WILHELM UND ELSE HERAEUS-STIFTUNG



659. WE-Heraeus-Seminar

Condensates of Light

January 14 – 17, 2018 at the Physikzentrum Bad Honnef/Germany

Introduction

The Wilhelm und Else Heraeus-Stiftung is a private foundation which supports research and education in science, especially in physics. A major activity is the organization of seminars. To German physicists the foundation is recognized as the most important private funding institution in their fields. Some activities of the foundation are carried out in cooperation with the German Physical Society (Deutsche Physikalische Gesellschaft).

Aims and scope of the 659. WE-Heraeus-Seminar:

Bose-Einstein condensation, the macroscopic ground state occupation of a system of integer spin particles (bosons), has been observed in the last twenty years for dilute cold atomic gases and in solid state systems with both exciton-polaritons and magnons. More recently, Bose-Einstein condensation of photons has been demonstrated in a dye-filled optical microcavity, where, unlike in a blackbody radiator, the photon number does not vanish at low temperature but instead exhibits condensation to the lowest energetic cavity mode. In other experiments, interesting other collective manybody effects of light have been observed in optical cavities, and both theorists and experimentalists are beginning to exploit novel quantum fluids of light.

The aim of the seminar is to illuminate the present status of Bose-Einstein condensates and other quantum fluids of light in different physical areas, ranging from atomic and molecular physics to solid state physics, including:

- Bose-Einstein condensation of light in dye-filled microcavities
- Condensates of semiconductor exciton-polaritons
- Organic polariton condensation
- Classical condensation phenomena in optics
- Superfluid light

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Sunday, January 14, 2018

17:00 - 21:00 Registration

18:00 *DINNER*

Monday, January 15, 2018

08:00	BREAKFAST		
09:00 - 09:50	Robert Nyman	Bose-Einstein condensation of just a few photons	
09:50 - 10:40	Henk Stoof	Broken symmetry in condensates of light	
10:40 - 11:10	COFFEE/ TEA BREAK		
11:10 – 12:00	Päivi Törmä	Bose-Einstein condensation in a plasmonic lattice	
12:00 – 12:50	Benjamin Lev	Strong, tunable-length, photon- mediated interactions between BECs in multimode cavity QED	
12:50	Conference Photo (in the front of the lecture hall)		
13:00	LUNCH		

Monday, January 15, 2018

14:50 – 15:40 Jacqueline Bloch Tailoring the stability of polariton condensates in semiconductor lattices

15:40 – 16:30 Natalia Berloff A polariton graph optimizer

16:30 - 17:00 COFFEE/ TEA BREAK

17:00 - 18:50 **Poster session**

19:00 *DINNER*

20:00 - 21:00 Poster Session

Tuesday, January 16, 2018

08:00	BREAKFAST	
09:00 - 09:50	Jan Klaers	Variable potentials for thermalized light and coupled condensates
09:50 - 10:40	Robin Kaiser	Non-equilibrium precondensation of classical waves in two dimensions propagating through atomic vapors
10:40 – 11:10	COFFEE/ TEA BREAK	
11:10 – 12:00	Jonathan Simon	Topological and strongly correlated photons
12:00 – 12:50	Sebastian Diehl	Probing the topology of density matrices
13:00	LUNCH	

Tuesday, January 16, 2018

14:50 – 15:15	Thorsten Ackemann	Optomechanical structuring in quantum-degenerate matter via diffractive light interactions
14:15 – 15:40	Hugo Terças	Bose-Einstein condensation of light in a plasma
15:40 - 16:05	Milan Radonjić	Interplay of coherent and dissipative dynamics in condensates of light
16:05 – 16:30	Refael Weill	Thermalization and Bose-Einstein condensation of photons in everyday fibers
16:30 – 17:00	COFFEE/ TEA BREAK	
17:00 – 17:50	Tal Goren	Topological Zak phase in strongly- coupled LC circuits
17:50 – 18:40	Marzena Szymanska	Polariton quantum fluids in and out of equilibrium
18:40 – 18:55	Stefan Jorda	About the Wilhelm and Else Heraeus Foundation
19:00	HERAEUS DINNER (social event with cold &	warm buffet with complimentary drinks)

Wednesday, January 17, 2018

08:00	BREAKFAST	
09:00 - 09:50	Dries Van Oosten	Effective interactions in a Bose- Einstein condensate of light
09:50 - 10:40	Peter Kirton	Superradiance and lasing in driven- dissipative Dicke models
10:40 – 11:10	COFFEE/ TEA BREAK	
11:10 – 12:00	Corinna Kollath	Cavity induced dynamic gauge fields in quantum gases
12:00 – 12:50	Thilo Stöferle	Strongly confined exciton-polariton condensates at room temperature
13:00	LUNCH	

Wednesday, January 17, 2018

14:50 – 15:15	Quentin Glorieux	Probing superfluid light in atomic vapor	
15:15 – 15:40	Chiao-Hsuan Wang	Photon condensation via laser cooling of atoms	
15:40 – 16:05	Jaime Gomez Rivas	Condensation of plasmon exciton polaritons in lattices of metallic nanoparticles	
16:05 – 16:30	André Eckardt	1-Large extent condensation of photons in a centre-axial symmetric photon container steady states of driven dissipative Bose gases	
16:30 – 16:40	Martin Weitz	Closing words	
16:40 – 17:00	COFFEE/ TEA		
17:00	End of the seminar and departure / Lab visit at Bonn University		

DINNER only in Bonn! For participants who take part at the lab visit.

Thursday, January 18, 2018

08:00 BREAKFAST

Departure

1st and 2nd-order coherence of a two-dimensional

Erik Busley photon gas Rhonda Au Yeung Effecting spontaneous coherence in hybridised cavityspin ensemble systems with incoherent driving Dario Ballarini Excitations of a long-lived polariton condensate Anton Baranikov Stimulated polariton condensation in an organic microcavity Johannes Beierlein Routing of exciton-polaritons in microcavity waveguides Matthieu Bellec Experimental evidences of light superfluidity in a bulk **Omar Boughdad** nonlinear crystal Claire Michel Robert Bennett QED treatment of the photon BEC in arbitrary Stefan Buhmann geometries: Coupled dissipative dynamics of dye Yaroslav Gorbachev molecules

Simon Betzold Tuneable light-matter hybridization in open organic

microcavities

The effective interaction strength in a photon Bose-Charly Beulenkamp

Einstein condensate in a dye-filled microcavity

Tom Bienaimé Superdluidity of ligth in a hot rubidium vapor:

Quentin Fontaine Sound-like excitation of a fluid of light in the propagating

Quentin Glorieux geometry

Hadiseh Alaeian

Paolo Comaron Dynamical critical exponent in a driven-dissipative

quantum system

Igor Dudas . Mean-field analysis and modelling of photon

condensation in a two-mode cavity

David Dung Christian Kurtscheid	Photon condensates in microstructured trapping potentials
Marco Dusel	Three-dimensional photonic confinement in imprinted liquid crystalline pillar microcavities
Uwe R. Fischer	Fragmentation of phase-fluctuating condensates
Francisco Garcia Flórez	Bose-Einstein Condensation of Light in Semiconductor Microcavities
Nikolay Gnezdilov	Dipolar quantum phase transition in the Dicke model with infinitely coordinated frustrating interaction
Sebastiaan Greveling	Polarization of a Bose-Einstein condensate of photons in a dye-filled microcavity
Henry Hesten Florian Mintert	Decondensation in non-equilibrium photonic condensates
Wassilij Kopylov	Numerical studies of dye-mediated photon-photon interaction in condensates of light
Johann Kroha Tim Lappe	Reservoir-induced collapse and revival of photon-BEC oscillations
Giovanni Lerario	Topological excitations and domain walls in a bistable polariton condensate
Calum Maitland	Exploring Superradiance in 2D Photon Fluid Rotating Black Holes

Bose-Einstein condensation in a plasmonic lattice

Exact analytical model for transport of 1D Bose- Einstein condensate under external waveguide

Antti Moilanen

Ajay Nath

Fahri Emre Öztürk Fluctuation-dissipation relations and finite

compressibility of a grand canonical Bose-Einstein

condensate

Axel Pelster Mean-field dynamics of a homogeneous photon Bose

Einstein condensate

Maciej Pieczarka Nonresonant manipulation of the photon propagation in a

1D microcavity laser

Radivoje Prizia Self-induced spatial decoherence in the Newton-

Schrödinger equation

Nick P. Proukakis Quench dynamics in driven-dissipative quantum systems

Milan Radonjic Modeling dye-mediated contribution to photon-photon

interaction in condensates of light

Marcel Scholten Bose-Einstein condensation of photons in various media

Vitaly Shumeiko Parametric resonance in superconducting resonator

Enrico Stein Collective frequencies of trapped photon Bose-Einstein

condensate

Artem Strashko Modelling the luminescence of exciton-polaritons with

organic molecules

Frank Vewinger Thermalization dynamics & calorimetry of a

Bose-Einstein condensed photon gas

Martina Vlaho Mode selection in a system of photons in a dye-filled

microcavity

Nina Voronova Anisotropic superfluidity of a photon condensate in a

periodically modulated microcavity

Christian Wahl Towards a photon Bose-Einstein condensate in the

vacuum-ultraviolet spectral regime

Maximilian Waldherr Suppression of inhomogeneous broadening of excitons

and trions in encapsulated MoSe₂ monolayers

Benjamin Walker Bose-Einstein condensation of just a few photon

Niclas Westerberg Self-bound droplets of light with angular momentum

Klaus Ziegler Superfluidity and collective properties of excitonic

polaritons in gapped graphene

Abstracts of Talks

(in alphabetical order)

Optomechanical structuring in quantum-degenerate matter via diffractive light interactions

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Supersolid phases are intriguing as they combine spatial structure with superfluidity. Early claims of observation in helium or cold atoms were controversial, but recent experiments demonstrated progress in 1D systems or intersected cavities [1].

We are considering a genuine 2D situation based on the single-mirror feedback scheme shown in Fig. 1. A detuned laser beam drives an expanded Bose-Einstein condensate at T=0. Most of the light is retro-reflected by a plane feedback mirror at some distance d. A modulation of the atomic density will cause a phase modulation of the transmitted wave. Diffraction in the feedback loop leads to conversion of phase-to-amplitude modulation on length scales $\sim \sqrt(\lambda d)$. This amplitude modulation leads to atomic bunching via dipole forces. Above some threshold, the density modulation and the optical lattice form spontaneously and sustain each other. For thermal atoms, this was demonstrated in [2].

A first treatment in [3] based on a Gross Pitaevskii equation with optical feedback confirmed the instability in quantum degenerate matter for vanishing atomic interactions and indicate a non-zero threshold even at T=0 due to the finite ground-state energy. Further analysis shows that repulsive atomic interactions hinder the instability whereas attractive ones enhance it. Numerical simulations in 1D [3] and 2D (Fig. 2) yield structured states in the atomic density, i.e. a supersolid phase. If the induced lattice would turn out to be strong enough to hinder tunneling, the resulting phase would be a highly interesting spontaneous Mott insulator.

- [1] J.-R. Li et al., Nature **543**, 91 (2017), J. Leonard et al., Nature **543**, 87 (2017)
- [2] G. Labeyrie et al., Nature Photonics 8, 321 (2014)
- [3] G. R. M. Robb et al., Phys. Rev. Lett. 114, 173903 (2015)

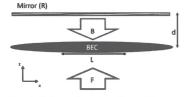
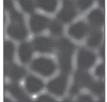


Fig. 1: Schematic setup. From [3].



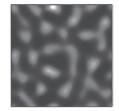


Fig. 2: Close to threshold phases of light intensity (left) and atomic density $|\psi|^2$ (right). Optical density in line center 200, detuning + 50 Γ .

A Polariton Graph Optimiser

K. Kalinin¹, P.G. Lagoudakis^{2,3}, Natalia Berloff^{1,3}

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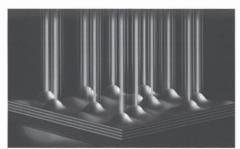
²Physics and Astronomy University of Southampton, UK

³Skolkovo Institute of Science and Technology, Russia

Recently we proposed polariton graphs as a novel platform for solving hard optimization problems that can be mapped into the XY model[1,2]. Polariton condensates can be imprinted into any two-dimensional graph by spatial modulation of the pumping laser as Fig. 1 illustrates. By controlling the pumping intensity and profile, the graph geometry and the separation distance between the lattice sites one can control the couplings between the sites and realise various phase configurations that minimize the XY model. This gives rise to the use of the polariton graph as an analogue XY Hamiltonian simulator. The search for the global minimum of the XY Hamiltonian is via a bottom-up approach which has an advantage over classical or quantum annealing techniques, where the global ground state is reached through either a transition over metastable excited states or via tunnelling between the states in time that depends on the size of the system.

In my talk I will discuss the range of optimization problems that can be efficiently solved by polariton graph. In particular, I will elucidate a relationship between the energy spectrum of the XY Hamiltonian and the total number of condensed polariton particles. Using as a test-bed the hexagonal unit lattice I will show that the lower energy states of the XY Hamiltonian are faithfully reproduced by mean-field numerical simulations utilising the Ginzburg—Landau equation coupled to an exciton reservoir[3].

Fig 1.
experiment:
condensates in the
and reading out
that minimize the



Schematics of the creating polariton vertices of a graph the phase differences XY Hamiltonian.

[1] N.G.Berloff, M. Silva, K. Kalinin, A. Askitopoulos, J D. Töpfer, P. Cilibrizzi, W. Langbein and P. G. Lagoudakis "Realizing the classical XY Hamiltonian in polariton simulators" Nature Materials 16, 1120–1126 (2017)

[2] P.G.Lagoudakis and N.G. Berloff, "A Polariton Graph SImulator," to appear in New Journal of Physics arXiv:1709.05498 (2017)

[3] K.Kalinin, P.G. Lagoudakis, and N.G.Berloff, "Simulating the spectral gap with polariton graphs,", arXiv:1709.04683 (2017)

Tailoring the Stability of Polariton Condensates in semiconductor lattices

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Quantum mechanics predicts that a homogeneous Bose-Einstein condensate at equilibrium cannot form in the presence of attractive interactions: they make the condensate to collapse so as to minimize its (negative) interaction energy. Cavity polaritons, exciton-photon hybrid quasi-particle generated in semiconductor microcavities, present strong mutual interaction (Kerr like non-linearity) which are essentially of repulsive nature, thus compatible with polariton condensate stability.

Under non-resonant excitation, a reservoir of uncondensed excitons is optically injected simulataneously to the polariton condensate. This reservoir is responsible for an effective attractive polariton-polariton interaction [1]. We experimentally demonstrate that indeed polariton condensates undergo a modulation instability, which is responsible for a fragmentation of the condensate and strongly reduces their spatial and temporal coherence [2]. Interestingly, depending of the sign of the polariton effective mass, intensity fluctuations in the excitonic reservoir are either amplified (for positive mass), or damped (for negative mass). When engineering the polariton band structure in a lattice, and triggering condensation in a negative mass quantum state, a stable regime with extended temporal and spatial coherence is evidenced [2].

A peculiar regime, where condensation is triggered in a band with infinite effective mass (flat band) will also be discussed [3].

This method for suppressing instabilities paves the way to a controlled exploration of long-range order in driven-dissipative condensates.

References

[1] T. C. H. Liew et al., Phys. Rev. B 91, 085413 (2015).

[2] F. Baboux et al., arXiv:1707.05798

[2] F. Baboux et al., Phys. Rev. Lett. 116, 066402 (2016)

Probing the topology of density matrices

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Topological concepts in many-body physics are so mainly formulated for ground states of Hamiltonians so far. The mixedness of a quantum state is usually seen as an adversary to topological quantization of observables. For example, exact quantization of the charge transported in a so-called Thouless adiabatic pump is lifted at any finite temperature in symmetry-protected topological insulators. Here, we show that certain directly observable many-body correlators preserve the integrity of topological invariants for mixed Gaussian quantum states in one dimension. Our approach relies on the expectation value of the many-body momentum-translation operator, and leads to a physical observable — the "ensemble geometric phase" (EGP) — which represents a bona fide geometric phase for mixed quantum states, in the thermodynamic limit. In cyclic protocols, the EGP provides a topologically quantized observable which detects encircled spectral singularities ("purity-gap" closing points) of density matrices. While we identify the many-body nature of the EGP as a key ingredient, we propose a conceptually simple, interferometric setup to directly measure the latter in experiments with mesoscopic ensembles of ultracold atoms.

References

 C. Barydn, L. Wawer, A. Altland, M. Fleischhauer, S. Diehl, arxiv:1706.02741 (2017)

Non-standard Bose condensation in non-equilibrium steady states of driven dissipative Bose gases

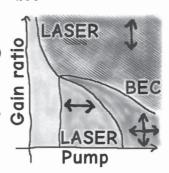
André Eckardt

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We investigate Bose-Einstein condensation in non-equilibrium steady states of driven-dissipative Bose gases, considering three basic scenarios: periodically forced (Floquet) systems coupled to a heat bath [1], systems coupled to two heat baths of different temperature [2], and pumped lossy photonic systems interacting with a heat bath [3]. Unlike equilibrium states, which are determined completely by a few thermodynamic variables like the bath temperature only, non-equilibrium steady states obey less restrictions and depend sensitively on the very details of the environment. This offers great freedom to tailor the properties of a quantum system by engineering its environment. Among others, we show that this freedom can be used for the robust preparation of excited-state and fragmented Bose condensates. We also demonstrate that Bose condensation can be induced by coupling a system to two competing baths, the temperatures of which both lie well above the equilibrium critical temperature [2]. Moreover, for a simple class of models describing a variety of complex photonic systems, we predict a cascade of transitions when the pump power is ramped up [3, 4]. First, above a threshold, the mode with the largest effective gain becomes macroscopically occupied (corresponding to simple lasing). Ramping up the pump further, further transitions can occur where single modes acquire or loose macroscopic occupation, eventually leading to a macroscopic occupation of the ground state alone (resembling equilibrium Bose condensation). Our theory describes, inter alia, a two-mode microcavity (see sketch) [3] and exciton-polaritons in a double well [Galbiati et al. PRL 108, 126403 (2012)] [4].

Based on references

- [1] D. Vorberg et al., PRL 111, 240405 (2013)& PRE 92, 062119 (2015)
- [2] A. Schnell et al., PRL 119, 140602 (2017)
- [3] A. Leymann et al., PRX 7, 021045 (2017)
- [4] D. Vorberg, R. Ketzmerick, A. Eckardt, in preparation



Probing superfluid light in atomic vapor

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In 2000, R. Chiao published a paper [1] asking if a photon fluid can be considered as a superfluid? Indeed, there are strong formal analogies between the Non-Linear Schrodinger Equation (NLSE) describing a beam of light propagating in a Kerr non-linear medium and the Gross-Pitaevskii Equation (GPE) describing the mean field behaviour of a quantum Bose gas.

Superfluidity is an universal phenomena which has been observed in a wide variety of systems from liquid Helium to atomic condensates. Superfluidity of light has also been reported using exciton-polaritons in a microcavity. Here we report the observation of a linear dispersion relation for a fluid of light propagating in a hot atomic vapor addressing the question ask by R. Chiao almost two decades ago. We believe that this demonstration can open the way for a new field of research on quantum fluid of light in the propagating geometry [2].

We have recently measured the dispersion relation by two complementary techniques and we observed a superfluid behavior at long wavelength as predicted. We compared our results with a microscopic theory and found an excellent agreement.

Finally, I will present future experiments that are highly counter-intuitive from a perspective of non-linear-optics and can be explain simply using the formalism of fluids of light, including the violation of Snell refraction law in degenerate four-wave-mixing.

References

- [1] R. Y. Chiao and J. Boyce, Bogoliubov dispersion relation and the possibility of superfluidity for weakly interacting photons in a two-dimensional photon fluid, Phys. Rev. A 60, 4114 (1999)
- [2] D. Vocke, D. Faccio and al., Experimental characterization of nonlocal photon fluids, Optica, vol 2 (2015).

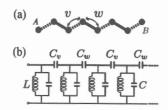
Topological Zak Phase in Strongly-Coupled LC Circuits

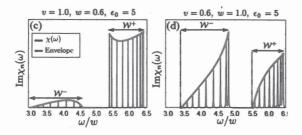
<u>Tal Goren</u>¹, Kirill Plekhanov¹², Felicien Appas¹, Karyn Le Hur¹
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Recently we have shown the emergence of topological Bogoliubov bosonic excitations in the relatively strong coupling limit of an LC (inductance-capacitance) one-dimensional quantum circuit [1]. This dimerized chain model reveals a Z2 local symmetry as a result of the counter-rotating wave (pairing) terms. The topology is protected by the sub-lattice symmetry, represented by an anti-unitary transformation.

The topology of a one dimensional system is defined by the Zak phase. We show how the winding of the topological Zak phase across the Brillouin zone can be measured by a reflection measurement of (microwave) light. Our method probes bulk quantities and can be implemented even in small systems. We study the robustness of edge modes towards disorder.





[1] Tal Goren, Kirill Plekhanov, Felicien Appas, Karyn Le Hur, arXiv: arXiv:1711.02034

Condensation of Plasmon Exciton Polaritons in Lattices of Metallic Nanoparticles

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Metallic nanostructures provide a toolkit for the generation of coherent light below the diffraction limit [1]. Strong light-matter coupling between excitons in organic molecules and the electromagnetic field generated by the plasmonic structures leads to the formation of hybrid quasi-particles known as plasmon-exciton-polaritons (PEPs). Due to the bosonic character of these quasi-particles, exciton-polariton condensation can lead to laser-like emission at much lower threshold powers than in conventional photon lasers. We have recently observed PEP lasing through a dark plasmonic mode in an array of metallic nanoparticles with a low threshold in an optically pumped organic system. Interestingly, the threshold power of the lasing is reduced by increasing the degree of light-matter coupling in spite of the degradation of the quantum efficiency of the active material [2]. Coherence measurements show a long-range coherence length, larger than the illumination area on the sample. Time-resolved experiments evidence the sub-ps formation dynamics of the condensate and a sizeable blueshift, thus measuring the effect of polariton interactions in plasmonic cavities [3]. Our results pave the way to the observation of room temperature condensation and novel nonlinear phenomena in plasmonic systems, challenging the common belief that absorption losses in metals prevent the realization of macroscopic quantum states.

References

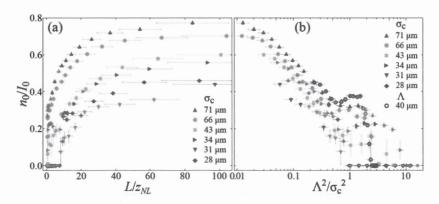
- W. Wang, M. Ramezani, A.I. Väkeväinen, P. Törma, J. Gómez Rivas, and T.W. Odom, The Rich Photonic World of Plasmonic Nanoparticle Arrays, Materials Today (in press).
- [2] M. Ramezani, A. Halpin, A.I. Fernández-Domínguez, J. Feist, S.R.K. Rodriguez, F.J. Garcia-Vidal, and J. Gómez Rivas, Plasmon-exciton-polariton lasing, Optica 4, 31-37 (2017).
- [3] M. De Giorgi, M. Ramezani, F. Todisco, D. Caputo, A. Halpin, A. Fieramosca, D. Sanvitto, and J. Gomez Rivas, Plasmon-exciton Polariton Condensation Evidenced Through Spatial Coherence and Interactions, submitted.

Non-equilibrium precondensation of classical waves in two dimensions propagating through atomic vapors

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The nonlinear Schrödinger equation, used to describe the dynamics of quantum fluids, is known to be valid not only for massive particles, but also for the propagation of light in a nonlinear medium, predicting condensation of classical waves. Here we report on the initial evolution of random waves with Gaussian statistics using atomic vapors as an efficient two dimensional nonlinear medium. Experimental and theoretical analysis of near field images reveal a phenomenon of nonequilibrium precondensation, characterized by a fast relaxation towards a precondensate fraction of up to 75%. Such precondensation is in contrast to complete thermalization to the Rayleigh-Jeans equilibrium distribution, requiring prohibitive long interaction lengths.



Precondensate fraction as a function of (a) propagation distance L/z_{NL} and (b) as a function of healing length $(\Lambda/\sigma)^2$

Superradiance and Lasing in Driven-Dissipative Dicke Models

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When the effective coupling between matter and light is large enough, a phase transition to a "superradiant" state can occur. The archetypal model for this is the Dicke model, describing N emitters coupled to a single photon mode. Such a transition has been realised in a driven system, with ultracold atoms in an optical cavity [1]. Similar physics can also be seen in superconducting qubits coupled to a microwave resonator, microcavity polaritons and several other experimental platforms. These experiments prompt questions about the effects of loss and dissipation on the phase transition. By using a combination of exact numerics based on permutation symmetry of the density matrix, and second order cumulant equations, we show that the presence of pure dephasing can kill the superradiance transition, but it can be restored by loss terms [2]. The fact that adding atom loss can restore a phase transition to an ordered state is a surprising result, which provides a crucial example where the precise form of dissipation dramatically changes the collective behaviour.

By adding incoherent driving to we are able to produce a model which crosses from superradiance to regular lasing [3]. We show that while the photon state in both phases is similar the emission spectrum of the cavity is able to clearly distinguish between them. This model gives rise to a complex non-equilibrium phase diagram which has regions of chaotic behaviour.

References

- [1] K. Baumann, C. Guerlin, F. Brennecke, and T. Esslinger, Nature **464**, 1301 (2010)
- [2] P. Kirton and J. Keeling, Phys. Rev. Lett. 118, 123602 (2017)
- [3] P. Kirton and J. Keeling, arXiv 1710.06212 (2017)

Variable potentials for thermalized light and coupled condensates

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We demonstrate a microstructuring technique that allows the generation of variable potentials for light within an optical high-finesse microcavity [1]. The long photon lifetime enables the thermalization of photons to the temperature of the environment and the formation of a microscopic photon condensate in a single localized site. We observe effective photon interactions as well as tunnel coupling between two microsites. The associated hybridization of eigenstates of the double-well system is monitored spectroscopically.

The investigated method is based on a thermo-sensitive polymer enclosed in an optical microresonator. In water, the polymer has a lower critical solution temperature of 32°C. Around this temperature, the polymer shows an extremely large thermo-optical coefficient: small (laser-induced) temperature changes lead to large changes in the index of refraction of the medium. This thermo-optical property can be used to spatially modulate the index of refraction of the optical medium in the resonator plane with micrometer resolution, which effectively introduces a fully tune-able trapping potential for the light in the microresonator. In particular, it is possible to capture photons onto periodic lattices sites. A unique feature of this technique is the fact that it is fully reversible: if the temperature drops below the lower critical solution temperature, the polymer recovers to its original water-solved state, which allows to successively implement an arbitrary number of different lattice geometries and tunnel coupling configurations in the same experimental system. We expect that the investigated system can used as a simulator for phenomena from solid-state physics.

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Cavity induced dynamic gauge fields in quantum gases

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The realization and control of topologically non-trivial quantum phases is currently of great interest after discovery of the topological insulators. The extended edge state existing in such materials are non-local and linked to the topological characteristics of the bulk. Therefore they are well protected against environmental influences and ideal candidates for quantum computation. It is not easy to manipulate the topologically protected quantities, which are typically of non-local character, via local and unitary operations. This difficulty can be overcome by coupling atoms to the cavity field which leads to an effective long-range interaction betweem atoms. We discuss how a quantum gas confined to optical lattices and coupled to an optical cavity together with a running pump laser beam, can organize into a topologically non-trivial state. The cavity field emerges spontaneously and induces a dynamical gauge field. This feedback leads to the self-organization of the topological quantum state which carries an extended edge state as the attractor state of a dissipative dynamics in a finite system.

Strong, tunable-length, photon-mediated interactions between BECs in multimode cavity QED

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We will present our first experiments involving multimode cavity QED, wherein the cavity-mediated atom-atom interaction is dramatically modified compared to that in a single-mode cavity. We will report on measurements to characterize the interaction between atoms in such a cavity and demonstrate control of the range of this interaction from long to short range. These results pave the way for future experimental access to nontrivial phase transitions in driven-dissipative quantum systems, synthetic dynamical gauge fields, and the ability to study novel non-equilibrium spin glasses and quantum neuromorphic computation.

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Bose-Einstein condensation of just a few photons Rob Nyman

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Bose-Einstein condensation (BEC) is well-defined for macroscopic numbers of particles at thermal equilibrium. Here we explore how that definition breaks down, for small numbers of particles in a driven-dissipative system where thermal equilibration is marginal. Photons in a dye-filled microcavity reach thermal equilibrium through multiple absorption and emission events from a fluorescent dye. Thermal equilibrium describes the photon distribution if the cavity lifetime is longer than the re-absorption time. With sufficient pump rate, BEC will occur [1,2]. By microfabricating mirrors with small radii of curvature we form harmonic traps for the photons with THz characteristic frequencies, and we can measure populations of individual modes.

Taking into account the distribution across all modes, the system is well-described by the Bose-Einstein distribution. The occupancy of the ground state as a function of total particle number indicates a threshold number for BEC of just 7 ± 1 photons, with up to 95% of all cavity photons in the ground state possible. The first-order correlation function for the ensemble shows decays in times $\sim\hbar/k_BT$, with revivals after trap periods, characteristic of equilibrium Bose gases in harmonic traps [3]. When the thermalisation rate is decreased, other modes reach threshold. The system can be understood as a multimode condensate or laser, for which we have numerically uncovered a rich phase diagram [4,5].

By contrast, individual modes may be understood as microlasers. In particular, the coherence time for each mode is much longer than the coherence time of the ensemble. As a driven-dissipative laser-like system, we unsurprisingly find increased coherence time for increased photon number, but decreased coherence time for increasing thermalisation rate. Thus thermalisation can both increase coherence (encouraging BEC) and decrease it (acting as a photon loss mechanism).

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Interplay of Coherent and Dissipative Dynamics in Condensates of Light

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Based on the Lindblad master equation approach, we obtain a detailed microscopic model of photons in a dye-filled cavity, which features condensation of light [1-3]. To this end, we generalize a recent non-equilibrium approach of Kirton and Keeling [4], such that the dye-mediated contribution to the photon-photon interaction in the photon condensate is accessible due to an interplay of coherent and dissipative dynamics [5]. We describe the steady-state properties of the system by analyzing the resulting equations of motion of both photonic and matter degrees of freedom. In particular, we discuss the existence of two limiting cases for steady states: photon Bose-Einstein condensate and laser-like regime. In the former case, we determine the corresponding dimensionless photon-photon interaction strength by relying on realistic experimental data and find a good agreement with previous theoretical estimates [6]. Furthermore, we investigate how the dimensionless interaction strength depends on the respective system parameters.

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Topological and Strongly Correlated Photons

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I will survey the ongoing photonic material work in my group, exploring both microwave photons in lattices and optical photons in the continuum. In the former case, we realize photonic Chern insulators by coupling each lattice unit-cell to a ferrimagnet, and photonic Mott insulators by coupling each unit cell to a Transmon qubit. In the latter case, we produce Landau levels for light by trapping the photons within a twisted (non-planar) optical resonator, and mediate interactions between individual photons by hybridizing them with Rydberg excitations of an atomic gas. I will conclude by discussing prospects for realizing topological many-body states of light, emphasizing their unique promise for understanding topological order and manipulating anyonic quasi-particles.

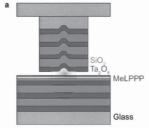
Strongly confined exciton-polariton condensates at room temperature

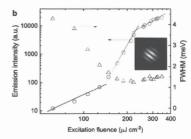
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We create cavity exciton-polaritons by placing a conjugated polymer inside an optical microcavity. At ambient conditions, we observe thermalization and non-equilibrium Bose-Einstein condensation, which is evidenced through nonlinear emission, interaction-induced blue-shift and long-range macroscopic coherence [1]. Harnessing recently developed tunable nanoscale defect cavities (left figure) to produce zero-dimensional polaritons [2] allows us to realize condensates with strong lateral confinement on the order of the wavelength scale [3].

Data on the condensation threshold, line narrowing, blue shift are analyzed as a function of excitation power (right figure) and as well as a function of the detuning of the cavity from the exciton energy. Furthermore, we explore the condensation in momentum space, exhibiting the distinct signature of the strong transversal confinement. First order coherence properties are measured with a Michelson interferometer (right figure, inset). In summary, we present the first tunable room-temperature polariton condensates with wavelength-scale confinement. This marks the initial steps for studying quantum fluids in extended, arbitrary potential landscapes at ambient conditions.





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Broken Symmetry in Condensates of Light

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The recent achievement of Bose-Einstein condensation of photons offers the exciting possibility to study the superfluid properties of light, both for fundamental light-matter research as well as for ultimately applying these properties to the creation of new optoelectronic devices.

In particular, the nonconservation of the number of photons leads to new fundamental questions about the intimate relationship between superfluidity and spontaneous symmetry breaking, which can for instance be addressed by studying the (global) phase dynamics of the Bose-Einstein condensate in an interference experiment.

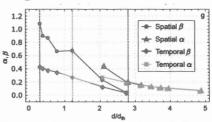
Spontaneously breaking the symmetry in a condensate of light requires (repulsive) interactions between the photons. These interactions are also crucial for a number of important equilibrium and dynamical properties of the light condensate. One of these is the superfluid-Mott-insulator transition and another the possible creation of a so-called radial vortex, which turns out to be the fluid analogue of a Reisnner-Nordström black hole known from classical gravity physics.

Polariton quantum fluids in and out of equilibrium [1,2]

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State-of-the-art semiconductor microcavities allowed recently to achieve fully thermalised photonic system analogous to cold atoms or liquid Helium. Here, we predict and observe the Berezinskii-Kosterlitz-Thouless transition for a 2D gas of exciton-polaritons with its clear signature in the first-order coherence both in space and time. We Fig 1: Algebraic exponents of spatial and temporal coherence. show that the mechanism of pairing of the



topological defects (vortices) is responsible for the transition to the algebraic order and achieve a thermodynamic equilibrium phase transition in an otherwise open driven/dissipative system [1].

At the same time, it has been shown that driven-dissipative polariton fluid could potentially exhibit a truly novel non-equilibrium order, where superfluidity is accompanied by stretched exponential decay of correlations. This celebrated Kardar-Parisi-Zhang (KPZ) phase has not been achieved in any physical system in 2D and even 1D realisations are not conclusive. Here, we show that driven microcavity polaritons in the OPO configuration can act as a natural and unifying laboratory for the exploration of a variety of intrinsic non-equilibrium phenomena, including the so far experimentally elusive KPZ phase in two dimensions [2]. Key features are the high tuneability of microscopic parameters in general, and of the spatial anisotropy of the microscopic physics in particular that allows one to move between different phases. We find that in the low pump power regime of the OPO configuration, the long-distance behaviour is governed by the non-equilibrium fixed point of the KPZ universality class. In the middle region of the OPO phase diagram the crossover length scale to the KPZ physics become small, approaching the order of the healing length and suggesting that the KPZ order in a quantum system might indeed be observed in experiments on semiconductor microcavities -- in stark contrast to incoherently pumped polaritons. On the other hand, in the same OPO configuration but at higher pump powers, we demonstrate that despite their intrinsic drivendissipative nature and highly non-thermal occupations, such systems can be driven to a phase characterised by the dynamical XY universality class. They then show emergent equilibrium behaviour at asymptotic length scales, with algebraic order and superfluidity even in the thermodynamic limit.

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Bose-Einstein Condensation of Light in a Plasma

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We study the Bose-Einstein condensation of photons in a plasma, where we include the cases of both transverse photons and plasmons. We consider four-wave mixing processes of photon and plasmon modes in a relativistic isotropic plasma to determine the coupling constant to lowest order. We further show that photon condensation is possible in an unbounded plasma because, in contrast with other optical media, plasmas introduce an effective photon mass. This guarantees the existence of a finite chemical potential and a critical temperature, which is calculated for both transverse photons and plasmons. By considering four-wave mixing processes, we derive the interactions between the photons in the condensate. We also study the elementary excitations (or Bogoliubov modes) of the condensed photon and plasmon gases, and determine the respective dispersion relations. Finally, we discuss the kinetics of photon condensation via inverse Compton scattering between the photons and the electrons in the plasma.

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Bose-Einstein Condensation in a Plasmonic Lattice

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Bose-Einstein condensation is a remarkable manifestation of quantum statistics and macroscopic quantum coherence. Superconductivity and superfluidity have their origin in Bose-Einstein condensation. Ultracold quantum gases have provided condensates close to the original ideas of Bose and Einstein, while condensation of polaritons, magnons and photons have introduced novel concepts of non-equilibrium condensation. We demonstrate a Bose-Einstein condensate (BEC) of surface plasmon polaritons in lattice modes of a metal nanoparticle array [1]. Interaction of the nanoscale-confined surface plasmons with a room-temperature bath of dye molecules enables thermalization and condensation in picoseconds. The ultrafast thermalization and condensation dynamics are revealed by an experiment that exploits thermalization under propagation and the open cavity character of the system. A crossover from BEC to usual lasing (such as [2]) is realized by tailoring the band structure. This new condensate of surface plasmon lattice excitations has promise for future technologies due to its ultrafast, room-temperature and on-chip nature.

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Effective interactions in a Bose-Einstein condensate of light.

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Since the achievement of Bose-Einstein condensation of photons by the Bonn group, several groups have been investigating the properties of such Bose-Einstein condensates, both theoretically and experimentally. In our group, we have focussed on studying whether photons in the condensate have some effective interaction and on what the polarization state of the condensate and thermal cloud is. In my talk, I will briefly address the subject of polarization, but mainly discuss the interactions.

In our experiment, we measure the radius of the condensate as a function of the number of photons in the condensate, on a single-shot basis. That is, we individually image and analyse each condensate we produce. By analysing many hundreds of condensates in this manner, we clearly observe that the condensate radius increases for increasing condensate photon number. This strongly hints at the presence of some effective photon-photon interactions. Surprisingly, the effective interaction strength we find seems to be orders of magnitude higher than what one would expect on the basis of the most likely interaction mechanisms (a Kerr-like nonlinearity or a thermo-optic effect).

Photon condensation via laser cooling of atoms C.-H. Wang^{1,2}, M. J. Gullans³, J. V. Porto^{1,4}, W. D. Phillips^{1,4}, J. M. Taylor^{12,4}

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In high optical depth atomic ensembles, we show that photons reemitted during the laser cooling process can equilibrate with the atomic motion and reach a steady state. We separate a set of long-lived (optically thick) photonic modes and study the atomic photon re-emission and absorption on top of the free-space cooling mechanism. In this regime, we find that a grand canonical ensemble of photons can arise directly via atomic laser cooling in an experimentally accessible regime, with a chemical potential controlled by the laser frequency [1]. Moreover, by placing the atoms in a curved cavity, the transverse modes in the cavity can be mapped into 2D massive bosons inside a parabolic well and can lead 2D Bose-Einstein condensate of light. We numerically studied the condensate fraction and the total photon number with simulated values appropriate for the Yb intercombination transition and constructed a phase diagram in the laser frequency and intensity parameter space showing the gain, condensate, thermal and quasi-thermal regimes for cavity photons.

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Thermalization and Bose-Einstein condensation of photons in everyday fibers

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Photons in laser cavities that are commonly not in thermal equilibrium (TE) were found to thermalize, show Bose-Einstein spectral distribution and even Bose-Einstein condensation (BEC) in one experimental system of a dye-filled microcavity [1]. It required strict conditions that included a micron-size cavity with a two-dimensional (2D) confinement of lateral modes, very high mirror reflectivities, and very low losses. It initiated theoretical studies, discussions and questions about the nature of photon-BEC in optical cavities and its relation to lasers [2].

In this work we find TE and BEC in standard erbium doped fiber (EDF) systems that can be a most simple platform for photon-BEC where most of the above restrictions in the microcavity experiment are relaxed.

The experiments were done in a standard, meters-long, double clad Er/Yb codoped fiber (EYDF) cavit. We used a double-clad EYDF, usually used as amplifiers for communication at the 1550nm wavelength regime that gives similar thermalization behavior to what we obtained in EDF [3] but allows relatively uniform pumping along the fiber. An important part of our work is the temperature dependence measurement that shows a close to linear decrease of the condensation power with temperature until it disappears at a critical temperature, in accordance with the theory for a photon gas in 1D.

The figure shows the measured spectra, for different pumping levels, showing thermalized spectra and BEC. The inset shows the theoretical BE (1D) distribution with a wavelength cutoff.



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