos, i.e., irregular regimes whose complexity scales linearly with the system size. Altogether, our work provides a unified analytical framework to understand the emergent dynamics of networks of oscillator populations, from chimera states to robust high-dimensional chaos.

References

[1] P. Floriach, J. Garcia-Ojalvo, and P. Clusella, Chaos **35** 023115 (2025)

Beyond Synchronization: Pattern Formation in Kuramoto-Type Systems

Albert Díaz-Guilera[1,2] and Conrad J. Pérez-Vicente[1,2]

[1]*Dep. Física de la Matèria Condensada, Universitat de Barcelona, SPAIN*
[2] *Universitat de Barcelona Institute of Complex Systems (UBICS), Barcelona, SPAIN*

The Kuramoto model is a standard framework for studying collective dynamics, but most work has focused on the onset of global synchronization. In this talk I will discuss two examples where the main question is not whether oscillators synchronize, but which *patterns* of phases and correlations emerge, and how they are regulated.

In the first part, we consider a modular neuronal network where persistent global synchronization is interpreted as a pathological state. We introduce a simple self-regulation mechanism inspired by the action of glial cells: when the local phase coherence between two nodes exceeds a given threshold, the corresponding link is temporarily disabled. The oscillators evolve on a modular network and follow Kuramoto dynamics, while the network itself becomes time dependent through this local inhibitory rule. We show that this feedback is sufficient to suppress global synchronization while preserving high levels of local coherence inside modules, leading to rich, dynamically regulated patterns of inter-module correlations.

In the second part, we analyze Kuramoto dynamics on a one-dimensional ring with both excitatory and inhibitory couplings. Even for identical oscillators, the presence of negative links qualitatively changes the attractor landscape: besides the fully synchronized state, the system supports a variety of stable and metastable phase patterns on the circle, which can be understood as competing ordered states. Combining analytical arguments and numerical simulations, we characterize these attractors and the transitions between them.

Together, these two examples illustrate how Kuramoto-type models with inhibitory interactions and simple structural features – modularity or a ring topology – naturally generate and select non-trivial patterns, suggesting a broader role of phase-oscillator models as tools for studying pattern formation beyond classical synchronization.

References

[1] Paula Pirker-Diaz, Albert Diaz-Guilera, Jordi Soriano. Self-regulation of a network of Kuramoto oscillators *Chaos Solitons & Fractals (2024)*

[2] Albert Diaz-Guilera, Dimitri Marinelli, Conrad J. Perez-Vicente Exploring the interplay of excitatory and inhibitory interactions in the Kuramoto model on circle topologies *Chaos (2024)*

Double explosive synchronization transition in hypergraphs

Sangita Dutta[1], Prosenjit Kundu[2], Pitambar Khanra[3], Ludovico Minati[4,5], Stefano Boccaletti[6,7,8], Pinaki Pal[1], and Chittaranjan Hens[9]

[1] *Department of Mathematics, National Institute of Technology, Durgapur 713209, India*
[2] *Dhirubhai Ambani Institute of Information and Communication Technology, Gandhinagar, Gujarat, 382007, India*
[3] *Department of Chemical and Biological Engineering, The State University of New York at Buffalo, Buffalo, New York 14260, USA*
[4] *School of Life Science and Technology, University of Electronic Science and Technology of China, 611731 Chengdu, China*
[5] *Center for Mind/Brain Sciences (CIMeC), University of Trento, I-38123 Trento, Italy*
[6] *Sino-Europe Complexity Science Center, North University of China, 030051 Taiyuan, China*
[7] *Research Institute of Interdisciplinary Intelligent Science, Ningbo University of Technology, 315104 Ningbo, China*
[8] *CNR - Institute of Complex Systems, Via Madonna del Piano 10, I-50019 Sesto Fiorentino, Italy*
[9] *Center for Computational Natural Science and Bioinformatics, International Institute of Informational Technology, Gachibowli, Hyderabad 500032, India*

Phase transitions are frequently seen in the networks of complex systems such as Kuramoto oscillators. In particular, the synchronization transitions can be continuous or discontinuous depending on several factors such as network structure, the coupling configuration and distribution of intrinsic natural frequencies etc. For example, adapting the Kuramoto order parameter with the coupling results in explosive or discontinuous transitions. In these case the system of the oscillators interacting through links and triangles can be governed by the following equations

$$\dot{\theta}_i = \omega_i + K_1 r_1^a \sum_{j=1}^{N} A_{ij} \sin(\theta_j - \theta_i) + K_2 r_1^b \sum_{j=1}^{N}\sum_{k=1}^{N} B_{ijk} \sin(2\theta_j - \theta_k - \theta_i), \qquad i = 1, 2, \ldots, N, \tag{1}$$

where K_1, K_2 are the couplings associated with pairwise and triangular interactions, θ_i is the phase of the i-th oscillator. A and B represents pairwise and triangular connections between nodes. ω_i is the natural frequency taken from a Lorentzian distribution. r_1 is the classical Kuramoto order parameter, the parameters a and b control the strength of adaptation in pairwise and triadic coupling respectively. Here we adapted the order parameter partially with the coupling strength and found that the oscillator system shows double explosive synchronization transition both in forward and backward direction. We tested this technique in degree-correlated uniform and power-law graphs. We also observed this phenomenon in uncorrelated random hypergraphs. The critical coupling for the onset of synchronization was also derived. We observed that the partial adaptation, the adaptation exponents and the coupling strengths tune and control the widths of the double explosive transition.

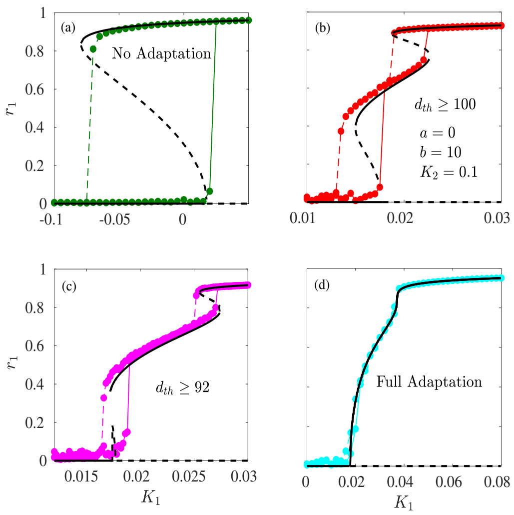


Figure 1: **Synchronization diagram.** Synchronization profiles showing r_1 as a function of K_1 for (a) no adaptation, (b) $d_{th} = 100$, (c) $d_{th} = 92$ and (d) full adaptation. Black solid and dashed lines indicate analytical curves. Colored filled circles joined with solid and dashed lines represent numerical data points for $a = 0,\ b = 10$ and $K_2 = 0.1$.

References

[1] S. Dutta, P. Kundu, P. Khanra, L. Minati, S. Boccaletti, P. Pal, and C. Hens, Physical Review Research 7, L022049 (2025).

Frequency Plateaus, Revisited

G. Zhang[1] , C. Watt[1], and B. Ermentrout[1]

[1]*University of Pittsburgh, Pittsburgh, USA*

Spatially coupled networks of oscillators that have frequency gradients will lock into traveling waves if the gradients are not too big. In networks of nearest neighbor coupled phase oscillators, the locking is lost through a saddle-node (SN) bifurcation leading to so-called frequency plateaus where clusters of oscillators have the same frequency with jumps between the clusters. In a recent paper, Sellier-Prono et al [1] showed that the loss of locking occurred via a SN in the continuous Ginzburg-Landau (GL) equation. If the dependence of the frequency of the GL oscillation on amplitude is not constant, we demonstrate that the loss of locking occurs via an Andronov-Hopf bifurcation rather than a SN.

References

[1] Sellier-Prono, Marie, et al. **PRL** 135.16 (2025): 168401.

A Novel Framework for Non-Local Hebbian Learning in Neural Fields

N. Fennelly[1], D. Avitabile[2] and Á. Byrne[1]

[1]School of Mathematics and Statistics, University College Dublin, Ireland. [2]Department of Mathematics, Faculteit der Exacte Wetenschappen, Vrije Universiteit Amsterdam, The Netherlands. MathNeuro Team, Inria branch of the University of Montpellier, France.

Synaptic plasticity, the mechanism underlying learning and memory, enables the brain to dynamically rewire in response to activity, reshaping synaptic landscapes and influencing the emergence of spatial patterns in neural activity. Neural field models are powerful tools for understanding how spatially organised patterns, such as bumps and waves of activity, develop and evolve in the brain. However, traditional neural field models typically assume spatially homogeneous interactions through the use of translationally invariant kernels. In contrast, synaptic plasticity is inherently nonlocal, as interactions between pre- and post-synaptic populations require a consistent description across distinct spatial locations.

In this work, we address this fundamental challenge by introducing a novel mathematical framework for analysing neural field models with non-local Hebbian learning, using an adapted Amari model as a baseline. This leads to a coupled system with non-local, mixed-dimensional interactions, which would typically preclude standard analytical treatment. Despite this, our framework enables a tractable Turing instability analysis, revealing the onset of pattern-forming instabilities. Numerical simulations are in agreement with the analytical predictions.

This framework also provides a foundation for incorporating more biologically realistic mechanisms of plasticity. In previous work, we introduced Kuramoto-based, synchrony-driven adaptive coupling in a network of theta-neurons, and derived a mean-field approximation for this spatially clamped system [1]. Applying the present framework to extend this next-generation model enables us to explore biophysically realistic plasticity in a spatial structure that captures population-level changes in synchrony.

This work advances our understanding of the interplay between plasticity, synchrony, and pattern formation, providing a more comprehensive framework for modelling neural self-organisation and its role in cognitive function.

References

[1] N. Fennelly, A. Neff, R. Lambiotte, A. Keane, Á. Byrne, Chaos 35, 013152 (2025).

Global stability of complex networks of Kuramoto (-like) oscillators

Kalel L. Rossi[1,2], Roberto C. Budzinski[3], Bruno R. R. Boaretto[4], Lyle E. Muller[5], Lukas Halekotte[1], Anna Vanselow[1], Ulrike Feudel[1]

[1]Theoretical Physics/Complex Systems, ICBM, Carl von Ossietzky University of Oldenburg, Oldenburg, Germany
[2]Max-Planck Institute for Neurobiology of Behavior, Bonn, Germany
[3]University of Lethbridge, Lethbridge,Canada
[4]Institute of Science and Technology, Federal University of São Paulo, São José dos Campos, São Paulo, Brazil
[5]Western Academy for Advanced Research, Western University, London, Ontario, Canada

Natural or technical systems possess often several possible stable states of operation. Linear stability theory is the appropriate tool to study the stability properties of such states with respect to small perturbations. However, in nature perturbations are not necessarily small but are finite in size. We use shock perturbations applied to either the state variables or the parameters of only one single node to show the sensitivity of complex networks. This sensitivity may be desired—for instance, in the brain, where a large repertoire of different dynamics, particularly different synchronization patterns, is crucial—or may be undesired—for instance, in power grids, where disruptions to synchronization may lead to blackouts. We show that networks of coupled phase oscillators, Kuramoto oscillators, with nonlinear interactions can acquire a very large and complicated sensitivity to changes made in either their units' parameters or in their network connections. Even modifications made to a parameter of a single node can radically alter the global dynamics of the network in an unpredictable manner. For the second example of a Kuramoto-like power grid model of Great Britain we find that the shock perturbations at many nodes only allow for a few outcomes, while the stability landscape of a few nodes show extreme complexity with more than a hundred basins of attraction. Particularly complex domains in the latter case can be related to unstable invariant chaotic sets of saddle type. Most importantly, we show that the characteristic dynamics on these chaotic saddles can be associated with certain topological structures of the network.

Synchronization with annealed disorder and higher-harmonic interactions in arbitrary dimensions: When two dimensions are special

R. Majumder[1] and S. Gupta[1]

[1]Tata Institute of Fundamental Research, Homi Bhabha Road, Mumbai 400005, India

The impact of disorder on collective phenomena depends crucially on whether it is quenched or annealed. In synchronization problems, quenched disorder in higher dimensional Kuramoto models is known to produce unconventional dimensional effects, including a striking odd even dichotomy: synchronization transitions are continuous in even dimensions and discontinuous in odd dimensions. By contrast, the impact of annealed disorder has received comparatively little attention. Here we study a D dimensional Kuramoto model with both fundamental and higher-harmonic interactions under annealed disorder, and develop an arbitrary dimensional center-manifold framework to analyze the nonlinear dynamics near the onset of collective behavior. We show that annealed disorder fundamentally alters the role of dimensionality. With fundamental coupling alone, it completely removes the odd even dichotomy, yielding continuous synchronization transitions with universal mean-field scaling in all dimensions. Higher-harmonic interactions preserve this universality while rendering the synchronization transition tunable between continuous and discontinuous. At the same time, they give rise to a novel, correlation-driven transition between a symmetry-protected incoherent phase and a symmetry broken state lacking global synchronization, which is therefore invisible to the conventional Kuramoto order parameter. This transition is continuous in two dimensions but discontinuous in higher dimensions, revealing an emergent and previously-unrecognized special role of two dimensions.

References

[1] R. Majumder and S. Gupta, arXiv:2601.10646 (2026)

Robustness To Disorder Using Complex Coupling In Laser Arrays

<u>T. Hacohen</u>, A. Pando, K. Manannikov, E. Bernstein, A. Poddubny,* A. Friesem *and N. Davidson*

Department of Physics of Complex Systems, Weizmann Institute of Science, Rehovot 7610001, Israel

Coupled laser arrays offer a highly versatile approach to experimentally study Kuramoto dynamics, where each laser acts as a phase oscillator and the coupling between them can be precisely controlled. Manipulating the phase and amplitude of light transferred between the lasers physically is equivalent to a complex coupling term in the Kuramoto model. Through the lens of non-Hermitian photonics, these systems can be described by a complex loss-frequency band structure which can be engineered and designed at will, enabling research and simulation of various new phenomena. Using a digital degenerate cavity [1], we implement an array of hundreds of lasers that obey Kuramoto dynamics [2] with precise digital control over the lattice geometry, frequency detuning and both dissipative and dispersive coupling. This allows us to experimentally probe the effects of complex coupling on synchronization of Kuramoto oscillators. For few lasers, I will present measurements of frequency pulling and pushing caused by dispersion, in agreement with theory. I will then focus on large arrays, describe a novel synchronization transition induced by a linear dispersion relation, and investigate it numerically and experimentally. We show that the system exhibits robustness to correlated disorder, a sharp transition out of synchrony at a much higher disorder threshold than other models, and unusual cluster structures different from purely dissipative models studied thus far. I will present preliminary experimental results and discuss our current efforts to measure these effects in the lab, leveraging fine control and tunability to investigate the mechanisms by which complex coupling and linear dispersion lead to better synchronization.

References:

[1] Chene Tradonsky, Simon Mahler, Gaodi Cai, Vishwa Pal, Ronen Chriki, Asher A Friesem, and Nir Davidson. High-resolution digital spatial control of a highly multimode laser. Optica, 8(6):880–884, 2021
[2] Hidetsugu Sakaguchi, Shigeru Shinomoto, and Yoshiki Kuramoto. Mutual entrainment in oscillator lattices with nonvariational type interaction. Progress of theoretical physics, 79(5):1069–1079, 1988

Internal reliability and antireliability in dynamical networks

S. Iubini[*,a,b], **T. Matteuzzi**[c], **F. Bagnoli**[c,b], **M. Baia**[c,b], **A. Pikovsky**[d]

[a]*Istituto dei Sistemi Complessi, Consiglio Nazionale delle Ricerche, via Madonna del Piano 10, I-50019 Sesto Fiorentino, Italy*

[b]*Istituto Nazionale di Fisica Nucleare, sezione di Firenze, via G.Sansone 1, I-50019, Sesto Fiorentino, Italy*

[c]*Department of Physics and Astronomy and CSDC, University of Firenze, via G.Sansone 1, I-50019, Sesto Fiorentino, Italy*

[d]*Department of Physics and Astronomy, University of Potsdam, Karl-Liebknecht-Str. 24/25, 14476, Potsdam-Golm, Germany*

ABSTRACT

We consider finite dynamical networks and define internal reliability based on the synchronization properties of a replicated unit or a set of units. If the states of the replicated units coincide with their prototypes, they are reliable; otherwise, if their states differ, they are anti-reliable. Quantification of reliability using the transversal Lyapunov exponent enables a straightforward analysis of different models. For a Kuramoto model of globally attractively coupled phase oscillators with a distribution of natural frequencies, we show that before the onset of synchronization, peripheral in frequency units (at the edges of the distribution) are anti-reliable, while central ones (with natural frequencies close to the mean frequency) are reliable. For repulsive coupling, the complementary configuration occurs, with central anti-reliable units and peripheral reliable ones. For this model, reliability can be expressed through phase correlations in a sort of fluctuation-dissipation relation. Sufficiently large sub-networks in the Kuramoto model are always anti-reliable; the same holds for a recurrent neural network, where individual units are always reliable. Furthermore, we show that other coupled oscillator models (Winfree-type coupled phase oscillators, coupled rotators, coupled Stuart-Landau oscillators) demonstrate patterns of reliability and anti-reliability similar to those of the Kuramoto setup.

REFERENCES

[1] T. Matteuzzi, F. Bagnoli, M. Baia, S. Iubini, A. Pikovsky Physical Review E, 112(4), 044216 (2025)

*Presenting author.

Transient times till synchronization in Reservoir Computing

Daniel Estevez-Moya[1], Misha Chai[1], Erick Alejandro Madrigal Solis[1],
Ernesto Estevez-Rams[2], Holger Kantz[1]

[1] Max Planck Institute for the Physics of Complex Systems, Dresden, Germany
[2] Universidad de La Habana, La Habana, Cuba

What is commonly called a reservoir computer (RC) is a driven dynamical system with network properties: The reservoir is a non-symmetric directed network with low link density, which is driven by some input. While the internal state of the network evolves according to its own dynamics and the input, output weights for a linear combination of the internal state variables are fitted so that the system can perform some forecasting task. When starting from a random initial state, the network's internal state first has to synchronize with the driver in the sense of generalized synchronization, before the training of the output weights can be started. This 'initial state forgetting' property, also called 'echo state property', is the key requirement for the system to work.

We study this initial transient behaviour in detail, with particular focus on transient times till synchronization. We show that for spectral radii of the adjacency matrix close to or even larger than unity, transient times follow distributions with long tails. This means that depending on the random realization of the adjacency matrix, time till synchronization can be very long, thereby wasting a large part of the input data. Further analysis demonstrates a paradoxical role of the network topology: While cycle-rich networks seem to offer more diverse dynamics which can improve the predictive power of the system after training, they possess reduced stability of the synchronized state.

References

Daniel Estevez-Moya et al., Chaos 36, 043104 (2026)

Collective Dynamics of Electrochemical Oscillator Networks Described by Higher-Order Kuramoto Models

Istvan Z. Kiss
Department of Chemistry, Saint Louis University, 3501 Laclede Ave, St Louis, MO, 63103, USA.

The Kuramoto model and its first-order phase-reduced extensions provide a foundational basis for understanding synchronization in weakly coupled oscillators. However, many experimental systems operate outside the regime in which purely pairwise sinusoidal coupling and small frequency detuning are sufficient. Electrochemical reactions on electrode arrays often exhibit nonlinear feedback mechanisms that generate robust oscillatory behavior. Understanding the network dynamics underlying the operation of such electrochemical systems is essential both for describing and controlling their behavior and for connecting experimental observations to phase-based theoretical frameworks. Using experiments on coupled electrochemical oscillators, we identify regimes in which standard first-order Kuramoto descriptions based on weak coupling and small detuning fail to reproduce observed phase-locking behavior.

For pairs of coupled electrodes, experiments demonstrate how changes in coupling strength systematically modify the stability of in-phase, anti-phase, and out-of-phase synchronized states, and how higher-order phase terms account for these transitions within a generalized Kuramoto description. In larger electrode networks, we observe a robust form of triplet phase locking, termed *hyperlocking*, that arises from nonlinear and resonant phase interactions and can be described using higher-order and nonpairwise coupling terms. We further present practical methods for inferring these generalized phase models directly from experimental measurements. Together, these results illustrate how electrochemical oscillator arrays inform the development of advanced Kuramoto-type descriptions for real, strongly coupled oscillatory systems.

Rethinking Phase Models: Data-Driven Pathways to Synchronization Analysis

H. Kori
Department of Complexity Science and Engineering, The University of Tokyo

This talk presents some complementary data-driven approaches that advance the modeling and inference of coupled oscillatory systems relevant to the Kuramoto framework.
First, we develop an isochron-free phase reduction based on generalized phases, yielding a closed stroboscopic circle-map description that supports robust coupling inference even under strong synchrony. Second, we propose a state-space modeling framework for estimating phase response curves (PRCs) via Kalman filtering, allowing noise-robust estimation and decomposition of multiple oscillatory components from a single waveform. Third, to address phenomena that cannot be captured by phase models alone, we introduce a method to infer Stuart-Landau phase-amplitude equations directly from waveforms, enabling quantitative prediction of amplitude-mediated synchronization transitions in both synthetic and experimental systems. Together, these methods provide a unified data-driven toolkit for reduced modeling and inference in complex oscillatory systems.

When amplitude counts:

Collective behavior of globally coupled Stuart-Landau oscillators

Nicolas Thomé, Yukiteru Murakami, Yannick Schöhs, Katharina Krischer

Physics Department, School of Natural Sciences, Technical University of Munich, Garching, Germany

In 1994, Nakagawa and Kuramoto published a paper in which they 'try […] to find new dynamical features as are absent in those models without amplitude degree of freedom' [1]. In this publication, they identify collective chaos of a few clusters, as well as of fused clusters where the oscillators distribute themselves along a closed loop, as examples of collective behavior requiring a degree of freedom in amplitude. In this talk, we will demonstrate that these cluster solutions are embedded in a hierarchy of cluster states, organized by codimension-2 points, coined cluster singularities. Furthermore, we explore how this organizational structure evolves when oscillator frequencies follow a unimodal distribution rather than being identical.

The coherent behavior predicted for the ensemble of oscillators with slightly heterogeneous frequencies was confirmed in electrochemical experiments. Furthermore, the experiments revealed a further evolution of the synchronized ensemble in a neighboring parameter range: it split into two clusters with different time-averaged frequencies. All experimental features of the transition to frequency-clusters could be reproduced in simulations of globally coupled Stuart-Landau oscillators. More detailed investigations showed that amplitude dynamics are also required for this diversification process.

[1] N. Nakagawa and Y. Kuramoto, Physica D 75 (1994) 74-80

Neural field models with delays

C. R. Laing[1] and O. E. Omel’chenko[2]

[1]Massey University, Auckland, New Zealand
[2]University of Potsdam, Potsdam, Germany

The theta neuron is a simple model for the onset of periodic firing through a SNIC bifurcation. Infinite networks of theta neurons can be studied using the Ott/Antonsen ansatz, resulting in complex-valued Riccati equations that describe the macroscopic dynamics, in the form of a neural field model. Periodic solutions of these equations can be found self-consistently in a computationally efficient manner. We extend results from [1] to include the effects of distributed delays and conduction delays, finding and following periodic solutions in next generation neural field models. We concentrate on travelling waves, “breathing” bump states, and spatially uniform periodic states. We also determine the stability of stationary states as a function of delay parameters and find that most periodic solutions are created in Hopf bifurcations of these stationary states.

References

[1] C. R. Laing and O. E. Omel’chenko, Biological Cybernetics **117**, 259–274, (2023)

The emergence of chaos in the Kuramoto model with random interactions.

I. León[1] and D. Pazó[2]

[1] Departamento de Matemática Aplicada y Ciencias de la Computación, Universidad de Cantabria, Santander, Spain
[2] Instituto de Física de Cantabria (IFCA), Universidad de Cantabria—CSIC, 39005 Santander, Spain

Frustrated random interactions play a central role in the physics of spin glasses and strongly influence the collective dynamics of interacting systems. Motivated by this perspective, we study the Kuramoto model with quenched random couplings, the simplest ensemble of oscillators with fully disordered interactions. Building on early work by Daido and by Stiller and Radons, which reported quasi-glassy behavior and chaotic dynamics in small systems, we revisit longstanding open questions concerning the thermodynamic limit and the scaling of chaos. Using extensive numerical simulations combined with a perturbative analytical approach based on the cavity method, we show that frequency entrainment does not persist in the thermodynamic limit. In the weak coupling regime, we derive closed-form expressions for the average frequency shift and the dissipative properties of the system. Our results further demonstrate that chaotic dynamics dominate a large portion of parameter space, including regions previously associated with glassy behavior. Remarkably, the largest Lyapunov exponent exhibits an unexpected universality: in the weak coupling limit, its asymptotic scaling is independent of the degree of coupling asymmetry, despite the strong impact of asymmetry on the microscopic dynamics. These findings provide a revised and more complete picture of randomly coupled oscillator populations and clarify the role of disorder in shaping their collective behavior.

Chaotic switching in Kuramoto model with inertia

Yuri Maistrenko

Institute of Mathematics and Technical Centre, National Academy of Sciences, Kyiv, Ukraine

and

PGI-14 Neuromorphic Compute Nodes, Forschungszentrum Jülich, Jülich, Germany

We report the *chaotic switching behavior* in small networks of globally coupled identical phase oscillators given by the Kuramoto model with inertia [1,2]. The switching is caused by the multiple co-existence of *weak chimera states*. Parameter regions for the chimeras are obtained, which are surrounded by the so-called riddled shadow, where the network behavior represents a heteroclinic switching between saddle chimeras. The switching arises in a riddling bifurcation. Basins of the chimera states become riddled and intermingled, causing an extreme sensitivity of individual trajectories to the initial conditions, system parameters, simulation algorithms, and noise. An additional puzzling of the problem is induced by a presumable presence in the model phase space of stable equilibriums or/and limit cycles, and the other type stable chimera states, co-existing with the switching. System trajectories can then be eventually captured by the stable states, which occurs in an unpredictable time moment after short or long transient.

We have also found this kind chaotic switching behavior in a similar model of coupled pendula, i.e., with an additional sinusoidal term [3]. Our finding indicates a common, probably universal phenomenon in networks of different nature, representing a novel challenging task of the Network Science.

References:

[1]. Y.Maistrenko, S.Brezetsky, P.Jaros, R.Levchenko, and T.Kapitaniak. Smallest chimera states, Physical Review **E 95**, 010203(R) (2017);

[2]. S.Brezetsky, P.Jaros, R.Levchenko, T.Kapitaniak, and Y.Maistrenko. Chimera complexity, Physical Review **E 103**, L050204 (2021);

[3]. P.Ebrahimzadeh, M.Schiek, and Y.Maistrenko. Chaotic switching in the minimal pendula network (submitted), arXiv preprint: 2602.24284 (2026).

Lie meets Network Dynamics: Exact macroscopic descriptions

E. A. Martens[1,2] and S. Dharmavaram[3]

[1]Dept. Of Science and Environment, Roskilde University, Denmark
[2]Centre for Mathematical Science, Lund University, Sweden
[3] Department of Mathematics and Statistics, Bucknell University, Lewisburg, USA

The Kuramoto model has become a paradigmatic framework for understanding synchronization and collective dynamics in coupled oscillator networks. In this talk, I will show how exact macroscopic descriptions of such systems arise naturally from the classical theory of Lie and Scheffers. This geometric perspective unifies several well-known results in synchronization theory and extends to a broader class of network dynamical systems. Time permitting, I will discuss applications and directions for future research.

The Kuramoto Model: From local to repressive to non-reciprocal couplings

H. Meyer-Ortmanns[1]

[1]*School of Science, Constructor University, Bremen, Germany*

We will review results for different versions of the Kuramoto model, ranging from Kuramoto oscillators, entrained by pacemakers in local attractive couplings to generalized Kuramoto models with adaptive and non-reciprocal couplings, where we observe metastable behavior in synchronization patterns [1]. A particularly interesting case of coupled Kuramoto oscillators is realized when repressive couplings are combined with frustration. In such ensembles, we find the “order-by-disorder” phenomenon, physical aging, the emergence of long periods of the Kuramoto order parameter and features of self-similarity in the attractor space. In these systems, under special conditions, even global couplings can be dynamically generated out of local ones with an ensuing dimensional reduction of the phase space in the sense of Watanabe-Strogatz. When the Kuramoto model is extended to include inertia and stochastic fluctuations, it may serve to predict blackout events in power grids. Our most recent reference related to the Kuramoto model is [1].

References

[1] S. Nag Chowdhury and H. Meyer-Ortmanns, Front. Netw. Physiol. 6, 1774273 (2026).

Synchronization modes in bipartite oscillator networks

P. Pomés, B. Pietras, and E. Montbrió

Deparment of Engineering, Pompeu Fabra University, Barcelona, Spain

Collective oscillations in neuronal systems often emerge from interactions between excitatory and inhibitory populations rather than from recurrent coupling within a single ensemble. Motivated by the coexistence of strongly and partially synchronous regimes in these systems, we study the canonical Kuramoto–Sakaguchi [1] model on a bipartite network. Despite its minimal structure, the model exhibits rich collective dynamics including both continuous and discontinuous transitions from full to partial synchrony (PS). In this PS regime, a global oscillation fails to entrain the oscillators in one of the two populations, which represents a paradigmatic example of self-organized quasiperiodicity [2]. Our results show that complex synchronization patterns naturally arise in multi-population oscillator networks interacting solely through cross-population, linear mean-field coupling.

References

[1] H. Sakaguchi and Y. Kuramoto, Prog. Theor. Phys. **76**, 576 (1986)
[2] M. Rosenblum and A. Pikovsky, Phys Rev Lett. **98**, 064101 (2007)

Phase Reduction Approach to Control and Design of Coupled Oscillators Networks

H. Nakao[1]

[1]*Institute of Science Tokyo, Tokyo, Japan*

The collective dynamics of coupled phase oscillators have been extensively studied for over 50 years since the pioneering work of Winfree and Kuramoto. Phase reduction theory, which systematically approximates the dynamics of limit-cycle oscillators under weak perturbations using simple phase equations, has long played a central role in the analysis, and is recently being applied also to control and design of oscillator networks. In this talk, I will discuss several topics regarding the control and design of collective dynamics in coupled oscillators via phase reduction, including the design of nonlinear oscillators with prescribed periodic orbits and phase response properties, optimization of coupling functions to accelerate synchronization, efficient synchronization based on Koopman-based phase-amplitude reduction, and data-driven mutual synchronization using phase autoencoder neural networks.

References

[1] Y. Kuramoto, *Chemical Oscillations, Waves, and Turbulence* (Springer, 1984)
[2] A. Pikovsky, M. Rosenblum, J. Kurths, *Synchronization: A Universal Concept in Nonlinear Sciences* (Cambridge, 2001)
[3] G. B. Ermentrout, D. H. Terman, *Mathematical Foundations of Neuroscience* (Springer, 2010)
[4] H. Nakao, Contemporary Physics **57**, 188 (2016)
[5] S. Takata, Y. Kato, H. Nakao, Chaos **31**, 093124 (2021)
[6] N. Namura, H. Nakao, Chaos, Solitons and Fractals **185**, 115080 (2024)
[7] N. Namura, H. Nakao, Nonlinear Dynamics **113**, 10105 (2025)
[8] K. Yawata, K. Fukami, K. Taira, H. Nakao, Chaos **34**, 063111 (2024)
[9] K. Yawata *et al.*, Phys. Rev. E **112**, 064211 (2025)

Mean-field approach to finite-size fluctuations in coupled oscillator systems

O. E. Omel’chenko[1] and **G. A. Gottwald[2]**

[1] *University of Potsdam, Potsdam, Germany*
[2] *University of Sydney, Sydney, Australia*

Networks of coupled phase oscillators are one of the most studied dynamical systems with numerous applications in physics, chemistry, biology, and engineering. A variety of methods exists to explain their properties and dynamics in the thermodynamic limit, when the network size tends to infinity. However, the behaviour of such systems in the more realistic case of a finite number of oscillators still remains poorly understood. In this talk, we revisit the paradigmatic Kuramoto-Sakaguchi model [1] describing synchronization transitions in networks of all-to-all coupled heterogeneous phase oscillators, and propose an ab initio approach for characterizing analytically the statistical properties of finite-size fluctuations in this system [2]. Our framework is applicable to any stationary partially synchronized state and does not require any prior knowledge about its structure. Moreover, it is sufficiently general such that it can be applied to a broader class of interacting particle systems.

References

[1] H. Sakaguchi and Y. Kuramoto, Prog. Theor. Phys. **76**, 576 (1986)
[2] O. E. Omel’chenko and G. A. Gottwald, arXiv:2511.03700 (2025)

Synchronization of oscillator lattices with quenched disorder using coupled laser arrays

A. Pando, T. Hacohen, S. Gadasi, E. Bernstein, N. Stroev, A. Friesem, and N. Davidson

Department of Physics of Complex Systems, Weizmann Institute of Science, Rehovot 7610001, Israel

We experimentally study the onset of synchronization in a coupled laser array that directly realizes Kuramoto dynamics. Our experimental system consists of arrays of hundreds of lasers where the geometry, coupling architecture, and disorder are highly controllable. This flexibility has previously allowed us to investigate a wide range of Kuramoto-related phenomena, including crowd synchrony, geometric frustration in Kagome lattices, and topological defects in oscillator rings. In this talk, I will focus on our recent work studying the interplay between disorder and synchrony[1]. We introduce controlled quenched disorder via frequency detuning with tunable magnitude and spatial correlations. For uncorrelated disorder, we observe how the steady-state synchronization is suppressed and compare our findings to theoretical predictions. For correlated disorder, we reveal non-monotonic behavior that depends on the ratio between the disorder's correlation length and the characteristic size of phase-locked clusters. Finally, I will discuss our current efforts to go beyond local dissipative coupling and implement long-range coupling to experimentally measure the canonical Kuramoto second-order synchronization phase transition.

Main References:

[1] A. Pando, S. Gadasi, E. Bernstein, N. Stroev, A. Friesem, and N. Davidson, Phys. Rev. Lett. 133, 113803 (2024).

Low-dimensional reductions and the analytic continuation problem

D. Pazó[1] and R. Cestnik[2]

[1]*Institute of Physics of Cantabria (U. Cantabria - CSIC), Santander, Spain*
[2] *Centre for Mathematical Science (Lund University), Lund, Sweden*

In a milestone paper [1], Ott and Antonsen proposed an ansatz to solve the continuity equation describing the Kuramoto model in the thermodynamic limit. The lower-dimensional description enabled by the Ott-Antonsen ansatz was subsequently applied to a number of systems consisting of globally coupled phase-like units. Building on this framework, the Lorentzian ansatz (a cousin of the Ott-Antonsen ansatz) extended this approach to populations of quadratic integrate-and-fire (QIF) neurons (or real Riccati equations) [2].

More recently, new ansatzes were introduced to reduce the dimensionality of populations composed of two-dimensional (or higher) units; namely, orientable agents [3] and complex Riccati equations [4]. The latter system may represent, for instance, a population of QIF neurons with clustered substructure [4].

In the schemes [1,2,4], complete reduction to ODEs was achieved when the heterogeneity (e.g., the natural frequencies) followed a rational density function (e.g., Cauchy/Lorentzian). Crucially, this required performing an analytic continuation of certain functions. When this fails, the dimensionality reduction can yield uncontrolled terms or can even become a complete failure.

In this contribution, we revisit the dimensionality reduction methods [1,2,4], and discuss when analytic continuation holds exactly, approximately, or not at all [5].

References

[1] E. Ott and T. M. Antonsen, Chaos **18**, 037113 (2008)
[2] E. Montbrió, D. Pazó, and A. Roxin, Phys. Rev. X **5**, 021028 (2015)
[3] S. Chandra, M. Girvan, and E. Ott, Chaos **29**, 053107 (2019)
[4] D. Pazó and R. Cestnik, Phys. Rev. E **111**, L052201 (2025)
[5] R. Cestnik and D. Pazó (in preparation).

Exact Mean-Field Model for Networks of Spiking Neurons with Quadratic Spike-Frequency Adaptation

B. Pietras[1-3], Pau Clusella[4] and E. Montbrió[1]

[1]*Department of Engineering, Universitat Pompeu Fabra, Barcelona, Spain*
[2] *NISYS GmbH, Frankfurt am Main, Germany*
[3] *Ernst Strüngmann Institute (ESI) gGmbH of the Max Planck Society, Frankfurt am Main, Germany*
[4] *EPSEM, Departament de Matemàtiques, Universitat Politècnica de Catalunya, Manresa, Spain*

A cornerstone of nonlinear dynamics is the Ott-Antonsen (OA) ansatz, which provides an exact low-dimensional description for the collective dynamics of heterogeneous phase oscillators. This framework has recently been extended to computational neuroscience via the Lorentzian ansatz, enabling an exact mean-field reduction for networks of Quadratic Integrate-and-Fire (QIF) neurons. These “next-generation” neural mass models bridge the gap between microscopic networks of spiking neurons and their macroscopic firing-rate dynamics. Yet, extending this tractability to neurons with internal degrees of freedom, such as spike-frequency adaptation, remains a theoretical challenge.

Here, we present an exact reduction for a large ensemble of QIF neurons incorporating a specific, biologically grounded form of adaptation: quadratic spike-frequency adaptation (QSFA). Unlike previous approximations that rely on a global adaptation variable, our approach preserves the neuron-specific nature of adaptation. By mapping the adaptive network to an effectively one-dimensional QIF model, we derive a low-dimensional system of firing rate equations (FRE) that remain analytically tractable.

A key conceptual result of our model is the discovery of an adaptation-induced reduction in frequency heterogeneity. We show that neurons with higher intrinsic firing rates undergo more pronounced adaptation, effectively “compressing” the population's frequency distribution and significantly promoting collective synchronization. While this bears resemblance to the frequency-adaptation mechanism investigated by Ott and Antonsen in 2017 for coupled phase oscillators, we emphasize that in our model, adaptation is independent of the level of synchrony and depends solely on individual neuron firing frequencies. Our FRE accurately capture the emergence of collective oscillations, bursting, and macroscopic chaos, offering a powerful tool for studying the interplay of adaptation and synchrony.

Properties of finite Kuramoto-Sakaguchi ensembles: chaos, coherence, reliability

A. Pikovsky

University of Potsdam, Potsdam, Germany

The Kuramoto-Sakaguchi model of globally coupled phase oscillators is well-studied in the thermodynamic limit of infinite population size. In this talk, I focus on the effects appearing in finite ensembles. First, the dynamics of oscillators is chaotic in a wide range of the coupling parameter. I describe properties of the largest Lyapunov exponent and its nontrivial dependence on the ensemble size. Second, due to the chaoticity of the dynamics, the mean field that appears beyond the synchronization transition is not purely periodic but exhibits fluctuations in phase and amplitude. These fluctuations lead to phase diffusion, which characterizes the coherence of the global mode. I discuss the dependence of the diffusion constant on the ensemble size. Finally, I discuss the internal reliability of the units in the ensemble, defined by the behavior of replicas of a particular oscillator (or a group of oscillators) that receive the same forcing as the replicated unit. This reliability can be defined for generic networks of dynamical units. I show that, for the Kuramoto-Sakaguchi ensemble, a variant of the fluctuation-dissipation relation holds, linking reliability to correlations among the oscillators.

Collective dynamics in the presence of long-range interactions

Antonio Politi [1,2]

[1)] ISC-CNR, Institute for Complex Systems, Sesto Fiorentino, Italy
[2)] University of Aberdeen, Aberdeen, UK, a.politi@abdn.ac.uk

Abstract
Networks of (nonlinear) dynamical systems often do not only display a microscopic chaotic evolution, but also a macroscopic irregular dynamics. This regime, which, strictly speaking, can be rigorusly defined only in the limit of infinitely many units, extends the concept of thermodynamic phase to stationary, but time-dependent, regimes. Its very existence is due to the non-equilibrium properties of the underlying dynamics.
This behavior has been observed in many different setups ranging from mean-field models, to networks with a finite connectivity - either in the presence or absence of heterogeneity.
Here, I mostly focus on two networks of Stuart-Landau oscillators, which represent a natural extension of Kuramoto models, where amplitude plays a non-trivial role.
The first setup consists of a one-dimensional chain of identical oscillators characterized by long-range interaction (power-law decay), showing that long-wavelength Fourier modes may behave chaotically. In other words a sort of hydrodynamic description seems appropriate even in the absence of underlying conservation laws.
The second setup is a standard network of globally coupled Stuart-Landau oscillators, previously studied by Kuramoto himself. I focus on a regime where the phases of the oscillators asymptotically align along a smooth closed curve, so that the amplitudes play a subtle but important role: the alignment of oscillator phases is indeed kept, but the curve itself evolve in time, contributing to an increased dynamical complexity
A similar phenomenology is finally discussed in a chain of truly phase oscillators: a Kuramoto-Daido model, where the coupling function is composed of three harmonics.s

References

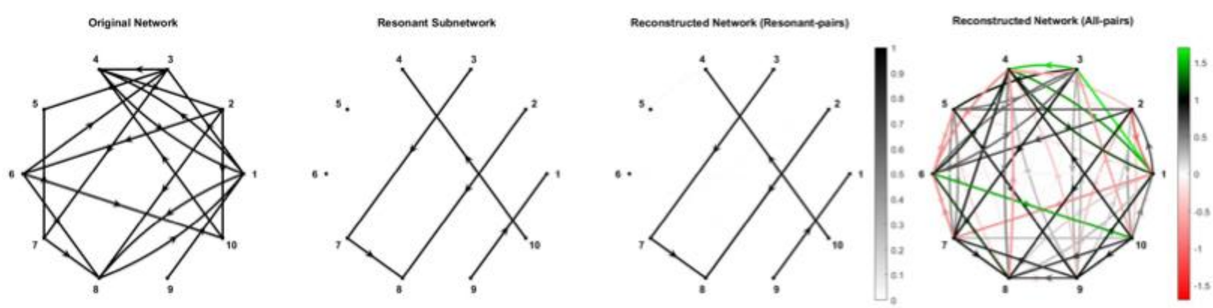
1. G. Robb and A. Politi, *Collective dynamics out of thermodynamic equilibrium*, Phys. Rev, E **95**, 040201 (2017).
2. P. Clusella and A. Politi, *Between phase and amplitude oscillators*, Phys. Rev. E, **99**, 062201 (2019).
3. P. Clusella and A. Politi, *Irregular collective dynamics in a Kuramoto-Daido system,* J. Physics: Complexity, **2**, 014002 (2021).

Reconstructing resonant oscillator interactions from noisy time series

Bengi Dönmez[1] and <u>Bob Rink</u>[2]

(1) Vrije Universiteit Amsterdam, Department of Mathematics, De Boelelaan 1111, 1081 HV Amsterdam, The Netherlands, b.donmez@vu.nl
(2) Vrije Universiteit Amsterdam, Department of Mathematics, De Boelelaan 1111, 1081 HV Amsterdam, The Netherlands, b.w.rink@vu.nl

Networks of coupled oscillators (such as neurons, chemicals, animal species) arise abundantly in biology. I will discuss the problem of inferring the differential equations that govern such oscillatory processes from time series observations: the so-called reconstruction problem. Reconstruction is typically done by first extracting phase signals from the time series, and subsequently fitting phase equations to these signals. A fundamental problem inhibiting the reliable reconstruction of phase equations is that they equations are typically not unique: distinct phase equations may generate the same or highly similar time series. As a result, common reconstruction methods such as SINDy are ill-posed in this context, sensitive to noise, and their outcomes are hard to interpret. Our solution to this problem is to reconstruct a "representative" differential equation, the so-called resonant normal form, from the observations. We prove that our method reconstructs the resonant normal form correctly and can hence learn the true dynamics of the oscillator network.



Network of Kuramoto oscillators (left panel) and the subnetwork of its resonant interactions (second panel). In the presence of observational noise, the original network is not correctly reconstructed (right panel), while our resonant reconstruction method correctly reconstructs the resonant subnetwork (third panel).

Synchronization patterns with topologically protected incoherent spots

M. Wolfrum

Weierstrass Institute for Applied Analysis and Stochastics, Berlin, Gemany

We study synchronization patterns with self-organized coherent regions of different rotation numbers, which can be found in a systems of active rotators with nonlocal attractive coupling and global repulsive coupling. We show that at the interface between the regions there appear small incoherent spots that are remarkably robust due to a specific topological protection mechanism. The incoherent spots are extremely small and, hence, appear as regions of extensive chaos only for sufficiently large system size. The incoherent motion also triggers an irregular lateral motion of the whole pattern. For smaller systems, the diffusive motion may halt through a pinning transition leading to a periodic pattern. This leads to an intriguing scenario where for varying system size time-periodic patterns alternate with chaotic motion. We show that this transition also induces a remarkable change in the scaling behavior of the diffusion process

Singular basins in multiscale systems

S. Yanchuk[1,2] , S. Wieczorek[1],

H. Jardón-Kojakhmetov[3], H. Alkhayuon[1]

[1]*University College Cork, Cork, Ireland*
[2] *Potsdam Institute for Climate Impact Research, Potsdam, Germany*
[3]*University of Groningen, The Netherlands*

Real-world systems often evolve on different timescales and possess multiple coexisting stable states. Whether or not a system returns to a given stable state after being perturbed away from it depends on the shape and extent of its basin of attraction. We show that basins of attraction in multiscale systems can exhibit special geometric properties in the form of singular funnels. Although singular funnels are narrow, they can extend to different regions of the phase space and, unexpectedly, impact the system's resilience to perturbations. Consequently, singular funnels may prevent common dimensionality reductions in the limit of large timescale separation, such as the quasi-static approximation, adiabatic elimination and time-averaging of the fast variables. We refer to basins of attraction with singular funnels as singular basins. We show that singular basins occur robustly in a range of multiscale systems: the normal form of a pitchfork bifurcation with a slowly adapting parameter, an adaptive active rotator, and an adaptive network of phase rotators [1].

References

[1] S. Yanchuk, S. Wieczorek, H. Jardón-Kojakhmetov, and H. Alkhayuon. "Singular Basins in Multiscale Systems: Tunneling between Stable States." Preprint arXiv:2601.02001 (2026)

Fluctuation-Induced Patterns in Power Networks — Localization, Resonances and Extreme Events

X. Zhang[1,2]

[1]*Tongji University, 200092 Shanghai, China*
[2] *Technical University Dresden, 01062 Dresden, Germany*

Understanding how self-organized patterns emerge in complex systems is central to predicting their dynamics and safeguarding their reliable operation. The Kuramoto model and its extensions provide a powerful framework for investigating these phenomena, revealing how coupled oscillators give rise to non-trivial heterogeneous structures—either in isolation or as responses to external signals. In this talk, we examine how the interplay of network topology, nodal dynamics, and external forcing shapes pattern formation in Kuramoto type models of power grids.

We first present network linear response theory and reveal and mechanistically explain three collective response regimes under external forcing. At intermediate driving frequencies network resonances emerge —non-trivial patterns that may bring system-wide risks by amplifying disturbances [1,2]. Second, we estimate nodal extreme events under colored noise excitation, identifying which nodes may amplify fluctuations into rare, large-amplitude frequency deviations that threaten reliable grid operation [3]. Third, we examine statistical predictability of grid frequency fluctuations resulting from $1/f^b$ noise, and demonstrate how the spectral exponent b influences the frequency fluctuations' stationarity, Gaussianity and spatiotemporal correlations [4].

References

[1] X. Zhang et al., Sci. Adv. **5**, eaav1027 (2019)
[2] X. Zhang and M. Timme, Eur. J. Appl. Math. **34**, 429-466 (2023)
[3] X. Zhang et al., Chaos **30**, 063111 (2020)
[4] L. Yang and X. Zhang, under review (2026)

Abstracts of Posters

(in alphabetical order)

Controlling Phases via Subsystem Resetting in Many-Body Interacting Systems

Anish Acharya[1], Rupak Majumder[1], Shamik Gupta[1]

[1]Department of Theoretical Physics, Tata Institute of Fundamental Research, Homi Bhabha Road, Mumbai 400005, India

For practical reasons, one is often interested in a phase of an interacting system which is thermodynamically unstable in certain parameter regime of interest. In traditional approaches, such phases are stabilised through intervening in the dynamics of all system constituents or introducing additional interactions. Recently, we have presented an alternative approach of resetting subsystems for stabilising and systematically manipulating phases in many-body interacting systems. The idea is simple: only a chosen subpart of the full system is stochastically reset to a fixed configuration, while the rest of the system evolves freely, and we ask what happens to a specific order parameter at steady state. This avoids global interventions and fine-tuning of couplings and results in macroscopic changes, including nontrivial restructuring of phase behaviours.

We have applied this protocol across a wide variety of models, namely, mean-field and non-mean-field spin models and a wide class of Kuramoto models. Here, I will focus on the application to the noisy Kuramoto model with single harmonic interaction and with higher harmonic interaction. I will demonstrate how resetting a subpart can have robust control of the phase diagram of the original model by adjusting the reset frequency, subsystem size, and reset configuration. Notably, for the model with both single and double harmonic interaction, the system under subsystem reset shows a distincpt reentrant phase transition. These results open up a versatile framework for stabilising or suppressing targeted phases in complex interacting systems.

References

1. Anish Acharya, Rupak Majumder, Shamik Gupta; Manipulating phases in many-body interacting system with subsystem resetting, PRL 135.127103

2. Rupak Majumder, Anish Acharya, Shamik Gupta; Analytical approach to subsystem resetting in generalised Kuramoto models, arxiv:2604.04769 (2026)

Characterizing brain-wide interactions in different states of consciousness: EEG analysis with ordinal patterns

Manuel Adams[1] and Klaus Lehnertz[1,2,3]

[1]Department of Epileptology, University of Bonn Medical Centre, Venusberg Campus 1, 53127 Bonn, Germany

[2]Helmholtz Institute for Radiation and Nuclear Physics, University of Bonn, Nussallee 14–16, 53115 Bonn, Germany

[3]Interdisciplinary Center for Complex Systems, University of Bonn, Brühler Straße 7, 53175 Bonn, Germany

Recent advances in neurophysiological brain network analysis have demonstrated novel potential for the diagnosis and prognosis of disorders of consciousness. Most previous studies derived networks from electroencephalographic (EEG) recordings and focused on the strength of interactions between pairs of sampled brain regions. However, to further support the Global Workspace Hypothesis that consciousness emerges through information processing, characterization of both the strength and direction of information flow is desirable.

To this end we make use of ordinal-pattern-based analysis techniques that have previously been employed in order to quantify the emergence of different synchronization phenomena in coupled complex dynamical systems. We derive ordinal-pattern-based quantifiers for the strength, complexity, and direction of interactions from multichannel multiday EEG data using a moving-window approach with non-overlapping 20 s windows. We investigate spatial distributions of group medians of the respective temporal mean of quantifiers of interactions in subjects with UWS and compare them with those of healthy controls during wakefulness and sleep.

The spatial distributions of the ordinal-pattern-based quantifiers differ between the investigated states of consciousness, although to varying degrees. Interestingly, parts of the distributions observed in healthy subjects during wakefulness resemble parts of the default mode network. These findings provide first promising evidence that ordinal-pattern-based investigations of brain-wide interactions may help disentangle spatial and temporal aspects of information flow across different states of consciousness.

Acknowledgements

This research was funded by Deutsche Forschungsgemeinschaft grant number LE 660/10-1.

Phase locking and multistability in the topological Kuramoto model on cell complexes

I. Bačić[1,2], M. T. Schaub[3], J. Kurths[4], and D. Witthaut[1]

[1]Forschungszentrum Jülich, Jülich, Germany
[2]Institute of Physics Belgrade, Belgrade, Serbia
[3]RWTH Aachen University, Aachen, Germany
[4]Potsdam Institute for Climate Impact Research (PIK), Potsdam, Germany

iva@ipb.ac.rs

Higher-order interactions fundamentally shape collective dynamics in oscillator networks. The topological Kuramoto model captures these effects by extending synchronization models to include interactions between cells of arbitrary dimension within simplicial and cell complexes. We introduce the topological nonlinear Kirchhoff conditions to characterize all phase-locked states in the topological Kuramoto model [1]. These conditions generalize Kirchhoff's current law (KCL) and Kirchhoff's voltage law (KVL) from circuit theory. The generalized KCL yields linear local balance conditions, whose solutions define a continuous family of candidate solutions. The generalized KVL imposes nonlinear global constraints that ensure that phases are single-valued up to integer multiples of 2π, thereby selecting a finite set of admissible phase-locked states.

Phase-locked states are organized by winding numbers associated with higher- dimensional cycles, which quantify how phases wind around higher-order cycles. Using rings, Platonic solids, and regular simplices as illustrative examples, we demonstrate a universal rule: boundaries must have at least five elements for multistability to arise. We further show that independent winding numbers associated with lower- and higher-dimensional boundaries generate cascades of multistability across dimensions. These results show how the topology and boundary structure of cell complexes influence phase locking and multistability, and provide a general framework for collective dynamics on cell complexes.

References

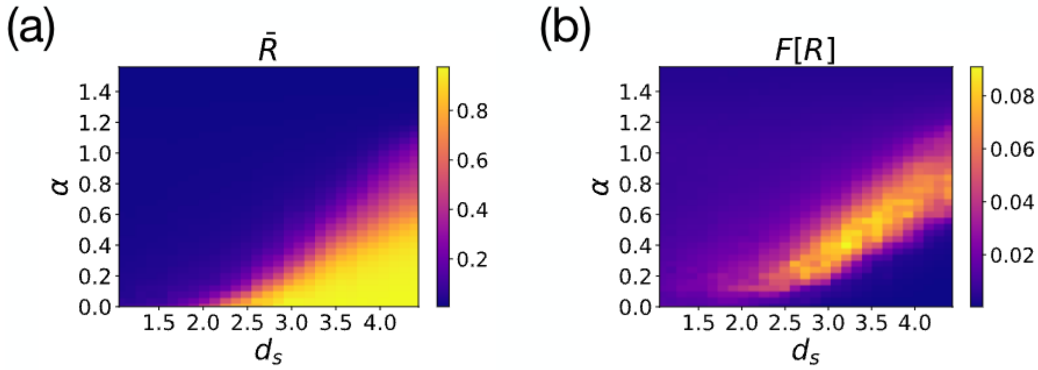
[1] I. Bačić, M. T. Schaub, J. Kurths, and D. Witthaut, *arXiv* 2510.05831v3 (2025), https://arxiv.org/abs/2510.05831v3.

Spectral dimension determines criticality in nonreciprocal phase oscillators

M. Bae[1] **and H. Kori**[1]

[1] *Department of Complexity Science and Engineering, Graduate School of Frontier Sciences, the University of Tokyo, Chiba 277-8561, Japan*

Spectral dimension is a key determinant of critical phenomena, but its role in nonreciprocal systems remains unexplored. We study noisy identical Kuramoto–Sakaguchi oscillators with phase lag $\alpha \in [0, \pi/2)$, where $\alpha > 0$ induces nonreciprocal interactions. Numerical phase diagrams in the (d_s, α) plane in complex networks, where d_s denotes the spectral dimension, reveal a critical phase lag α_c, below which spontaneous synchronization occurs. This critical phase lag appears only for $d_s > d_s^c$, where $d_s^c = 2$ is the critical spectral dimension, and increases monotonically with d_s. We analytically derive the criticality using the dynamical renormalization group method.



Metabolic gels: from chemical microreactors to polymeric actuators

Baptiste Blanc

Université Côte d'Azur, INPHYNI

email: baptiste.blanc@univ-cotedazur.fr

The existence of living organisms is evidence that functional motile autonomous materials can be built robustly using macromolecules. However, the challenge of building from the molecule to the macroscopic scale a purely synthetic functional chemomechanical material that harness chemical energy to produce motion is unmet. Inspired by Yoshida's work, we create synthetic autonomous chemomechanical polymeric materials made of acrylamide hydrogels doped with the catalyst of the Belousov Zhabotinsky(BZ) reaction . We identify the role of the surrounding medium chemistry and gel radius for the occurrence of BZ gel oscillations, quantified by the Damkohler number, ratio of chemical reaction to diffusion rates. We also argue that the gel chemomechanics arises from the change in solvation energy. These findings contribute to the understanding of Nature's ability to harvest chemical energy for actuation.

Structural, functional, and effective connectivity in chimera states

M. Brešar[1], J. Epifanio[2] and R. G. Andrzejak[2]

[1]*Jožef Stefan Institute, Department of Systems and Control, Ljubljana, Slovenia*
[2] *Universitat Pompeu Fabra, Department of Engineering, Barcelona, Catalonia, Spain*

Chimera states occur in networks of identical oscillators which spontaneously segregate into a high coherence group (HCG) and a low coherence group (LCG). We ask how the interactions in chimera states are reflected in the different variables observed from the system, and what dynamical mechanisms give rise to these inferred interactions. We study a chimera in a nonlocally coupled Kuramoto model using a data-driven connectivity approach. Following a convention from neuroscience, we use the notions of structural, functional, and effective connectivity. Structural connectivity is directly given by the coupling kernel. Starting from phase velocity time series of individual oscillators, we estimate functional connectivity using linear cross-correlation, and effective connectivity using the cross-rank vector measure [1], a directed interdependence measure based on reconstructed state-spaces. We observe a first symmetry break in functional connectivity, which is a direct consequence of the existence of the HCG and LCG. A further symmetry break is found in the effective connectivity, where the dependence of LCG on HCG is substantially stronger than vice versa. The mechanism leading to this directional dependence is discussed.

We expand the analysis to group mean field phase velocities of the HCG and LCG, defined by their local Kuramoto order parameters. We find that the inferred directionality is the opposite of that on the individual oscillator scale: there is a stronger dependence of HCG mean field on LCG mean field than vice versa. We also observe a gradual switching of directions by individually adding oscillators to the mean fields. Together, our results show that the inferred directionality can depend strongly on the observational scale. This is especially relevant for neuroscience, where recording scales span from the subcellular level up to brain-wide measurements of neural population activity.

References

1. M. Brešar, R. G. Andrzejak, P. Boškoski, Chaos **35**, 013130 (2025)

Emergence of a Tipping Subnetworks During a Critical Transitions in Coupled Oscillator Networks

T. Bröhl[1] and K. Lehnertz[1]

[1]Department of Epileptology, University Hospital Bonn, Bonn, Germany

Understanding critical transitions that culminate in extreme events — rare yet large-amplitude collective deviations — remains a major challenge in the study of coupled oscillator networks. We investigate such transitions in networks of diffusively coupled self-excitable FitzHugh-Nagumo oscillators across paradigmatic coupling topologies: small-world, scale-free, random, and all-to-all. These systems self-generate and self-terminate extreme events without changes to control parameters, with generating mechanisms intimately linked to phase synchronization phenomena.

We construct time-evolving functional networks via phase-based estimators for pairwise interaction strength and track the structural integration of each network constituent (vertices and edges) over time using multiple centrality concepts. We identify tipping elements — constituents whose structural integration changes significantly and reproducibly during the critical transition — and show that they form a larger connected "tipping subnetwork".

The tipping subnetwork undergoes topology-dependent modifications largely associable with a structural homogenization and increase in rigidity. The tipping subnetwork is already discernible during normal dynamics and becomes progressively more secluded as an extreme event approaches.

These findings establish the tipping subnetwork as a novel meso-scale structural unit encapsulating key mechanisms of emergent extreme events in coupled oscillator systems, with implications for theory and prediction of critical transitions.

Synchronization of higher-dimensional Kuramoto oscillators on networks: from scalar to matrix-weighted couplings

Anna Gallo[1,2], Renaud Lambiotte[3] and **Timoteo Carletti**[4]
[1] Centro Studi e Ricerche 'Enrico Fermi' (CREF), Via Panisperna 89A, 00184 Rome (Italy)
[2] IMT School for Advanced Studies, Piazza San Francesco 19, 55100 Lucca (Italy)
[3] Mathematical Institute, University of Oxford, Woodstock Rd, Oxford OX2 6GG (United Kingdom)
[4] Department of mathematics and Namur Institute for Complex Systems, naXys, University of Namur
Rue Grafé 2, B5000 Namur (Belgium)
timoteo.carletti@unamur.be

The Kuramoto model is the paradigmatic model to study synchronization in coupled oscillator systems. In its classical formulation, the oscillators move on the unit circle, each characterized by a scalar phase and a natural frequency, by interacting through a sinusoidal coupling. In this work, we propose a d–dimensional generalization in which oscillators are represented as unit vectors on the $(d-1)$–sphere and interact through a matrix-weighted network (MWN), a recently introduced framework where links are endowed with a matrix weight instead of a scalar one. We derive necessary conditions for global synchronization via a Master Stability Function approach: the existence of a synchronous solution requires identical frequency matrices across nodes and, in the MWN case, a coherence condition on the network structure. Through a suitable change of variables, the stability analysis reduces the full Nd–dimensional problem to a family of d–dimensional eigenvalue problems, each one parametrized by the eigenvalue of a suitable scalar weighted Laplacian, showing that the synchronous solution is locally stable for any positive coupling strength K on any connected network. Analytical results are complemented by numerical simulations.

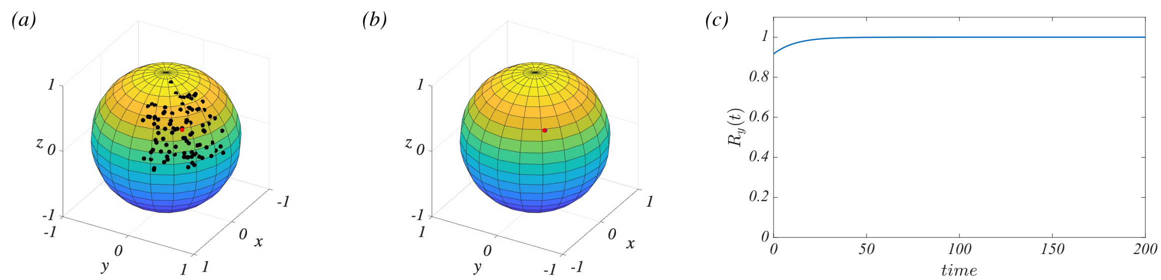


FIG. 1: **Dynamics for the d-Kuramoto model in the "rotated" variables, $\vec{y}_i$.** We report the initial conditions, $\vec{y}_i(0)$, (black dots in panel (a)) and the configuration at some future time t_{fin}, i.e., $\vec{y}_i(t_{fin})$, (blue dot in (panel (b)) of the solution of the3-Kuramoto model, i.e., on the 2–sphere. The MWN is build by using an Erdős-Rényi network with $N = 50$ nodes and probability $p_{ER} = 0.5$ to have a link among a couple of nodes; the matrices $\mathbf{R}_{ij}$ are rotations about the vector $\vec{u} = (0,0,1)^\top$ and the matrix Ω is the generator of a rotation about the same vector. The synchronous solution (red dot in both panels) is given by $\vec{s}(t) = e^{\Omega t}\vec{s}(0)$, with $\vec{s}(0) = (\sqrt{3}/3, \sqrt{3}/3, \sqrt{3}/3)^\top$. Being $K = 0.1$ the system synchronizes as we can appreciate by looking at the order parameter, $R_y(t)$ (panel (c)) that reaches quite fast the value 1.

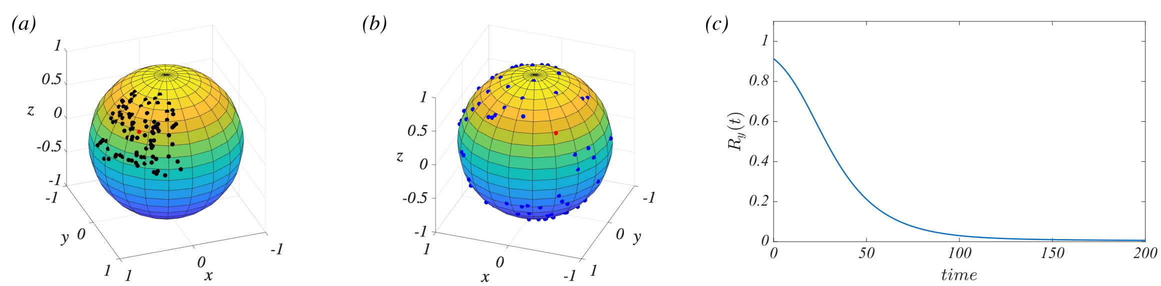


FIG. 2: **Dynamics for the d-Kuramoto model in the "rotated" variables, $\vec{y}_i$.** We report a simulation for the case $K = -0.1$. Initial conditions, $\vec{y}_i(0)$, (black dots in panel (a)) are randomly distributed about $\vec{s}(0)$; because $K < 0$, the vectors $\vec{y}_i(t)$ cannot synchronize and are dispersed on the sphere (blue points in panel (b)). The order parameter, $R_y(t)$ converges to zero, thus testifying the absence of synchronization. The simulation refers to $N = 50$, 3-Kuramoto oscillators coupled with a MWN is build by using an Erdős-Rényi network with probability to have a link given by $p_{ER} = 0.5$. The matrices $\mathbf{R}_{ij}$ are rotations about the vector $\vec{u} = (0,0,1)^\top$ and the matrix Ω is the generator of a rotation about the same vector. The synchronous solution (red dot in both panels) is given by $\vec{s}(t) = e^{\Omega t}\vec{s}(0)$, with $\vec{s}(0) = (\sqrt{3}/3, \sqrt{3}/3, \sqrt{3}/3)^\top$.

Effect of kinetics on non-equilibrium phases of a driven Potts model

Maciej Chudak[1], Massimiliano Esposito[2], and Krzysztof Ptaszyński[1]

[1]Institute of Molecular Physics, Polish Academy of Sciences, Smoluchowskiego 17, 60-179 Poznań, Poland
[2]University of Luxembourg, 30 Avenue des Hauts-Fourneaux, L-4362 Esch-sur-Alzette, Luxembourg

The Potts model is a generalization of the Ising model where spins can take more than two states. We study a driven three-state nonequilibrium Potts model with homogeneous all-to-all coupling. This is the minimal model, which at the mean-field level exhibits complex behavior, such as synchronization that leads to the emergence of persistent macroscopic oscillations (limit cycles). (*1*)

We first consider the mean-field limit, where the model dynamics is described via deterministic equations of motion for macroscopic variables of the system. The choice of the transition rate function can drastically reshape the phase diagram of the model, generating much richer behavior than that originally reported in Ref. (*1*).

Beyond the mean-field analysis, we further characterize the effect of rare fluctuations on the model behavior. Using the method of Gaspard (*2*), we determine the coherence lifetime of the oscillations and compare it to a thermodynamic bound given by the entropy production per cycle. (*3*, *4*) In this way, we show that the trade-off between coherence lifetime and entropy production can be tailored by adjusting the parameters of the transition rate model. We also investigate the scaling of fluctuations with system size.

Using the instanton approach (*5*, *6*), we characterize rare stochastic transitions among the coexisting mean-field attractors, which tend to relax the system to a single "most likely" attractor that determines the macroscopic behavior of the model. In particular, we show that in the region of multistability, the ordered state is usually more likely than disordered or oscillating state.

References

1. T. Herpich, J. Thingna, M. Esposito, *Phys. Rev. X* **8**, 031056 (3 Sept. 2018).
2. P. Gaspard, *Journal of statistical physics* **106**, 57–96 (Jan. 2002).
3. L. Oberreiter, U. Seifert, A. C. Barato, *Phys. Rev. E* **106**, 014106 (1 July 2022).
4. D. Santolin, G. Falasco, *Dissipation bounds the coherence of stochastic limit cycles*, 2025, arXiv: 2501.18469.
5. M. I. Dykman, E. Mori, J. Ross, P. M. Hunt, *J. Chem. Phys.* **100**, 5735–5750, ISSN: 0021-9606 (Apr. 1994).
6. R. Zakine, E. Vanden-Eijnden, *Phys. Rev. X* **13**, 041044 (4 Dec. 2023).

Order amidst a world of chaos: recent developments in cluster synchronization

G. Contreras-Aso[1]

1. Grupo Interdisciplinar de Sistemas Complejos (GISC), Department of Mathematics, Universidad Carlos III de Madrid, Spain

Synchronization is one of the most striking examples of emergent collective behavior in complex systems, particularly in the case of chaotic oscillators. From the firing of neurons to the operation of power grids, coordinated dynamics can arise spontaneously from simple local interactions. While the phenomenon of complete synchronization (where all nodes in a network evolve identically) has been thoroughly studied using the Master Stability Function formalism [1], many real-world systems display richer and more structured patterns of coordination. In these cases, the network self-organizes into clusters of nodes that synchronize internally but not with each other, a state known as cluster synchronization.

Here, I will present recent theoretical and computational developments in understanding how these cluster states emerge in networks of coupled chaotic oscillators. I will show how the spectral properties of the graph Laplacian reveal the building blocks of synchronization, introducing the concept of localized spectral blocks as a key mathematical tool to detect and predict cluster formation [2]. These insights naturally lead to the graph-theoretic notion of equitability, which we can generalize to include dynamical information [3].

Finally, I will discuss extensions of these ideas to multilayer and higher-order (hypergraph) networks [3], where interactions go beyond simple pairwise couplings. These generalizations uncover new mechanisms for coordination and provide a unified perspective on synchronization phenomena across networked systems, while explaining the ubiquitous lack of cluster synchronization in these network models.

[1] L. M. Pecora, T. L. Carroll. *Master Stability Functions for Synchronized Coupled Systems*. Phys. Rev. Lett. 80 (1998).

[2] A. Bayani, F. Nazarimehr, S. Jafari, K. Kovalenko, G. Contreras-Aso, K. Alfaro-Bittner, R. J. Sánchez-García, S. Boccaletti. *The transition to synchronization of networked systems*. Nat. Comm. 15, 4955 (2024).

[3] K. Kovalenko, G. Contreras-Aso, C. I. del Genio, S. Boccaletti, R. J. Sánchez-García. *Equitability and explosive synchronisation in multiplex and higher-order networks*. Comm. Phys, 9, 117 (2026).

Designing Networks of Coupled Oscillators for Computing

Thomas Geert de Jong[1], Hirofumi Notsu[2], and Kohei Nakajima[3]

[1]Temple University, Japan Campus,, 1-14-29 Taishido, Setagaya-ku, Tokyo 154-0004,
[2]Faculty of Mathematics and Physics,, Institute of Science and Engineering,, Kanazawa University,, Kakuma, Kanazawa 920-1192
[3]Graduate School of Information Science and Technology,, The University of Tokyo,, Bunkyo-ku, Tokyo 113-0033

Computing in silica comes with drawbacks, including high energy consumption, susceptibility to critical failures in extreme environments such as high-radiation areas, as well as computational speed limitations that can be troublesome when processing real-time photonic data. This has sparked significant interest in discovering alternative physical systems that can overcome these limitations. A framework that addresses these issues is physical reservoir computing, which allows us to directly harness the computational capabilities of a physical system without requiring a specific architecture, greatly reducing engineering overhead.

We show how coupled oscillators can be used as a computational resource within the reservoir computing framework. In the simplest setting, the oscillators can be taken to be Kuramoto oscillators. This Kuramoto reservoir computer can perform tasks similar to those of classical reservoir computers. By considering suitable network structures or parameters, we can engineer the Kuramoto reservoir computer to excel at certain tasks. Inspired by the tentacle reservoir computer, we show how a network of coupled oscillators can be designed such that the reservoir computer exhibits high memory capacity.

References

1. T. G. de Jong, H. Notsu, and K. Nakajima (2025). *Harnessing omnipresent oscillator networks as computational resource.* arXiv:2502.04818.

2. T. G. de Jong (2025). *Designing learning in high dimensional oscillator networks with low dimensional read-out.* arXiv:2509.00848.

3. K. Nakajima et al. (2015). *Information processing via physical soft body.* Scientific Reports.

Collective drift and pinning in Kuramoto-type active rotator networks with mixed-sign local feedback

Arpan Dey [1]

[1] Laboratoire Charles Coulomb, Université de Montpellier, Montpellier, France
Email: arpand2004@gmail.com

Abstract

Active rotator models provide a minimal phase description of systems that exhibit excitable and oscillatory dynamics, and have long been studied in the context of mutual entrainment, synchronization, and phase transitions. Building on Kuramoto-type coupling with an additional Adler-like local term, we numerically investigate finite oscillator networks governed by:

$$\dot{\theta}_i = \omega - A_i \sin\theta_i + \frac{K}{k_i}\sum_{j\in N_i} \sin(\theta_j - \theta_i)$$

where $A_i \sim \mathcal{N}(0, \sigma_A)$ and ω is fixed for all oscillators. The isolated dynamics exhibits a pinned-to-drifting transition when the intrinsic drive exceeds the local locking scale. In coupled networks, however, this local criterion can compete with collective phase alignment. We study this competition through finite-size numerical regime maps, using locked fraction, late-time drift, and the direction of phase drift as diagnostics. Preliminary simulations indicate that when the local Adler coefficient is drawn from a mixed-sign distribution, the system shows a nontrivial competition between local pinning and collective drift. As σ_A/ω is increased, different oscillators experience local feedback of different strengths and signs, which slows their phase motion unevenly. At weak coupling, this stronger local feedback can suppress phase drift, while stronger coupling can restore coherent drift. This ongoing work will further compare the effects of distributed natural frequencies, system size, and randomly-connected network structures.

References

[1] Shinomoto, S., & Kuramoto, Y. (1986). *Phase transitions in active rotator systems*. Progress of Theoretical Physics, 75(5), 1105–1110.

[2] Sakaguchi, H., & Kuramoto, Y. (1986). *A soluble active rotator model showing phase transitions via mutual entrainment*. Progress of Theoretical Physics, 76(3), 576–581.

[3] Acebrón, J. A., Bonilla, L. L., Vicente, C. J. P., Ritort, F., & Spigler, R. (2005). *The Kuramoto model: A simple paradigm for synchronization phenomena*. Reviews of Modern Physics, 77(1), 137–185.

[4] Lopes, M. A., Lopes, E. M., Yoon, S., Mendes, J. F. F., & Goltsev, A. V. (2016). *Synchronization in the random-field Kuramoto model on complex networks*. Physical Review E, 94(1), 012308.

Characterizing resonant solitary states as relative equilibria via symmetry reduction

<u>Bengi Dönmez</u>[1], Bob Rink[1]

[1]*Vrije Universiteit Amsterdam, Amsterdam, The Netherlands*

We investigate solitary states in coupled oscillator networks, where a single oscillator detaches from a synchronized cluster and evolves at a distinct mean frequency. We focus on resonant solitary states, in which the detached oscillator's mean frequency is shaped by resonance with a mode of the cluster.

Here, we introduce a normal-form and symmetry-based approach to identify such states. We reduce the high-dimensional network dynamics to a two-dimensional second-order system that couples a rotator representing the solitary oscillator to a harmonic oscillator representing the dominant Laplacian mode of the cluster. We use normal-form transformations to introduce symmetry into this system. Using this symmetry, we reformulate the frequency selection problem as the search for relative equilibria of the action of the normal form symmetry. We derive explicit algebraic conditions for the existence of these relative equilibria, and hence for resonant solitary states and their effective frequencies.

Chaotic switchings in the minimal pendula network

<u>P. Ebrahimzadeh</u>[1], M. Schiek[1], and Y. Maistrenko[2]

[1]*Institute of Neuromorphic Compute Nodes (PGI-14), Forschungszentrum Jülich, Germany*

[2]*Institute of Mathematics and Technical Center, NAS of Ukraine, Kyiv, Ukraine*

Collective behavior of coupled identical oscillators exhibits various dynamics ranging from simple phase-locked synchrony patterns, spatio-temporal chimera states to chaos. In pendula networks, so-called *mixed-mode chimera states* are found characterized by the coexistence of different synchronous clusters with zero and non-zero average frequencies.[1]

In this talk, we report different types of collective dynamics for the minimal pendula network (Kuramoto oscillators with inertia and gravitation) in the repulsive coupling regime $\alpha > \pi/2$. Our main result consist of three scenarios for the transition to the spontaneous chaotic switching between the chaotic chimera states: 1) starting with a riddling bifurcation where an unstable periodic orbit inside the chaotic chimera becomes transversally unstable while the chaotic chimera itself on average remains transversely stable 2) a blowout bifurcation, where the chaotic chimera becomes transversally unstable in average and 3) a chaotic chimera becomes a laminar chaotic saddle within a turbulent attractor.

The existing regimes of different chimera states, chaotic switchings and stable fixed-points are obtained in the parameter plane. Large coexisting regions for these behaviors are found, where the system dynamics is multistable. In the parameter regions where a stable fixed point or a stable chimera coexist with the chaotic saddles, the chaotic switching trajectories eventually fall into the stable attractor at an unpredictable time. This causes an additional uncertainty to the chaotic switching dynamics observed, so that the limiting system behavior becomes even less predictable.

[1] P. Ebrahimzadeh, M. Schiek, Y. Maistrenko; Mixed-mode chimera states in pendula networks. Chaos 1 October 2022; 32 (10): 103118

Anticipating explosive synchronization in networks of Kuramoto oscillators

Juan Gancio,[1] Giulio Tirabassi,[2] Cristina Masoller,[1]

1. Universitat Polintècnica de Catalunya, Departament de Fisica, Rambla Sant Nebridi 22, Terrassa 08222, Barcelona, Spain
2. Universitat de Girona, Departament de Informàtica, Matemàtica Aplicada i Estadística, Carrer de la Universitat de Girona 6, Girona 17003, Spain.

Many complex systems experience sudden changes in their dynamics as they transition between different states when a control parameter is varied. The threshold value of the parameter for which an abrupt transition occurs is know as "tipping point". For real–world systems, such as animal populations, vegetation fields, and the climate, there is a particular interest in being able to detect if a tipping point is approaching, since once crossed the systems may not recover the previous stable state [1]. An example of a sudden change is explosive synchronization (ES), an abrupt transition between a disorder state and a synchronous one, characterized by being of first–order [2].

In this work, we propose several quantifiers based on the permutation entropy (PE) that present a particular behavior when the network approaches a synchronous state. PE is a time series analysis technique that uses symbols (ordinal patterns) defined by the ordering of the data points, disregarding the actual values [3], and has recently been proposed as a new indicator for critical transitions [4,5]. We test this new method in different networks of Kuramoto phase oscillators linked in such a way that they present ES. We observe substantial and characteristic changes in these quantities as the networks approach synchronization, even when the synchronization is explosive. Therefore, PE-based quantities can serve as early warning signals (EWS) of this class of transitions. Our results complement previous findings regarding PE methods for EWS of critical transitions, expanding their applications to systems characterized by complex temporal behaviors.

[1] Scheffer, M., Bascompte, J., Brock, W. A., Brovkin, V., Carpenter, S. R., Dakos, V., ... & Sugihara, G. (2009). *Early-warning signals for critical transitions.* Nature, 461(7260), 53-59.
[2] Leyva, I., Navas, A., Sendina-Nadal, I., Almendral, J. A., Buldú, J. M., Zanin, M., ... & Boccaletti, S. (2013). *Explosive transitions to synchronization in networks of phase oscillators.* Scientific reports, 3(1), 1281.
[3] Bandt, C., & Pompe, B. (2002). *Permutation entropy: a natural complexity measure for time series.* Physical review letters, 88(17), 174102.
[4] Tirabassi, G., & Masoller, C. (2023). *Entropy-based early detection of critical transitions in spatial vegetation fields.* Proceedings of the National Academy of Sciences, 120(1), e2215667120.
[5] Tirabassi, G. (2024). *Linear theory of the spatial signatures of critical slowing down.* Physical Review Research, 6(2), 023228.

Nonequilibrium criticality in the dynamics of synchronization in one dimension

Ricardo Gutiérrez and **Rodolfo Cuerno**

Complex Systems Interdisciplinary Group (GISC), Department of Mathematics, Universidad Carlos III de Madrid, Leganés (Madrid), Spain

The study of synchronous dynamics has mostly focused on the transition to synchronization from a static (steady-state) perspective, while the dynamical process whereby systems of oscillators synchronize at long times has received much less attention. One might expect this process to be strongly system-dependent, yet we have recently shown in a series of publications [1-4] that it displays robust universal features previously observed in the context of nonequilibrium critical dynamics, as suggested by a mathematical connection between spatially discrete oscillator models and the continuum equations of surface kinetic roughening. By means of detailed numerical studies of 1D systems of phase [1,3,4] and certain limit-cycle oscillators [2,3] in the presence of quenched [1,2] and time-dependent noise [3] or a combination of them [4], we provide evidence confirming that the synchronization process in these systems is characterized by forms of generic scale invariance associated with kinetically rough interfaces, such as that of the Kardar-Parisi-Zhang (KPZ) universality class. In fact, the precise form of the coarse-grained dynamical equations and the role of symmetries and randomness can be analytically understood by a combination of continuum approximations and phase-reduction methods [2,4], and the existence of well-known crossover and finite-size effects [5]. We moreover find that fluctuations around the average growth appear to generically follow a Tracy-Widom distribution, frequently associated with the KPZ nonlinearity. Synchronization and surface growth are more closely related than previously anticipated due to such robust universal features, which make the experimental observation of the nonequilibrium criticality of synchronization an alluring possibility.

References

[1] R. Gutierrez and R. Cuerno, Phys. Rev. Research **5**, 023047 (2023).
[2] R. Gutierrez and R. Cuerno, Phys. Rev. Research **6**, 033324 (2024).
[3] R. Gutierrez and R. Cuerno, Phys. Rev. E **110**, L052201 (2024).
[4] R. Gutierrez and R. Cuerno, Physica D **473**, 134552 (2025).
[5] R. Gutierrez and R. Cuerno, arXiv:2604.06040, (2026).

Symbolic and statistical structure of chaotic neuronal bursting due to slow oscillating currents

C. Hinsley[1] and A. L. Shilnikov[1,2]

[1] Department of Mathematics & Statistics, Georgia State University, Atlanta, GA, USA
[2] Neuroscience Institute, Georgia State University, Atlanta, GA, USA

We study chaotic bursting activity arising in a six-dimensional fast-slow interneuron model of Hodgkin-Huxley type. In the two-slow-current case, and in the singular limit of timescale separation, a symbolic description based on membrane-potential traces appears to induce a generating partition of the underlying strange attractor. This symbolic encoding is used to construct a kneading diagram that illustrates the rich bifurcation structure governing the behavior of the chaotic attractor, organized around a codimension-two Shilnikov-Hopf point.

The introduction of a third slow current changes this organization: preliminary computations suggest that the stability windows present in the two-current regime may become hidden or suppressed. Numerical bifurcation continuation, symbolic dynamics, and statistical methods are used to make sense of this proliferation of complex bursting activity throughout the parameter space.

References

1. J. Scully, C. Hinsley, D. Bloom, H. G. E. Meijer, A. L. Shilnikov, Chaos **35**, 033120 (2025)

Gradient Kuramoto-type model for energy optimization in frustrated spin systems

A. Hrabovets[1], O. Burylko[1,2], B. Zwicknagl[2]

[1]Institute of Mathematics of the NAS of Ukraine, Kyiv, Ukraine
[2] Institut für Mathematik, Humboldt-Universität zu Berlin, Berlin, Germany

We consider a gradient oscillator model designed to study the collective behavior and energy landscape of frustrated spin systems. The oscillator model is of the Kuramoto type defined on a ring with competing interactions between nearest and next-nearest neighbors. The symmetry of the ring is reflected in a circulant coupling matrix, providing an analytical framework for the study of different collective regimes and their stability. A significant portion of this research is dedicated to the phenomenon of high multistability. Due to the frustrated nature of the couplings, the system supports numerous coexisting equilibria, including synchronization, rotating waves, and anti-phase regimes, along with more asymmetric fixed points and manifolds of equilibria. The high multistability of this system varies significantly as a result of regular and degenerate bifurcations.

We examine how the coupling strength between the oscillators governs the overall energy configuration of the frustrated system. A central component of this analysis is the description of the Maxwell sets. These sets represent the critical parameter values where a "global" transition occurs, specifically, where the global energy minimum shifts from one configuration to another. By identifying these sets, we map the boundaries of the energy landscape, providing a clear picture of which phase configurations are energetically optimal under varying coupling conditions. Our results demonstrate that in the thermodynamic limit, the system's behavior aligns with established findings for spin chains with specific boundary conditions [1, 2]. This correspondence may offer a helpful perspective for energy optimization in systems with different or more complex structures.

References

[1] M. Cicalese, F. Solombrino, J. Nonlinear. Sci. **25**, 291-313 (2015)
[2] J. Ginster, B. Zwicknagl, J. Nonlinear Sci. **33**, article 20 (2023)

Taming Synchronization Transitions: Tiered and Double Explosive Transitions

Pitambar Khanra[1]

[1]*UPES Dehradun, Uttarakhand, India*

We explore how synchronization emerges in networks where oscillators interact not just in pairs, but also in groups. By introducing an adaptive coupling [1] that depends on the system's global synchrony, we uncover exotic transition paths. Using a reduced model and simulations, we demonstrate how to engineer tiered synchronization and, more strikingly, a double explosive transition [2] with two discontinuous jumps, revealing new principles for controlling collective dynamics in complex systems [3]. This work provides a general framework for designing and predicting complex synchronization pathways in adaptive systems with multi-body interactions. The dynamics of the system is governed by the equations:

$$\dot{\theta}_i = \omega_i + \frac{K_1 r_1^a}{N}\sum_{j=1}^{N} \sin\left(\theta_j - \theta_i - \beta\right) + \frac{K_2 r_1^b}{N^2}\sum_{j=1}^{N}\sum_{k=1}^{N} \sin\left(2\theta_j - \theta_k - \theta_i - \beta\right), i = 1, \dots, N$$

Where θ_i is the phase and ω_i is the natural frequency of the oscillator i. K_1 and K_2 are the coupling strengths corresponding to pairwise and triadic interaction respectively.

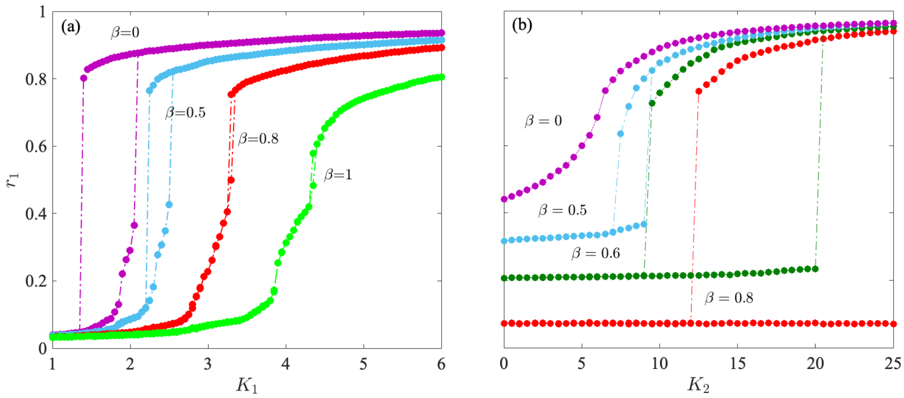


FIG. 1. Synchronization profile as obtained from the forward (filled circles connected by solid lines) and backward (filled circles connected by dashed dot lines) numerical continuation of the solutions of the Kuramoto system for a = 0 and b = 2. (a) Variation of r_1 with K_1 for $K_2 = 10$ and different β. (b) Variation of r_1 with K_2 for $K_1 = 2.5$ and different β. The purple, sky blue, red, green, and light green colors respectively represent the curves for β = 0, 0.5, 0.6, 0.8, and 1. The emergence of tiered synchronization in the system is evident in both the cases.

Reference:

1. S. Dutta, A. Mondal, P. Kundu, P. Khanra, P. Pal, C. R. Hens, "*Impact of phase lag on synchronization in frustrated Kuramoto model with higher-order interactions*," **Physical Review E 108, 034208** (2023).
2. S. Dutta, P. Kundu, P. Khanra, C. R. Hens, P. Pal, "*Transition to synchronization in adaptive Sakaguchi- Kuramoto model with higher-order interactions*," **Physical Review E 110, 064317** (2024).
3. S. Dutta, P. Kundu, P. Khanra, L. Minati, S. Boccaletti, P. Pal, C. Hens, "*A double explosive Kuramoto transition in hypergraphs*," **Physical Review Research (Letter) 7, L022049** (2025).

Analysis of the synchronization and stability of a wind power network subjected to strong perturbations

Nadia Kouonang[1,2], Sandrine Mabekou[2], Thierry Njougouo[3], Timoteo Carletti[1]

[1]Department of Mathematics and Namur Institute for Complex Systems, naXys, University of Namur, Belgium

[2]Mechanics and Modeling of Physical Systems Research Unit (UR-2MSP), Department of Physics, University of Dschang, Cameroon

[3]IMT School for Advanced Studies Lucca, Italy

The exponential growth of the world population is accompanied by a significant increase in energy demand. However, fossil resources present major limitations, notably their progressive depletion, their increasing exploitation cost, and their significant contribution to greenhouse gas emissions. In this context, renewable energies appear as a sustainable and environmentally friendly alternative. Among them, wind energy stands out for its ability to efficiently convert the kinetic energy of the wind into clean electricity. In this work, we analyze the synchronization and stability of a network of interconnected wind generators subjected to significant disturbances, as studying synchronization between wind turbines amounts to ensuring the stability, safety, and energy efficiency of the network. Without it, the network risks experiencing frequency imbalances, production losses, and material damage. To better capture the impact of natural wind fluctuations on this dynamic, particular attention is paid to the stochastic modeling of wind variability, described using the Ornstein–Uhlenbeck process. The dynamics of each generator are modeled using the Kuramoto model with inertia, as it makes it possible to take into account the mechanical inertia of the generator rotors as well as the coupling interactions between them. Furthermore, the global stability of the network is studied through the concept of the basin of stability, which represents the set of initial conditions leading to a synchronized state. The results show that the network can lose synchronization for high wind speeds associated with weak coupling between generators. They also highlight the determining role of key model parameters such as inertia, damping, coupling strength, the intensity of wind fluctuations, and its correlation time on the stability of the network. In addition, an enlargement of the basin of stability appears as an essential factor to improve the robustness of the network against disturbances and to ensure the maintenance of synchronization.

Keywords: synchronization, renewable energy, wind energy, basin of stability

Collective behaviour of driven Potts models

Jan Meibohm

Institute for Physics and Astronomy, Technische UniversitÃ¤t Berlin, Germany

The driven q-state Potts model is a generalisation of the classic equilibrium Potts model to a non-equilibrium setting, in which a drive breaks detailed balance while the dynamics can be made thermodynamically consistent. With global coupling, the model serves as a paradigmatic system for synchronisation in discrete-state non-equilibrium systems.

Starting from a general microscopic dynamics, we derive the normal form of the high-dimensional Hopf bifurcation underlying the synchronisation transition. These normal-form equations are exact in the thermodynamic limit and close to the bifurcation, and their symmetry allows us to solve them analytically, revealing a rich phase diagram with multiple coexisting oscillating states. Strikingly, the collective behaviour depends sensitively on the parity of q: for odd q, the transition is into a synchronised oscillating phase, whereas for even q, an additional stable mode gives rise instead to a static non-equilibrium steady state [1,2]. This odd-even asymmetry is a hallmark of the driven Potts model and distinguishes it sharply from the Kuramoto model.

Monte Carlo simulations on a nearest-neighbour lattice confirm the mean-field picture and reveal critical behaviour belonging to equilibrium universality classes: the Ising class for even q and the XY class for odd q, in three spatial dimensions, with upper critical dimension $d=4$. We conjecture that this emergence of equilibrium universality in a driven model reflects the identical symmetries of the corresponding bifurcations of the most unstable modes.

[1] Meibohm, Jan, and Massimiliano Esposito. "Small-amplitude synchronization in driven Potts models." Physical Review E 110.4 (2024): 044114.
[2] Meibohm, Jan, and Massimiliano Esposito. "Minimum-dissipation principle for synchronized stochastic oscillators far from equilibrium." Physical Review E 110.4 (2024): L042102.

A route to higher-order Kuramoto models through phase reduction theory

I. León[1], R. Muolo[2,3], S. Hata[4], H. Nakao[3]

[1]Department of Applied Mathematics and Computer Science, Universidad de Cantabria, Santander 39005, Spain
[2] iTHEMS, RIKEN, Saitama 351-0198, Japan
[3] Department of Systems and Control Engineering, Institute of Science Tokyo, Tokyo 152-8552, Japan
[4] Graduate School of Science and Engineering, Kagoshima University, Kagoshima 890-0065, Japan

Synchronization is a widespread emergent phenomenon where interacting units behave in unison [1]. Understanding its mechanisms is crucial, especially since interaction structure strongly influences the transition to synchronization. Phase models, which describe oscillators solely by their phase, are useful when coupling is weak [2]. The classic Kuramoto model captures synchronization via pairwise interactions, but many natural systems involve higher-order (many-body) interactions [3]. Moreover, higher-order interactions naturally emerge when phase reduction is performed beyond the first order [4]. Extensions of the Kuramoto model incorporating these interactions can lead to explosive synchronization or collective chaos [5]. Even the simplest minimal extension of Kuramoto-type phase model with higher-order interactions exhibits rich dynamics, as we showed in a previous work [6]. In this work [7], we provide a general theory of phase reduction for systems with higher-order interactions, yielding higher-order Kuramoto-like models. We start from a population of oscillators coupled via 3-body interactions and we show that, although higher-order connection topology is preserved in the phase reduced model, the interaction topology generally changes, due to the fact that the hypergraph is likely to turn into a simplicial complex in the reduced Kuramoto-type model. Furthermore, we show that, when the oscillators have certain symmetries, even couplings are irrelevant for the dynamics at the first order.

References

[1] Y. Kuramoto, Springer-Verlag (1984)
[2] H. Nakao, Cont. Phys. 57, 188 (2016)
[3] F. Battiston et al., Phys. Rep. 84, 1 (2020)
[4] I. León and D. Pazó, Phys. Rev. E 100, 012211 (2019)
[5] P. S. Skardal and A. Arenas, Phys. Rev. Lett. 122, 248301 (2019)
[6] I. León et al., Chaos 34, 013105 (2024)
[7] I. León et al., Physica D 482, 134858 (2025)

Synchronization of Mechanical Arms Driven by Josephson Junction Circuits: Mimicking Myriapod Locomotion

Isidore Komofor Ngongiah[1], Gaetan Fautso Kuiate[2,3], and Sifeu Takougang Kingni[1]

[1]Department of Mechanical, Petroleum and Gas Engineering, National Advanced School of Mines and Petroleum Industries, University of Maroua, P.O. Box 46, Maroua, Cameroon

[2]Department of Physics, Higher Teacher Training College, University of Bamenda, P.O. Box 39 Bamenda, Cameroon

[3]Department of Mechanical and Industrial Engineering, National Higher Polytechnic Institute, University of Bamenda, P.O. Box 39 Bamenda, Cameroon

This study explores the synchronization of mechanical arms driven by Josephson junction (JJ) circuits, focusing on their capability to mimic myriapod locomotion. Stability analysis reveals two equilibrium points, with stability switching influenced by stimulation current. Numerical simulations demonstrate synchronized movement of multiple arms, paralleling myriapod motion. This electromechanical system addresses challenges in the coupling of JJ circuits for effective actuation. The progressive excitation and collective behavior observed showcase emergent order-forming dynamics, contributing to advances in nonlinear systems theory and applications relevant to current research initiatives in synchronization and collective behavior in networked systems.

Beyond Synchronization: Pattern Formation in Kuramoto-Type Systems

Albert Díaz-Guilera[1,2] and Conrad J. Pérez-Vicente[1,2]

[1]*Dep. Física de la Matèria Condensada, Universitat de Barcelona, SPAIN*
[2] *Universitat de Barcelona Institute of Complex Systems (UBICS), Barcelona, SPAIN*

The Kuramoto model is a standard framework for studying collective dynamics, but most work has focused on the onset of global synchronization. In this talk I will discuss two examples where the main question is not whether oscillators synchronize, but which *patterns* of phases and correlations emerge, and how they are regulated.

In the first part, we consider a modular neuronal network where persistent global synchronization is interpreted as a pathological state. We introduce a simple self-regulation mechanism inspired by the action of glial cells: when the local phase coherence between two nodes exceeds a given threshold, the corresponding link is temporarily disabled. The oscillators evolve on a modular network and follow Kuramoto dynamics, while the network itself becomes time dependent through this local inhibitory rule. We show that this feedback is sufficient to suppress global synchronization while preserving high levels of local coherence inside modules, leading to rich, dynamically regulated patterns of inter-module correlations.

In the second part, we analyze Kuramoto dynamics on a one-dimensional ring with both excitatory and inhibitory couplings. Even for identical oscillators, the presence of negative links qualitatively changes the attractor landscape: besides the fully synchronized state, the system supports a variety of stable and metastable phase patterns on the circle, which can be understood as competing ordered states. Combining analytical arguments and numerical simulations, we characterize these attractors and the transitions between them.

Together, these two examples illustrate how Kuramoto-type models with inhibitory interactions and simple structural features – modularity or a ring topology – naturally generate and select non-trivial patterns, suggesting a broader role of phase-oscillator models as tools for studying pattern formation beyond classical synchronization.

References

[1] Paula Pirker-Diaz, Albert Diaz-Guilera, Jordi Soriano. Self-regulation of a network of Kuramoto oscillators *Chaos Solitons & Fractals (2024)*

[2] Albert Diaz-Guilera, Dimitri Marinelli, Conrad J. Perez-Vicente Exploring the interplay of excitatory and inhibitory interactions in the Kuramoto model on circle topologies *Chaos (2024)*

Origin of Frequency Clusters and Self-Organized Triplet Locking in the Kuramoto Model with Inertia

Yannick Schöhs[1], Nicolas Thomé[1] and Katharina Krischer[1]

[1]*Technical University of Munich, Garching bei München, Germany*

We investigate the origin of frequency clusters – states in which multiple groups of oscillators with different mean frequencies coexist – in the globally coupled Kuramoto model with inertia and identical oscillators. Two frequency clusters are studied in the thermodynamic limit, three frequency clusters with a system of seven oscillators. Using bifurcation analysis, we demonstrate that in both cases the frequency clusters emerge through homoclinic bifurcations.

In the case of three frequency clusters, the homoclinic bifurcation necessarily induces a locking of the frequency differences at rational ratios:

$$\frac{\langle \Delta\dot{\phi}_1 \rangle}{\langle \Delta\dot{\phi}_2 \rangle} = \frac{\langle \dot{\phi}_1 \rangle - \langle \dot{\phi}_2 \rangle}{\langle \dot{\phi}_1 \rangle - \langle \dot{\phi}_3 \rangle} = \frac{k}{l} \quad \text{for } k, l \in \mathbb{Z},\ k \neq l.$$

This relation implies the following condition characterizing triplet locking:

$$m\langle \dot{\phi}_1 \rangle - l\langle \dot{\phi}_2 \rangle + k\langle \dot{\phi}_3 \rangle = 0 \quad , m = l - k \in \mathbb{Z}.$$

Regarding the transversal stability of two- and three-cluster states, we find that stable solutions may destabilize as individual clusters lose phase synchrony through either period-doubling or transcritical bifurcations. While these bifurcations lead to a loss of phase synchrony of a cluster, they preserve equal average frequencies within each cluster.

Synchronization of nonlinearly coupled Stuart–Landau oscillators on networks

W. Segnou[1], R. Muolo[2,3], M. Dorchain[1], H. Nakao[3,4], T. Carletti[1]

[1] Department of Mathematics and Namur Institute for Complex Systems, naXys, University of Namur, Rue de Bruxelles 61, B-5000 Namur, Belgium
[2] RIKEN Center for Interdisciplinary Theoretical and Mathematical Sciences, 2-1 Hirosawa, Wako, Saitama 351-0198, Japan
[3] Department of Systems and Control Engineering, Institute of Science Tokyo, I1-20, 2-12-1 Ookayama, Meguro-ku, Tokyo 152-8552, Japan
[4] International Research Frontiers Initiative, Institute of Science Tokyo, Tokyo, Japan

The dynamics of coupled Stuart–Landau (SL) oscillators plays a central role in the study of synchronization phenomena. Previous works have focused on linearly coupled oscillators in different configurations, such as all-to-all or generic complex networks, allowing for both reciprocal and non-reciprocal links. The emergence of synchronization can be deduced by proving the linear stability of the limit-cycle solution for the SL model; the linear coupling assumption allows for a complete analytical treatment of the problem, mostly because the linearized system turns out to be autonomous.

In this work, we analyze SL oscillators coupled through nonlinear functions on both undirected and directed networks. Synchronization now depends on the study of a non-autonomous linear system, and thus novel tools are required to tackle the problem. We provide a complete analytical description of the system behaviour for some choices of the nonlinear coupling, for instance in the resonant case. Otherwise, we develop a semi-analytical framework based on the Jacobi–Anger expansion and Floquet theory, which allows us to derive precise conditions for the emergence of complete synchronization. The obtained results extend the classical theory of coupled oscillators and pave the way for future studies of nonlinear interactions in networks of oscillators and beyond.

A robust method to identify chimera states

S. Nirmala Jenifer[1,2], **Riccardo Muolo**[3], **Paulsamy Muruganandam**[2], **and Timoteo Carletti**[1]

[1] *Department of Mathematics and naXys, University of Namur, Namur, Belgium*
[2] *Department of Physics, Bharathidasan University, Tamilnadu, India*
[3] *RIKEN Center for Interdisciplinary Theoretical and Mathematical Sciences, Japan*

Chimera states represent one of the most intriguing phenomena emerging in complex systems. Characterized by the coexistence of coherent and incoherent behavior, they have been observed in a wide range of natural systems and experimental devices. Despite extensive studies and numerous observations in different settings, the development of reliable and systematic methods to classify chimera states and distinguish them from other dynamical patterns remains a challenging task. Existing approaches are often limited in scope and lack robustness, being grounded on the definition of thresholds whose value remains somehow unjustified. In this work, we propose a method based on Fourier analysis combined with statistical classification to identify chimera behavior, allowing us to tackle the above issue. The method is applied to a system of topological signals coupled via the Dirac operator, where it successfully captures and clearly distinguishes the rich dynamical regimes exhibited by the model. The proposed method can be summarized with three steps: from the time series obtained from numerical simulations, or experiments, extract relevant dynamical information about phases, amplitudes, and frequency. Then compute the (normalized) total variations to measure the spatial regularity of the features computed before. Eventually apply a statistical classification scheme to data obtained at step two, to extract clusters, each one characterized by a distinguished dynamical behavior [1]. We demonstrate that the proposed approach is robust with respect to variations in network topology and system parameters, moreover it is general enough to be performed with different methods. For those reasons we believe that the proposed method can be applied to a large and diverse class of dynamical systems and provide a general and automated tool for distinguishing different dynamical regimes in complex systems.

References

[1] S. N. Jenifer, R. Muolo, P. Muruganandam, and T. Carletti, *A robust method for classification of chimera states*, arXiv:2603.22026 (2026).

Chimera states on m-directed hypergraphs

Rommel Tchinda Djeudjo[1], Timoteo Carletti[1], Hiroya Nakao[2,3], Riccardo Muolo[2]

[1] Department of Mathematics & naXys, Namur Institute for Complex Systems, University of Namur, B5000 Namur, Belgium
[2] Department of Systems and Control Engineering, Institute of Science Tokyo (former Tokyo Tech), Tokyo 152-8552, Japan
[3] International Research Frontiers Initiative, Institute of Science Tokyo (former Tokyo Tech), Kanagawa 226-8501, Japan

rommel.tchindadjeudjo@unamur.be

Chimera states are synchronization patterns in which coherent and incoherent regions coexist in systems of identical oscillators. This elusive phenomenon has attracted significant interest and has been widely analyzed, revealing several types of dynamical states. Most studies involve reciprocal pairwise couplings, where each oscillator exerts and receives the same interaction from neighboring ones, thus being modeled via symmetric networks. However, real-world systems often exhibit non-reciprocal, non-pairwise (many-body) interactions. Previous studies have shown that chimera states are more elusive in the presence of non-reciprocal pairwise interactions, while they are easier to observe when the interactions are reciprocal and higher-order (many-body). In this work, we investigate the emergence of chimera states on non-reciprocal higher-order structures, called m-directed hypergraphs, which we compare with their corresponding networks, and we observe that some types of chimera states can emerge due to directionality, which had not been previously observed in its absence. We also compare the effect of non-reciprocal interactions between higher-order and pairwise couplings, and we find numerically that chimera states appear over a broader parameter range when considering higher-order interactions than in the corresponding network case, demonstrating the impact of directionality and the effect of higher-order interactions. Finally, the nature of phase chimeras has been further validated through phase reduction theory.

Synchronisation in Polar Active Solids

Zeynep Naz Tuna, Qingju Qi, Michel Fruchart, Olivier Dauchot

Gulliver UMR CNRS 7083, ESPCI Paris, PSL Research University, 10 rue Vauquelin, 75005 Paris, France

Active solids are nonequilibrium materials where internally driven units are embedded in an elastic medium, leading to a nontrivial feedback between activity and mechanical constraints. These systems sustain solid-like structure while exhibiting collective dynamical phenomena, one prominent example being the emergence of a collective actuation phase in which internal actuation variables synchronize across the material.

Motivated by this observation, we investigate to what extent synchronization in active solids can be understood within extensions of the Kuramoto model. We consider a Kuramoto-type model with a bimodal, symmetric distribution of intrinsic frequencies and show that it reproduces the synchronization transition observed in active solids, including the crossover from a discontinuous transition with hysteresis at low noise to a continuous transition at higher noise. However, a bimodal frequency distribution alone turns out to be insufficient to account for the spontaneous chiral symmetry breaking observed in active solids: while synchronized Kuramoto oscillators converge to a frozen, non-rotating state, active solids develop a finite collective rotation in the synchronized phase. This points to the need for additional dynamical variables.

To identify the origin of this discrepancy, we derive a reduced mean-field, phase-only description directly from the microscopic equations of active solids. Although the resulting equations recover the sinusoidal coupling characteristic of Kuramoto models, the term analogous to intrinsic frequencies is dynamical rather than quenched. A linear stability analysis shows that this additional dynamics destabilizes the zero-rotation synchronized state and gives rise to two symmetric stable fixed points, hinting at the the emergence of chiral rotating states that we observe in simulations. These results highlight a fundamental distinction between synchronization with quenched disorder and synchronization with dynamical frequency variables, and suggest that the latter may be relevant more broadly in active systems.

References

[1] P. Baconnier, O. Dauchot, V. Démery, G. Düring, S. Henkes, C. Huepe, and A. Shee, Rev. Mod. Phys. **97**, 015007 (2025).

[2] P. Baconnier, D. Shohat, C. Hernández López, C. Coulais, V. Démery, G. Düring, and O. Dauchot, Nat. Phys. **18**, 1234 (2022).

[3] J. A. Acebrón, L. L. Bonilla, C. J. Pérez Vicente, F. Ritort, and R. Spigler, Rev. Mod. Phys. **77**, 137 (2005).

[4] S. H. Strogatz, Sync: The Emerging Science of Spontaneous Order, Hyperion, New York (2003).

Extreme synchronization and hysteresis in phase oscillator networks

Cyriel C. van Velzen[1], Bob Rink[1], and Christian Bick[1]

[1] *Vrije Universiteit Amsterdam, Amsterdam, the Netherlands*

We investigate the collective behaviour of N all-to-all coupled phase oscillators θ_j. Writing $Z_n = \frac{1}{N}\sum_{k=1}^{N} e^{in\theta_k}$ for the Daido order parameters, we consider the system

$$\dot{\theta}_j = \omega + \Im(fe^{-i\theta_j}),$$

with $\omega \in \mathbb{R}$ and $f = Z_1 e^{-i\beta} + \varepsilon Z_p^q \overline{Z_k}^m e^{-i\gamma}$, which is an extension of the Kuramoto-Sakaguchi model by non-linear mean-field coupling terms, giving rise to "higher order interactions". Here $p, q, k, m \in \mathbb{N}$ satisfy $pq - km = 1$, $\varepsilon \in \mathbb{R}_{\geq 0}$, and $\beta, \gamma \in \mathbb{R}$ denote frustration parameters.

Then we ask: does the system synchronize ($|Z_1| \xrightarrow{t\to\infty} 1$) or rather tend to an incoherent state ($|Z_1| \to 0$), depending on $\beta, \gamma, \varepsilon$. For certain parameters, we find bistability of (near-)incoherence and synchrony and a sharp transition from one to the other as $\cos\beta$ crosses 0, which we call *extreme synchronization*. Consequently, we also find hysteresis in this system.

In our analysis, we use the Watanabe-Strogatz framework [1,2]. We also consider the asymptotic limit using the Ott-Antonsen Ansatz [3]. Furthermore, we quantify the finite-size effects in the dynamics. Comparison of analytical predictions with numerical results shows a high degree of agreement.

References

1. S. Watanabe and S. H. Strogatz, Physica D 74, 197–253 (1994)
2. S. A. Marvel, R. E. Mirollo, and S. H. Strogatz, Chaos 19, 043104 (2009)
3. E. Ott and T. M. Antonsen, Chaos 18, 037113 (2008)