Delayed Complex Systems

675. WE-Heraeus-Seminar

July 2 - 5, 2018
at the Physikzentrum Bad Honnef/Germany

WILHELM UND ELSE
HERAEUS-STIFTUNG

Introduction

The Wilhelm and Else Heraeus Foundation (Wilhelm und Else Heraeus-Stiftung) is a private foundation which supports research and education in science, especially in physics. A major activity is the organization of seminars. To German physicists the foundation is recognized as the most important private funding institution in their fields. Some activities of the foundation are carried out in cooperation with the German Physical Society (Deutsche Physikalische Gesellschaft).

Scope of the 675. WE-Heraeus-Seminar:

Time delays play an important role in many applications in physics, biology and engineering. They appear, for example, in systems where signal propagation, memory effects or processing delays cannot be neglected. As a result these systems are described by delay differential equations.

The topics include but are not limited to:

- Laser dynamics with external (delayed) feedback
- Time delayed feedback control
- Complex dynamics and synchronization in delay coupled systems
- The role of delays in population dynamics and epidemiology
- Effects of nonlinearities and delays in engineering applications
- Fundamental aspects of systems with variable or state-dependent delay

Scientific Organizers:

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Program

Program

Sunday, July 1, 2018

17:00 – 21:00	Registration	
from 18:30	*BUFFET SUPPER / Informal get together*	

Monday, July 2, 2018

08:00	BREAKFAST	
09:00 – 09:15	Scientific organizers	**Opening and welcome**
09:15 – 10:00	Rajarshi Roy	**Photons, Networks and Synchrony in Complex Delayed Systems**
10:00 – 10:45	Yuliya Kyrychko	**Exploring the wonderland of systems with dirstributed delays**
10:45 – 11:15	*COFFEE BREAK*	
11:15 – 12:00	Luca Giuggioli	**Analytic investigations of probability distributions for linear delayed Langevin equations with additive Gaussian noise**
12:00 – 12:45	Andre Longtin	**Distributed delays in stochastic neural systems**
12:45	*LUNCH*	

Program

Monday, July 2, 2018

14:15 – 15:00	Bernd Krauskopf	**State-dependent delayed feedback in the El Niño Southern Oscillation system**
15:00 – 15:30	Tony Humphries	**Bifurcations of Scalar DDE Cell-Cycle Models**
15:30 – 16:00	David Müller	**Laminar Chaos**
16:00 – 16:30	COFFEE BREAK	
16:30 – 18:00	**Poster Flashes**	
18:00 – 19:00	**Poster Session**	
19:00	*DINNER*	

Program

Tuesday, July 3, 2018

07:30	*BREAKFAST*	
08:45 – 09:00	Stefan Jorda	**About the Wilhelm and Else Heraeus Foundation**
09:00 – 09:45	Rifat Sipahi	**Effects of Delays on the Dynamics of Multi-Agent Systems**
09:45 – 10:30	Wim Michiels	**Design of structured controllers for network of delay-coupled systems**
10:30 – 10:45	**Conference Photo**	
10:45 – 11:15	*COFFEE BREAK*	
11:15 – 12:00	Sue Ann Campbell	**Symmetry, Hopf Bifurcation and the Emergence of Cluster Solutions in Time Delayed Neural Networks**
12:00 – 12:45	Eckehard Schöll	**Partial synchronization patterns - interplay of time delay, dynamics, and network topology**
12:45	*LUNCH*	
14:15 – 18:30	**Excursion:** • **Boat trip on the Rhine to the City of Linz** • **Stay in Linz** • **Boat trip back to Bad Honnef**	
19:00	*DINNER*	

Program

Wednesday, July 4, 2018

08:00	*BREAKFAST*	
09:00 – 09:45	Serhiy Yanchuk	**Dynamical patterns in networks of excitable systems with coupling delays**
09:45 – 10:30	Vladimir Klinishov	**Destabilization of regular spiking in networks with pulse delayed coupling**
10:30 – 11:00	*COFFEE BREAK*	
11:00 – 11:45	John Milton	**Hybrid systems with delay in human balance control**
11:45 – 12:30	Michael Mackey	**Mathematical models of gene regulation: Biology drives new mathematics**
12:30	*LUNCH*	
14:00 – 14:45	Laurent Larger	**Delay dynamics emulating network of coupled oscillators: from chimera states to brain-inspired processing**
14:45 – 15:30	Ingo Fischer	**Employing Delay-Systems for Fast Neuro-Inspired Information Processing**
15:30 – 16:00	Thomas Jüngling	**Consistency properties of chaotic systems driven by time-delayed feedback**
16:00 – 16:30	*COFFEE BREAK*	
16:30 – 17:15	Andrei Vladimirov	**Time-delay systems in multimode laser dynamics**
17:15 – 18:00	Jan Sieber	**Spectrum of periodic orbits with long delay and slightly longer period**
18:00 – 18:45	Kathy Lüdge	**Routes to complexity in the light output of mode-locked lasers**
19:00	*HERAEUS DINNER at the Physikzentrum (cold & warm buffet, free beverages)*	

Program

Thursday, July 5, 2018

08:00	BREAKFAST	
09:00 – 09:45	Svetlana Gurevich	**Delayed feedback control of self-mobile cavity solitons in a wide-aperture laser with a saturable absorber**
09:45 – 10:30	Thomas Erneux	**Fronts, pulses, and wave trains of delayed PDEs**
10:30 – 11:00	COFFEE BREAK	
11:00 – 11:45	Fatihcan Atay	**Conserved Quantities in Network Dynamics under Time Delays**
11:45 – 12:30	Spase Petkoski	**Heterogeneity of time delays determines synchronization of coupled oscillators**
12:30 – 12:45	Scientific organizers	**Closing remarks and Poster Awards**
12:30	LUNCH	

End of the seminar and FAREWELL COFFEE **/ Departure**

Please note that there will be **no** dinner at the Physikzentrum on Thursday evening for participants leaving the next morning.

Posters

Posters

01	Hiroyasu Ando	**Time-delayed feedback control of diffusion in random walkers-Phenomena**
02	Tom Birkoben	**Time delay induced dynamic changes in coupled relaxation-type oscillators**
03	Konstantin Blyuss	**Time-delayed models for the impact of awareness on the spread of epidemics**
04	Philipp C. Böttcher	**Delayed Control of the Synchronous European Electricity Grid**
05	Svitlana Bugaychuk	**Longitudinal solitons in time-delayed nonlinear systems**
06	Daniel Canaday	**Rapid Time Series Predictions with Hardware-Based Reservoir Computing**
07	Huseyin Ersin Erol	**Decentralized time-delay controller design**
08	Joseph Hart	**Experiments with arbitrary networks in time-delayed systems**
09	Altug Iftar	**Decentralized Control of Time-delay Systems**
10	Firas Khasawneh	**Utilizing Topological Data Analysis for Studying Signals of Time- Delay Systems**
11	Miki Kobayashi	**Time-delayed feedback control of diffusion in random walkers-Analysis**
12	David Lipshutz	**Exit time asymptotics for small noise stochastic delay differential equations**

Posters

13	Toru Ohira	**Delayed Stochastic Resonances**
14	Suleyman Mert Özer	**New methods and a software to design decentralized stabilizing controllers for time-delay systems**
15	Gabriela Petrungaro	**Mobility-induced chimera states**
16	Alexander Pimenov	**Analysis of temporal localized structures in a time delayed model of a ring laser**
17	Xavier Porte Parera	**Controlling switching dynamics in micropillar lasers with time-delayed optical feedback**
18	Dmitry Puzyrev	**Bound pulse trains in coupled spatially extended dynamical systems**
19	Courtney Quinn	**Exploration of bistability in a paleoclimate delay model**
20	Alexandre René	**Stochastic delay systems tend to be effectively low-dimensional**
21	Stefan Ruschel	**Beyond Square Waves: Stable asymmetric spatiotemporal patterns in singularly perturbed Delay Differential Equation**
22	Arindam Saha	**Extreme events in delay coupled systems: Generating mechanism, precursors and baisins of attraction**
23	Sergey Sobolev	**Discrete model for spatially extended systems with time delays: Application to rapid alloy solidification**

Posters

24	Pankaj Wahi	**State Dependent Delayed models for drill-string Dynamics: Stability and Bifurcation Characteristics**
25	Hendrik Wernecke	**Tests for chaos and partial predictability with an application to time delay systems**
26	Lucas Wetzel	**Robust synchronization in networks of spatially-distributed delay-coupled electronic clocks**

Abstracts of Lectures

(in chronological order)

Photons, Networks and Synchrony in Complex Delayed Systems

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Delays appear naturally in optical systems when propagation times are included in models of lasers and resonators. They made significant entries into the dynamical behavior of paradigmatic models through the insights of Ikeda and coworkers and of Lang and Kobayashi almost forty years ago [1, 2, 3] and continue to reveal new aspects today in both basic science and engineering applications. Unexpected features are revealed as we learn to fabricate new sources and observe them with unprecedented (in time and space) resolution. The dynamics of single devices, the appearance of synchrony between periodic and chaotic dynamics of coupled systems, and the design and creation of networks for communication and information processing are all frontier research areas. Homogeneous and heterogeneous delays and distributions of delays occur naturally in experimental systems that are investigated. We will discuss experimental systems that we have studied recently to quantify the emergence of chaotic attractors from the stochastic dynamics of single photon events using information theoretic ideas [4]. Applications to the study of chimera states [5], dynamical computation, random number generation [6] and synchronization patterns will be outlined. An application of the space-time mapping ideas for delay systems will be described that leads to the design and creation of arbitrary networks through the use of field programmable gate array technology [7].

References

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Exploring the wonderland of systems with distributed delays

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Many physical, biological and engineering processes can be represented mathematically by models of coupled systems with time delays. Time delays in such systems are often either hard to measure accurately, or they are changing over time, so it is more realistic to take time delays from a particular distribution rather than to assume them to be constant. In this talk, I will show how distributed time delays affect the stability and synchronisation properties, and give rise to a new type of solutions in systems of coupled oscillators. Furthermore, I will present a system with distributed delays and Gaussian noise, and illustrate how to calculate the optimal path to escape from the basin of attraction of the stable steady state, as well as how the distribution of time delays influences the rate of escape away from the stable steady state. Throughout the talk, analytical calculations will be supported by numerical simulations to illustrate possible dynamical regimes and processes.

Analytic investigations of probability distributions for linear delayed Langevin equations with additive Gaussian noise

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The linear delayed Langevin equation with additive Gaussian noise is studied analytically in two ways. (1) Using characteristic functionals it is possible to construct the full time dependence of the multivariate probability distribution of the stochastic process as a function of the delayed and nondelayed random variables. (2) And by deriving Fokker-Planck equations for the one and two-time probability distribution which subsequently can be solved exactly.

References

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Distributed delays in stochastic neural systems

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Distributed delay systems are dynamical descriptions that take into the account that feedback pathways are made up on heterogeneous sub-pathways, each one with its own delay. The combination of noise and distributed delays is presented in two contexts. First, an analysis of the stochastic distributed delayed dynamics in one variable is given that reduces the full dynamics into a exact mode decomposition. It is shown that the dominant part of the solution evolves on a smaller effective dimension [1]. Second, a description of a neural network with delays is presented, with the goal of understanding how mild oscillations can arise in a new feedforward context, one where excitation is paired with delayed inhibition. Linear response theory can be used to analytically calculate the oscillatory strength and the role of delay distribution [2]. Finally we present preliminary results linking distributed delay dynamics to systems with a large number of randomly chosen delays, which have been proposed in the context of random number generation.

References

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State-dependent delayed feedback in the El Niño Southern Oscillation system

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Global climate phenomena generally feature feedback loops with delay times on the order of months, which arise from the transport of energy through the atmosphere and/or oceans over large distances. This is why their mathematical description by delay differential equations (DDEs) has proved successful at a conceptual level. So far, the delays are generally assumed to be constant. The question is whether this is realistic, and what behaviour may arise from state dependence of delays. As a concrete example we consider a DDE model for the El Niño Southern Oscillation climate system. We present physical arguments for the state-dependence of relevant delays and conduct a bifurcation analysis that demonstrates its effects for the overall dynamics of the system.

Bifurcations of Scalar DDE Cell-Cycle Models

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We consider the Mackey-Glass Equation and a second DDE which represents the Burns-Tannock model of the cell cycle. Both models result in nonlinear scalar DDEs with a single constant delay. Although both are well known and studied, aspects of their dynamics have yet to be fully explored. We are particularly interested in bifurcations which are already known in higher dimensions in ODEs, but which have not (as far as we are aware) been studied in scalar DDEs. Although the infinite-dimensionality of the DDE phase space allows for the possibility of such bifurcations occurring in scalar equations, analysing them can be non-trivial. In particular we will focus on an apparent canard explosion that arises in the Burns-Tannock model. Previous explorations of canards in DDEs considered equations with at least two spatial dimensions. We will also reveal flip-fold bifurcations that occur in the Mackey-Glass equation.

Laminar Chaos

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Time delay systems can be found in various fields such as engineering, biology and physics. In the scientific literature many results exist for systems with constant delay. However, in reality delays are typically not constant but rather time-varying. Recently we found that there are two fundamental classes of time-varying delays [1, 2]. Systems with conservative delay are equivalent to systems with constant delay. On the other hand, there are systems with dissipative delay, which cannot be transformed to systems with constant delay and are associated with fundamentally different dynamic properties.

In this talk we show that nonlinear systems with large dissipative delay can exhibit a new kind of chaotic behavior characterized by laminar phases, which are periodically interrupted by irregular bursts [3]. In particular, the output intensity during the laminar phases remains almost constant, but its level varies chaotically from phase to phase, see Fig. 1. The periodic dynamics of the lengths and the chaotic dynamics of the intensity levels can be understood and also tuned via two one-dimensional maps, which can be deduced from the nonlinearity of the delay equation and from the delay variation, respectively.

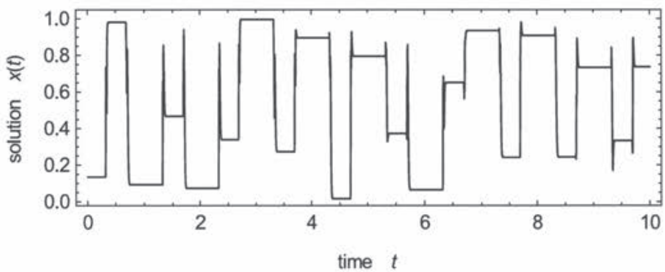

Figure 1: Laminar chaotic time series

References

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Effects of Delays on the Dynamics of Multi-Agent Systems

Rifat Sipahi

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In this talk, we will present how the delay margin of a class of linear time-invariant multi-agent system (MAS) can be increased by removing certain edges from the underlying graph of the MAS. This result will then be demonstrated on a self-regulation mechanism where delay-induced instability can be remedied as the agents split into stable subgroups. We will conclude the talk with a demonstration of how one can utilize delays intentionally to design distributed controllers for the agents in order to achieve fast consensus-reaching while effectively suppressing noise present in measuring agents states.

Design of structured controllers for network of delay-coupled systems

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A methodology is proposed for the design of robust structurally constrained controllers for linear time-delay systems, focusing on decentralized and overlapping fixed-order controllers for Multiple Input Multiple Output (MIMO) systems. The methodology is grounded in a direct optimization approach and relies on the minimization of the spectral abscissa and H-infinity cost functions, as a function of the controller for design parameters. First, an approach applicable to generic MIMO time-delay systems is proposed, which is based on imposing a suitable sparsity pattern with the possibility of fixing elements in the controller parameterization. Second, if the delay system to be controlled has by itself the structure of a network of coupled identical subsystems, this structure can be exploited by an improved algorithm for the design of decentralized (or overlapping) fixed-order controllers for the infinite-dimensional system, thereby increasing the computational efficiency and scalability with the number of subsystems. The two approaches, which have been implemented in publicly available software, support system models in terms of delay differential algebraic equations. They allow to model interconnected systems in a systematic way, and include retarded and neutral systems with delays in state, inputs and outputs. Several numerical examples illustrate the effectiveness of the methodology, as well as its extension towards consensus type problems.

Symmetry, Hopf Bifurcation and the Emergence of Cluster Solutions in Time Delayed Neural Networks

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We consider networks of N identical oscillators with time delayed, global circulant coupling, modeled by a system of delay differential equations with Z_N symmetry. We first study the existence of Hopf bifurcations induced by coupling time delay, and then use symmetric Hopf bifurcation theory to determine how these bifurcations lead to different patterns of symmetric cluster oscillations [1, 2]. We apply the theory to the Morris Lecar neural oscillator and compare our results with observations and results from phase model theory.

References

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Partial synchronization patterns - interplay of time delay, dynamics, and network topology

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Chimera states are an intriguing example of partial synchronization patterns emerging in networks of identical oscillators. They consist of spatially coexisting domains of coherent (synchronized) and incoherent (desynchronized) dynamics [1]. Here we focus on the role of time-delayed coupling and time-delayed feedback in controlling these patterns, and on their interplay with the network structure. We analyze chimera states in networks of Stuart-Landau [2], Van der Pol [3] and FitzHugh-Nagumo [4] oscillators, in particular with fractal (hierarchical) connectivities [5, 6]. In the parameter plane of coupling strength and delay time we find tongue-like regions of existence of chimera states alternating with regions of synchronization. We demonstrate that by varying the time delay one can deliberately stabilize desired spatio-temporal patterns in the system.

Furthermore, we show that chimera patterns can be induced by noise in FitzHugh-Nagumo networks in the excitable regime (coherence resonance chimeras) [7], and that time-delayed feedback allows for the control of these chimeras [8].

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Dynamical patterns in networks of excitable systems with coupling delays

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In this presentation I start with a short review of a general delay-reduction technique in networks of dynamical systems [1, 2]. Further, I show an example how this technique can be used for constructing delay-coupled networks of excitable elements with prescribed properties [3]. If the delays are tuned appropriately, the network elements can either stay in the steady state or, alternatively, exhibit a desired spiking pattern. It is shown that such a network can be used as a pattern-recognition system. As excitable elements, we employ a minimal coincidence detector model as well as Hodgkin-Huxley neuron model. Moreover, the system was implemented experimentally with a Field-Programmable Gate Array (FPGA).

References

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Destabilization of regular spiking in networks with pulse delayed coupling*

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Networks of pulse-coupled oscillators have been widely studied as models for numerous physical and biological systems such as spiking neurons, communicating fireflies, impacting mechanical oscillators, electronic oscillators, optical systems, etc. In realistic networks of various physical nature, pulses propagate with finite speed leading to nonzero coupling time delays. The focus of the present research is the dynamics of oscillatory networks with pulse delayed coupling, particularly, the scenarios of destabilization of regular spiking. By regular spiking we mean the regimes when the oscillators produce spikes periodically with the constant inter-spike intervals, for example in global synchrony. We show that destabilization of the regular spiking relates to the slope of the phase response curve (PRC) characterizing the oscillators interaction. We describe two different destabilization scenarios. The first scenario is the so-called multi-jitter bifurcation which takes place for the PRCs with large negative slopes. In this scenario the destabilization of the regular spiking leads to the emergence of the so-called jittering regimes characterized by unequal inter-spike intervals. These intervals may form various complex sequences giving rise to many long-periodical regimes and extreme multistability. The second scenario is observed for positive PRC slopes and relates to a homoclinic bifurcation of the regular spiking solution. The regular spiking destabilizes through the so-called phase slip patterns manifesting themselves as a repetitive process when the oscillators leave the synchronized cluster and then return back. We present the analytic results on the both scenarios and discuss their relation.

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Hybrid systems with delay in human balance control

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One of the most important regulatory mechanisms is time-delayed feedback. Problems related to human balance control provide experimentally accessible paradigms to explore how the nervous system uses time-delayed feedback to stabilize an unstable fixed point. Three curious dynamical properties have been identified: 1) control is exerted intermittently, 2) Weibull-type survival curves for stick balancing, and 3) microchaotic dynamics for human postural sway. These dynamical behaviors are related to the effects of a sensory dead zone. A dead zone means that there is a range of the controlled variable for which no change in feedback occurs. Intermittent control arises because the feedback switches ON or OFF as the controlled variable crosses the sensory threshold. When the control is tuned towards the edge of stability to minimize energetic costs, the dead zone produces an uncertainty in the initial conditions which is manifested by Weibull-type stick balancing survival times. Finally, the combination of time-delayed feedback, a sensory dead zone and frequency-dependent encoding of force can produce microchaotic fluctuations. The benefits of sensory dead zones in other biological contexts are briefly discussed.

References

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Mathematical models of gene regulation: Biology drives new mathematics

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Simple bacterial gene regulatory motifs can be viewed from the perspective of dynamical systems theory, and mathematical models of these have existed almost since the statement of the operon concept. In this talk I will review the three basic types of these regulatory mechanisms and the underlying dynamical systems concepts that apply in each case. In the latter part of the talk I will discuss the exciting mathematical challenges that arise when trying to make honest mathematical models of the underlying biology. These include transcriptional and translational delays, and the fact that these delays may be state dependent, as well as the interesting and often unsolved problems of characterizing the noise inherent in bacterial dynamics.

Delay dynamics emulating network of coupled oscillators: from chimera states to brain-inspired processing

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Space-time analogy for delay systems is known for at least 25 years. Beyond being a tool for a visually convenient 2D representation of complex temporal solutions developed in delay systems, it was recently further investigated for the discovery of new phenomena and features now available through delay dynamics. New complex patterns known in spatio-temporal dynamics as chimera states, have been indeed found in delay systems as well. They consist in temporally stable coexisting spatial regions, which are each filled by very different patterns. These complex coexisting patterns were initially found in network of identically coupled oscillators, and spatio-temporal analogy of delay dynamics helped to demonstrate their existence in complex delayed systems as well. An unusual way of modeling such delay dynamics through a nonlinear convolution operation instead of a nonlinear delay differential equation, allowed to reveal in a meaningful way the space-time analogy with a network of continuously distributed coupled Kuramoto oscillators, highlighting the role of the delay dynamics impulse response as the equivalent of the coupling function between virtual oscillators. From a more applied perspective, the same analogy was used to explore the potential of delay systems for emulating a novel computational neural network paradigm known as Reservoir Computing. Again, it was found that delay dynamics can provide features that have been initially observed in recurrent neural network (a spatio-temporal dynamical system). Moreover, the concept could have been physically implemented through traditional photonic delayed optoelectronic feedback architectures that are well known to be accurately modeled by nonlinear delay differential equation. As for the chimera state discovery, a robust photonic experiment led to successful physical implementation, moreover with unprecedented speed performances of 1 Million words per second achieved while performing a speech recognition task, thanks to the use of broadband optical telecommunication devices in the delay dynamics setup.

Employing Delay-Systems for Fast Neuro-Inspired Information Processing

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A property of delay-dynamical systems that has attracted a lot of interest, but also has made their analysis so challenging for a long time, is their high-dimensionality, i.e., the potentially large number of dynamical degrees of freedom. In optical systems, such delays occur generically in feedback or coupling and can be discrete, distributed, modulated and even state-dependent. The ease of implementation, the various kinds of delays and the vital role of optics for technology make such systems particularly interesting for applications.

A recently identified field of application is neuro-inspired information processing. The brain, and, inspired by it also machine learning techniques use the mapping of input information onto high-dimensional state spaces. While the complex interacting neurons provide the many dynamical degrees of freedom in the brain, in machine learning the neurons of a network are usually simulated in software. Hardware-implementations suffer from the technical difficulty to provide many coupled dynamical degrees of freedom. That is, where delay-dynamical systems offer attractive properties, since they provide high-dimensional dynamical properties, without the need for complicated hardware.

In this presentation, I will introduce, how the many degrees of freedom of delay-dynamical systems can be employed to realize the mapping of input information onto a high-dimensional state space [1]. Exploiting this, I will explain how to implement neuro-inspired information processing concepts. Indeed excellent computing performance can be obtained at highest speeds and with rather simple hardware. Besides benchmark tests, I will discuss applications to optical communication systems and in particular, how this could contribute to extending their transmission range [2].

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Consistency properties of chaotic systems driven by time-delayed feedback

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Consistency of an externally driven dynamical system refers to the property of responding in similar ways to similar inputs. In a delay system, one can consider the delayed feedback signal as an external drive to the undelayed subsystem. We analyse the degree of consistency in a generic chaotic delay system by means of the auxiliary system approach, in which an identical copy of the dynamical node is driven by exactly the same signal as the original. This scheme allows for the verification of complete consistency via complete synchronisation. We develop a theory of three basic correlation signatures in this setup based on the distinction between consistency and nonlinearity [1]. The analytical framework is supported by numerical calculations of chaotic maps with delay and stochastic functional maps. An experiment involving a semiconductor laser with optical feedback is employed to further illustrate and verify the theoretical results and provide a physical interpretation.

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Time-delay systems in multimode laser dynamics

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Multimode lasers are widely used in medical, industrial, and scientific applications. In particular, mode-locked semiconductor lasers are low cost, compact, and efficient sources of short optical pulses with high repetition rates suitable for application in telecommunication networks. A common approach to study these lasers theoretically is based on numerical integration of a certain set of PDEs, known as travelling wave equations. In the talk an alternative method to describe multimode laser dynamics is discussed, based on the use of delay differential equations (DDEs). DDE models of different multimode laser devices: mode-locked semiconductor lasers generating short optical pulses, frequency swept lasers used in optical coherence tomography, and broad area external cavity lasers capable of generating 3D localized structures of light (the so-called light bullets) are developed and studied numerically and analytically in the large delay time limit. Distributed and nonlocal delay models for modelling the effects of dispersion and transverse diffraction on the laser dynamics are proposed and investigated.

Spectrum of periodic orbits with long delay and slightly longer period

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A common phenomenon in delay differential equations (DDEs) with long delay are periodic orbits that have periods close to the delay (or fractions of it). While the orbit itself can often be efficiently computed using simulations or numerical continuation, its spectrum is often difficult to find accurately. Similar to the case of equilibria with large delay we find that the spectrum has a pseudo-continuous and a uniformly (in tau) discrete component. We present approaches to computing both components with expressions that are asymptotically accurate for large delays.

Routes to complexity in the light output of mode-locked lasers

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Integrated laser structures, in this case two section mode-locked laser with optical self feedback, can be both a test bed for nonlinear dynamics as well as a guide for optimization of pulsed light sources. In this talk we discuss the multitude of dynamics that can be observed in this specific laser system due to the presence of multiple delays. We use a delay differential equation system to describe the dynamics of the electric field while normal rate equations are used for the total gain and total losses within the two sections of the active quantum-well medium during one roundtrip. Besides the desired mode-locked and harmonic-mode-locked dynamics large regions of irregular self pulsations and quasi-periodic dynamics can be found, in very good agreement to experimental results. In order to unravel the underlying bifurcation scenario we use numerical path continuation tools and map the bifurcation structure as a function of the number of longitudinal modes within the device. We find the intersection of two Hopf-bifurcations, i.e. the bifurcation points where first and second order mode-locking are born, to be the organizing center for the quasi-periodic pulsations. Nevertheless, we also find that the multistability induced by larger delay times leads to a shadowing of the complex dynamics and eventually stabilizes the fundamental mode-locking over wide parameter ranges.

The rate equation approach used so far assumes instantaneous absorption or emission processes. It becomes problematic in the limit of large longitudinal mode numbers. In this limit very short output pulses emerge and consequently also very fast changes of gain and loss variables occur. As a consequence the rate equation modeling is not sufficient any more. Instead, coherent effects induced by the dynamics of the material polarisation inside the gain medium become important and need to be modeled by optical Bloch equation. We present numerical results for the light output in this limit and discuss the impact of the additional dynamical degree of freedom on pulse shape and stability of the laser output.

Delayed feedback control of self-mobile cavity solitons in a wide-aperture laser with a saturable absorber

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We investigate the spatiotemporal dynamics of cavity solitons in a broad area vertical-cavity surface-emitting laser with saturable absorption subjected to time-delayed optical feedback in a self-imaging configuration where diffraction in the external cavity is negligible. Using bifurcation analysis, direct numerical simulations, and numerical path-continuation methods, we identify the possible bifurcations and map them in a plane of feedback parameters. We show that for both the homogeneous and localized stationary lasing solutions, the time-delayed feedback induces complex spatiotemporal dynamics, in particular a period doubling route to chaos, quasiperiodic oscillations, and multistability of the stationary solutions. Finally, we show that the delay impacts both stationary and moving solutions either causing drifting and wiggling dynamics of initially stationary cavity solitons or leading to stabilization of intrinsically moving solutions.

Fronts, pulses, and wave trains of delayed PDEs

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Delayed feedbacks are known to induce spontaneous motion of 2D cavity solitons in optical resonators [1]. They also substantially modify wave propagation speeds in delay reaction-diffusion systems appearing in biology [2, 3, 4]. Although important contributions in pure analysis have been devoted to models in population biology, we concentrate here on constructive analytical (exact or asymptotic) techniques. Our goal is to derive valuable informations on wave speeds and large delay effects. We first consider scalar delayed reaction-diffusion equations and contrast the front propagation speed of a model with a delayed threshold nonlinearity and a model exhibiting bistability with a weak delayed feedback. We then examine the cases of pulses and wavetrains of delayed two-variable FitzHugh-Nagumo (FHN) equations. We consider both the FHN model with delayed threshold nonlinearity and the continuous FHN system subject to weak delayed feedback.

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Conserved Quantities in Network Dynamics under Time Delays

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The synchronization or consensus behavior in coupled systems is geometrically characterized with the convergence of the dynamics to a low-dimensional subspace associated with the null space of the coupling matrix. Algebraically, the presence of a zero eigenvalue suggests a conserved quantity. In this talk, we consider a general setting of networks of coupled nonlinear systems whose dynamics are subject to time delays, and derive a general formula for the conserved quantity. We apply the results to coordination and agreement problems with a generalized consensus subspace and calculate the consensus value in the presence of arbitrary multiple time delays. As a particular application, we study consensus in a network model of anticipating agents.

Heterogeneity of time delays determines synchronization of coupled oscillators

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Network couplings of oscillatory large-scale systems, such as the brain, have a space-time structure composed of connection strengths and signal transmission delays. We provide a theoretical framework, which allows treating the spatial distribution of time delays with regard to synchronization, by decomposing it into patterns and therefore reducing the stability analysis into the tractable problem of a finite set of delay-coupled differential equations. We analyze delay-structured networks of phase oscillators and we find that, depending on the heterogeneity of the delays, the oscillators group in phase-shifted, anti-phase, steady, and non-stationary clusters, and analytically compute their stability boundaries. These results find direct application in the study of brain oscillations [1].

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Abstracts of Posters

(in alphabetical order)

Time-delayed feedback control of diffusion in random walkers-Phenomena

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Time delay generally leads to instability in dynamical systems, while a specific feedback scheme with time delay can control irregular fluctuation in nonlinear deterministic systems, namely control chaotic dynamics to a stable regular state. In this presentation, we consider a stochastic process, i.e., a random walk, instead of deterministic dynamical behavior, and observe its diffusion phenomenon with the time delayed feedback control for chaotic systems. As a result of numerical simulations of one-dimensional Wiener processes, the diffusion coefficient decreases with increasing the delay time, especially with a power-law decay as shown in Fig. 1. We analytically illustrate this suppression of diffusion by using stochastic delay differential equations and check if this suppression is able to be observed in a practical case by applying the time-delay feedback to a molecular dynamics model with a few particles.

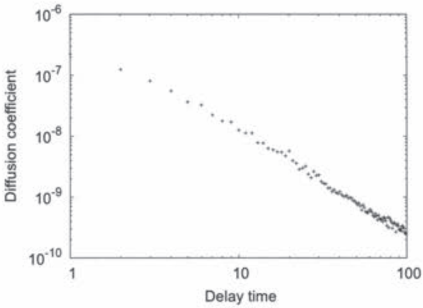

Figure 1: Power law decay of diffusion coefficient for stochastic processes.

Time delay induced dynamic changes in coupled relaxation-type oscillators

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Mutual coupled relaxation type oscillators can exhibit a rich variety of dynamical behavior and encompasses a broad range of phenomena in science and engineering [1]. A general consideration thereby is an instantaneous coupling scheme, where the signal propagation speed between individual oscillators is considered as infinitely fast. However, over the last decades it became clear that signal time delay can play a significant role in complex systems such as gene-regulator networks or coupled neuronal oscillator networks [2]. Moreover, interesting phenomena like multijitter bifurcations can be observed [3]. Here we present our results on relaxation-type oscillators comprising a finite time delay during pulse coupling. The oscillators are based on programmable unijunction transistor circuits. They offer a simple read out of the pulse trains (spike patterns) as well as the realization of excitatory and inhibitory coupling schemes. Adjustable time delays were generated using a Field Programmable Gate Array (FPGA, Cyclon V). Biological relevant times in the order of several tens of milliseconds were used for both the intrinsic period of the oscillator and the signal time delay. The spikes of a relaxation oscillator were delayed and feed back to the excitatory and inhibitory input of the same oscillator. This self-projected system led to different firing patterns. Its equivalence can be found in more complex systems like the neuronal system which exhibits mechanisms like reentry or delayed feedback through poly-synaptic loops [5,6].

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Time-delayed models for the impact of awareness on the spread of epidemics

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Spread of infectious diseases often results in behavioural changes in the population, which, in turn, affect the disease dynamics. Our work looks at the effects of simultaneous spread of an infectious disease and disease awareness, arising from personal contacts and public awareness campaigns. Time delay is associated with the delayed response of individuals to available information. We investigate different dynamical scenarios depending on system parameters, and also analyse optimal strategies for disease control [1,2].

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Delayed Control of the Synchronous European Electricity Grid

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The transition towards a power system that relies on renewable resources presents a major challenge to the energy system. During the transition, highly volatile energy sources (i.e. wind and photovoltaics) will be introduced to a system built with conventional energy sources in mind. At present, conventional energy sources with large rotating generator masses provide stabilising inertia to the system. Removing these conventional energy sources and replacing them with fluctuating renewable generation that does not provide inertia could lead to a system that is vulnerable to disturbances. Presently, the power frequency control operated by the 'European Network of Transmission System Operators for Electricity' (ENTSO-E) guarantees that the system remains stable. The control mechanisms can only be employed by accurately measuring the system state (i.e. frequencies and load flows) and by correctly communicating theses values. Miscommunication can have far reaching effects, such as the deviation grid frequency in Continental Europe, observed in 2018, caused by a dispute between Serbia and Kosovo. The delay associated with the measurement, communication and the deployment of control might play an increasingly important role in a system that relies on feed-in that is fluctuating on small time scales.

The control mechanisms operated by the ENTSO-E are integrated into a model of coupled oscillators which resembles the second order Kuramoto model. This integrated model is used to investigate the behaviour of the grid frequency of the interconnected electricity grid. To identify regions in parameter space that make stable grid operation possible, the linearised system is analysed to create the system's stability chart. The influence of magnitude of delay, parameters of control and underlying network topology on the grid frequency is of special interest.

Longitudinal solitons in time-delayed nonlinear systems

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A number of nonlinear self-action phenomena are known occurring due to interaction of laser pulses inside an extended dynamic nonlinear medium. The time delay in a Kerr-type nonlinear medium is well-known for generation of a nonlocal response, usually studied in the frames of four-wave mixing. We investigate a degenerate case of this process with two interacting waves and demonstrate for the first time that this system also possesses solitons.

The light-matter coupling process is then described by a parametric nonlinear Schrödinger equation (pNLS), [1] (the reflection geometry):

$$i\frac{\partial \mathbf{A}}{\partial \eta} + \frac{2\gamma_N}{q^3}\frac{\partial^2 \mathbf{A}}{\partial \zeta^2} - \frac{\gamma_N}{q^3}\exp\left(-2T_0/\tau\right)|\mathbf{A}|^2\mathbf{A} = 0. \tag{1}$$

where $\mathbf{A}$ is the grating amplitude, and the parameters of the equation are explicit experimental parameters: γ_N is the maximum value of the nonlinear response, τ is the time relaxation constant, T is a real time and q is the grating period.

Coherent laser beams form spatially periodic interference pattern (the light lattice), which induces a refractive index grating. Due to relaxation of the nonlinearity inside the medium this grating appears to be dynamic both in space and in time. A longitudinal envelope soliton is formed inside this medium as a result of nonlinear coupling between the light lattice and the dynamic grating. The envelope soliton describes the form of the intensity distribution in the light lattice maxima.

Stability analysis of the pNLS solutions demonstrates that various soliton states can emerge during self-action processes, with the crucial parameter being the ratio of the duration of the laser pulses to the time relaxation constant τ. This finding can be used e.g. in active fiber systems for substantial amplification of a weak signal pulse via energy transfer from the pump beam to the signal beams.

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Rapid Time Series Predictions with Hardware-Based Reservoir Computing

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The reservoir computing (RC) paradigm demonstrates state-of-the-art performance in a number of benchmark machine learning tasks while maintaining remarkably low training time. Advances in the performance of field-programmable gate arrays (FPGAs) have allowed for high-performance hardware implementations of RC. A low footprint, easy reconfigurability, and the ability to rapidly transform the reservoir state are desirable features in such implementations. We present a method for RC with an autonomous, time-delay, Boolean network realized on an FPGA. We observe the capability of the time-delay network to synchronize, in a generalized sense, to an input signal - an essential ingredient for successful RC. We find the use of incommensurate time-delays between nodes to be critical for successful operation. These delays allow the network to perform complex machine learning tasks. We apply our technique to the prediction of the Mackey-Glass chaotic system and find comparable performance to software-based techniques of similar network size. Further, we take advantage of the Boolean nature of the reservoir state to achieve a record-high rate of real-time prediction at 160 MHz.

Decentralized time-delay controller design*

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For many large-scale systems, a centralized control approach, which requires gathering all the information in a centralized place, processing it there, and dispatching the commands from there, is either impractical or impossible. Decentralized control is either preferable or necessary for such systems because it attempts to avoid difficulties in information gathering, processing, and structuring. Furthermore, many control systems, especially large-scale control systems, may experience time-delays. Even if the plant does not have any time-delays, it is probable that, after the implementation of a feedback control system, time-delays will occur because of actuators, sensors, field networks that are involved in feedback loops. Several works have already been devoted to the subject of decentralized control of time-delay systems ([1, 2] and references therein). This topic has also been the main subject of our study in recent years [3–5].

The existence of time-delays in the plant and/or anywhere else in the control loop may cause undesirable system transient response or even instability. However, delay properties can be beneficial for control in some cases. Although it is known that a linear time-invariant (LTI) time-delay system can be stabilized by an LTI time-delay controller only if it can be stabilized by an LTI finite-dimensional controller, time-delay controllers can sometimes be advantageous. In this regard, time-delayed feedback control has been used in a large variety of systems. In this context, the problem of constructing decentralized LTI stabilizing time-delay controllers for a class of LTI retarded time-delay systems is considered here. Recently, we proposed a time-delay compensator design approach in [6] which can be considered as a preliminary to the study discussed here.

In this study, the stabilizing decentralized time-delay controller design problem for LTI retarded time-delay systems with no direct connection from the inputs to the outputs is considered. For this purpose, a design approach, based on the continuous pole placement algorithm and the decentralized pole assignment algorithm, is proposed.

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Experiments with arbitrary networks in time-delayed systems

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We report a new experimental approach using an optoelectronic feedback loop to investigate the dynamics of oscillators coupled on large complex networks with arbitrary topology. Our implementation is based on a single optoelectronic feedback loop with time delays. We use the space-time interpretation of systems with time delay to create large networks of coupled maps. Others have performed similar experiments using highpass filters to implement the coupling; this restricts the network topology to the coupling of only a few nearest neighbors

In our experiment, the time delays and coupling are implemented on a field-programmable gate array, allowing the creation of networks with arbitrary coupling topology. This system has many advantages: the network nodes are truly identical, the network is easily reconfigurable, and the network dynamics occur at high speeds. We use this system to study cluster synchronization and chimera states in both small and large networks of different topologies [1,2].

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Decentralized Control of Time-delay Systems*

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It is impractical, if not impossible, to control many large-scale complex systems from a centralized location. Decentralized control structure is, thus, necessary for such systems. Furthermore, in a large-scale system, time-delays in general appear due to delays in actuators, sensors, communication networks, and data processing and computation processes. In addition to such delays, the physical processes themselves may involve inherent time-delays. Therefore, decentralized control of time-delay systems is an important topic from both theoretical and practical views. By this motivation, for the past six years, this topic has been chosen as the main research topic by our research group. As a result of our efforts, a number of theoretical results on stabilizability of time-delay systems by finite-dimensional and time-delay controllers have been obtained [1,2]. A number of algorithms have also been proposed to design decentralized controllers for time-delay systems [3-7] and a MATLAB-based software package has been developed [8] to implement these algorithms. Furthermore, a number of results on overlapping decompositions and expansions of time-delay systems for decentralized controller design have been obtained [9-12]. In this talk, all these results will be summarized together with some newer results on decentralized time-delay controller design for time-delay systems.

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Utilizing Topological Data Analysis for Studying Signals of Time- Delay Systems

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This work describes a new approach for studying the stability of stochastic delay equations by investigating their time series using topological data analysis (TDA). The approach is illustrated utilizing two stochastic delay equations. The first model equation is the stochastic version of Hayes equation - a scalar autonomous delay equation - where the noise is an additive term. The second model equation is the stochastic version of Mathieu's equation - a time-periodic delay equation. In the latter, noise is added via a multiplicative term in the time-periodic coefficient. The time series is generated using Euler-Maruyama method and a corresponding point cloud is obtained using the Takens-embedding. The point cloud is then analyzed using a tool from TDA known as persistent homology. The results of this study show that the described approach can be used for analyzing datasets of delay dynamical systems that are described using constant as well as time-periodic coefficients. The presented approach can be used for signals generated from both numerical simulation and experiments. It can be used as a tool to study the stability of stochastic delay equations for which there are currently a limited number of analysis tools.

Time-delayed feedback control of diffusion in random walkers-Analysis

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Time delayed feedback can control chaotic motion to periodic motion by stabilizing an unstable periodic orbit that is embedded in a chaotic set. We have found that the time delayed feedback control method can be applied to a non-stationary stochastic process with Gaussian noise, and it can control the diffusion processes in a stochastic system. In this poster, we give a mathematical explanation for the diffusion control.

Exit time asymptotics for small noise stochastic delay differential equations

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Dynamical system models with delayed dynamics and small noise arise in a variety of applications in science and engineering. In many applications, stable equilibrium or periodic behavior is critical to a well functioning system. Sufficient conditions for the stability of equilibrium points or periodic orbits of certain deterministic dynamical systems with delayed dynamics are known and it is of interest to understand the sample path behavior of such systems under the addition of small noise. We consider a small noise stochastic delay differential equation (SDDE). We obtain asymptotic estimates, as the noise vanishes, on the time it takes a solution of the stochastic equation to exit a bounded domain that is attracted to a stable equilibrium point or periodic orbit of the corresponding deterministic equation. To obtain these asymptotics, we prove a sample path large deviation principle (LDP) for the SDDE that is uniform over initial conditions in bounded sets. The proof of the uniform sample path LDP uses a variational representation for exponential functionals of strong solutions of the SDDE.

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Delayed Stochastic Resonances*.

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The main theme of this paper is the presentation of a simple model which shows that delay and noise can have a resonance by themselves without an external periodic driving force. We will mainly discuss two models.

The first model is a two-state system whose dynamics is governed by combinations of its state at some fixed interval in the past and noise with a certain strength. With this model, one can calculate the resonant curve exactly, showing the interplay between delay and noise.

The second model is a collection of random walkers which relay a signal. When there exist delays in each transaction for the passing of the signal, there exists an optimal number of walkers which minimize the total time required for relaying of the signal.

Though simple, these phenomena may find wide applications both theoretically and experimentally, some of which will be also presented.

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New methods and a software to design decentralized stabilizing controllers for time-delay systems*

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For many large-scale systems, it may be very costly, even impossible, to collect all the information in a centralized place, process it there, and dispatch the control commands from there. For such systems, decentralized control is necessary or preferable. As a result, such systems are no longer controlled by a centralized controller but by several independent controllers. In this structure, these local controllers, all together, represent a decentralized controller. Furthermore, many large-scale systems may involve time-delays.

Even though, many methods have been proposed to design centralized controllers for time-delay systems, not much work has been done to develop decentralized controller design methods until recently. In this work, we propose two decentralized controller design approaches to stabilize linear time-invariant (LTI) time-delay systems. The first one [1] is based on the decentralized pole assignment algorithm, which was originally proposed for finite-dimensional systems. In this algorithm, a centralized controller is designed for each control agent sequentially. To design centralized controllers, we use a non-smooth optimization based fixed-order controller design method, which has already been developed. In the sequential design, an important point is the controller design order. It has been shown that, by changing the design order, it may be possible to obtain lower dimensional controllers which is quite beneficial from the cost point of view. On the other hand, designing the decentralized controllers simultaneously, rather than sequentially, may lead to even better controllers (in terms of lower controller dimensionality and/or achieving more stable closed-loop systems and/or achieving better performance). Therefore, as a second approach, we propose an alternative design procedure, in which the decentralized controllers are designed simultaneously [2]. In this approach too, we use the non-smooth optimization based fixed-order controller design method. This method relies on minimizing the real part of the right-most closed-loop mode, i.e., spectral abscissa, as a function of the controller parameters.

Besides developing new algorithms, we have also developed a new software package to implement these algorithms [3]. This software can be used to design centralized or decentralized controllers (with both simultaneous and sequential design techniques) for time-delay systems. In this presentation we will introduce both the design algorithms and the software package we have developed.

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Mobility-induced chimera states

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Chimera states are complex spatio-temporal patterns in which order and disorder coexist within the same state in systems of identical oscillators. Since their discovery in 2002, they have demonstrated to be broadly encountered in multiple systems with distinct coupling schemes, both in theoretical and experimental systems. Despite the great progress made in understanding chimera states, the mechanisms and conditions for their existence are still not clear. In addition, although they are thought to play a role in natural systems, most of the experimental evidence is found in carefully designed laboratory setups.

In this work we study chimera states that spontaneously emerge in a system of mobile, locally coupled identical oscillators in the presence of coupling delays. The system has its main motivation in the vertebrate segmentation clock, a tissue generating cyclic patterns that is thought to be a population of coupled oscillators where significant coupling delays occur together with cell movements. In addition to well-known chimera states where in-phase order coexists with disorder, we find other kinds: antichimera states where antiphase order coexists with disorder and dual-chimera states where both in-phase and anti-phase order coexist with disorder. These chimera states are dynamic and can persist for long time for intermediate mobility values. We analyse their occurrence and persistence and discuss the mechanisms leading to the formation of these chimera states in different mobility regimes [1].

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Analysis of temporal localized structures in a time delayed model of a ring laser

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Various kinds of temporal localized structures were recently observed experimentally in fiber and semiconductor ring lasers with long cavities as short periodic bursts or drops in the optical field intensity for sufficiently weak pump current under or near the lasing threshold. These bright or dark pulses were mostly predicted and analysed theoretically as the so-called dissipative soliton solutions of complex Ginzburg-Landau-type partial differential equations. These models are well suited for the analysis of the pulse propagation and stabilization along the cavity under various distributed physical effects, however their application to various important and standard ring laser device configurations is often possible only under very limiting assumptions on the dynamics of the laser such as neglect of multiple longitudinal modes. For that, typically the simulations of travelling wave partial differential equations are used (e.g., [1]), however, their deep analysis is limited to the steady states and constant wave solutions (relative equilibria), whereas true (or relative) periodic solutions like the temporal localized structures are out of reach. We consider a delayed model of a multi-mode semiconductor ring laser under injection of external optical field and perform stability analysis of the constant wave and injection-locked steady state solutions in the limit of large delay, accompanied by extensive numerical simulations (see Fig. 1) and numerical bifurcation analysis using DDE-BIFTOOL, to study appearance and destabilization of the localized structures such as dissipative phase solitons [1] and dissipative solitons. Finally, we discuss novel type of stable solutions with almost periodic intensity drops in the same ring laser without optical injection.

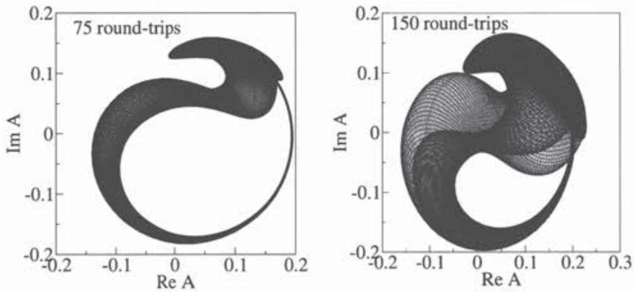

Figure 1: Numerical phase portraits of quasi-periodic dissipative phase solitons in a ring laser under optical injection for large values of the linewidth enhancement factor in the active medium, where A represents the electrical field.

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Controlling switching dynamics in micropillar lasers with time-delayed optical feedback

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Microlasers provide a platform for the realization of delay-coupled experiments in the field of quantum optics, where single emitter effects and high spontaneous emission noise become prominent. It is particularly attractive the possibility of tailoring the emission properties of microlasers via non-invasive optical coupling. However, until now scarce research has been done in this direction [1, 2]. Here, we aim at the control of the emission properties of a microlaser via time-delayed optical feedback.

We experimentally and theoretically investigate the feedback effects on the mode-switching dynamics of an electrically driven bimodal quantum-dot micropillar laser. In these structures two linear, orthogonally polarized lasing modes compete for the common gain medium, resulting in characteristic switching dynamics above the lasing threshold with sub-μW emission powers. We characterize the impact of delayed optical feedback on different properties of the microlaser like optical spectrum, output power and photon statistics. We support our experimental findings with numerical modeling based on semi-classical rate equations [3]. Delayed optical feedback is found to influence the switching dynamics and time scales of the micropillar. Moreover, we directly measure the photon number distribution with a photon-number resolving sensor to identify the regimes in which feedback leads to stabilization and destabilization of the modes. Our results expand the comprehension of the behavior of micropillar lasers when subject to time-delayed optical feedback and demonstrate the possibility to tailor the properties of microlasers via non-invasive external control.

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Bound pulse trains in coupled spatially extended dynamical systems

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We study the dynamics of an array of nearest-neighbor coupled spatially distributed systems each generating a periodic sequence of short pulses. One example of such setup is an array of coupled mode-locked lasers, where each element is described by a set of delay-differential equations.

We demonstrate that unlike a solitary laser generating a train of equidistant pulses, laser array can produce a sequence of clusters of closely packed pulses, with the distance between individual pulses depending on the coupling phase. This new regime associated with the formation of locally coupled pulse trains due to a balance of attraction and repulsion between them is different from the soliton bound states reported earlier in different laser, plasma, chemical, and biological systems. We propose an analytical description of the observed phenomenon, which is in a good agreement with the results of direct numerical simulations of a model system describing an array of coupled mode-locked lasers.

Exploration of bistability in a paleoclimate delay model

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We present a study of a delay differential equation (DDE) model for the Mid-Pleistocene Transition (having occurred between 1200 and 800 kyr BP). The model has a bistable region consisting of a stable equilibrium along with a large amplitude stable periodic orbit. We analyse the behaviour of the model within this region when subjected to periodic and quasiperiodic (astronomical) forcing, and how the forcing amplitude affects the response. In the periodically forced case, there is a sharp transition in responses when increasing the parameter for forcing strength. We show that the forcing amplitude at which we observe this transition is far away from any bifurcations. The only alternative mechanism is that the initial condition of our simulation moves from the basin of attraction of the small-amplitude response to the basin of the large-amplitude response. When using the forcing known from historical data for solar insolation we see a transition in amplitude and at a specific time. We show through finite-time Lyapunov exponents that there is an attracting response prior to any transition. This makes the transition independent of initial history. We also explore the sensitivity of the phase of oscillation to noise, and introduce the idea of noise-induced desynchronisation.

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Stochastic delay systems tend to be effectively low-dimensional

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Although dynamical models are often required to incorporate both delays and noise, the inherently infinite-dimensional nature of delay equations makes formal solutions to stochastic delay differential equations (SDDEs) challenging. Here we show that SDDEs in fact tend to have a very small effective dimensionality, in part due to the noise itself (Fig. 1). We provide numerical evidence in the case of a nonlinear system with saturating feedback, showing that very few PCA components suffice to explain most of the variance. We explain this observation by solving the linearized system and writing the solution for the moments as sums over the system's modes. Noise preferentially excites more fundamental modes, thus partially driving the collapse of the dynamics to these modes.

Our solution approach is similar in spirit to the analysis of functional differential equations, but based on finite-dimensional matrix operators[1]. This results in a method for obtaining both transient and stationary solutions that is directly amenable to computation, and applicable to first order differential systems with either discrete or distributed delays. We make fewer assumptions on the system's parameters than other current solution methods and do not require to be near a bifurcation.

Moments of a frequently appear in expressions for the original nonlinear system and are used to quantify temporal structure. Since our approach solves for the moments, it may be applicable to a wide range of linearizable systems.

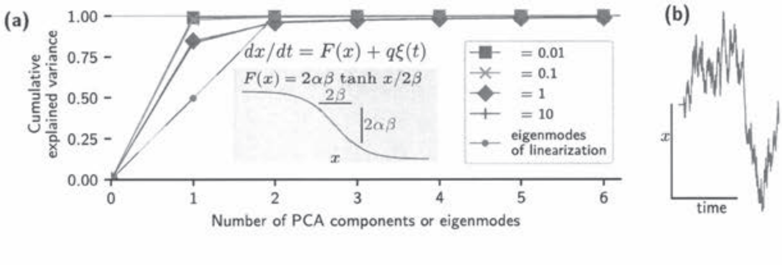

Figure 1: **(a)** PCA estimation of the effective dimensionality of a nonlinear system. Feedback $F(x)$ has slope α and is approximately linear on the range $[-\beta, \beta]$; parameters α and q are set to -1 and 1 respectively. **(b)** Typical realization of the dynamical system.

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Beyond Square Waves: Stable asymmetric spatiotemporal patterns in a singularly perturbed Delay Differential Equation

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We study a singularly perturbed Delay Differential Equation exhibiting solutions of periodically repeating patterns of coherent, almost constant plateaus and possibly incoherent chaotic phases connected by sharp transition layers with period T slightly larger than the delay. We show that under some conditions, this class of coherence-incoherence patterns is also present in a set of reduced equations and stable under the time-T solution map of the Delay Differential Equation.

Extreme events in delay coupled systems: Generating mechanism, precursors and basins of attraction

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In a system of two identical FitzHugh-Nagumo oscillators coupled to each other by multiple delay-diffusive couplings, we show that extreme events can be generated in parameter regimes sandwiched between a period adding cascade and a period doubling cascade. In contrast to the other known mechanisms of extreme event generation, the extreme events generated in this system appear in two distinct categories — synchronous and asynchronous — depending on the distance of the trajectory relative to the invariant synchronization manifold. We also identify the role of this manifold in the underlying mechanism of the dynamics including the observed in-out intermittency. In the poster, we also present our investigations regarding the basin structure of the system when multiple attractors are present simultaneously. Our results show that if one of the attractors present in such a parameter regime corresponds to extreme events, the bains are riddled. In such a scenario, the phase space can be partitioned into pure and mixed regions, where initial conditions in the pure regions certainly avoid the generation of extreme events, while initial conditions in the mixed region may or may not exhibit such events. This implies that any tiny perturbation of initial conditions in the mixed region could yield the emergence of extreme events because the latter state possesses a riddled basin of attraction.

Discrete model for spatially extended systems with time delays: Application to rapid alloy solidification

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One of the most simple and effective approaches to describe heat-mass transfer in the spatially extended systems with time delays is the discrete variable model (DVM), which discretizes the transport process in space and time by defining the minimum lattice size h to which the local temperature T (or concentration) can still be assigned and the minimum time τ, which is the delay time between the successive events of energy (mass) exchange. The model provides the transfer equations directly in the discrete form, which is particularly convenient for computer simulations. In the continuum limit when τ and h tend to zero, the model leads to a hierarchy of delay differential equations of increasing order. The type of the equations (parabolic or hyperbolic) depends on an invariant of the continuum limit, which conserves a desirable property of the transport process. If the continuum limit keeps a finite value of the diffusivity D, the discrete model gives a hierarchy of parabolic differential equations of increasing order. In such a case the signal propagation velocity v, which is the ratio of h to τ, tends to infinity. If the continuum limit keeps a finite value of the signal propagation velocity v, then the discrete model gives a hierarchy of hyperbolic differential equations of increasing order. It has been demonstrated that the delays plays a critical role in some engineering applications, such as reaction-diffusion front propagation, rapid alloy solidification, ultrafast laser heating. For example, the delay effects lead to an upper limit for the propagation velocity of the reaction-diffusion front independently of how fast the reaction rate is. Moreover, the delay effects result in the sharp transition from diffusion regimes of binary alloy solidification with solute partition in the solid-liquid interface to diffusionless and partitionless solidification when the interface velocity passes through the critical point, which depends on the value of the delay time. The results corresponds to experimental data and cannot be explained within usual approach with zero delay time [1-5].

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State Dependent Delayed models for drill-string Dynamics: Stability and Bifurcation Characteristics

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Vibrations in drill-strings adversely affect the performance of deep drilling systems which are important for oil and gas industry. A slenderical structure of drill-string typically gives rise to high flexibility in the torsional, axial as well as flexural mode making it susceptible to self-excited vibrations. The coupling of axial and torsional motions gives rise to a regenerative cutting effect which is best described by a state-dependent delayed term [1, 2]. In this work, we will present some typical lumped parameter models for the drill-string which include only the axial and torsional modes and discuss the possible dynamics associated with these models. We will first consider an idealized system wherein the drill-string at the top table has a constant rate of progression [2]. For this system we observe an interesting finding that internal resonance between the various lumped nodes can lead to a significant improvement in the stability of steady drilling under this assumption [3]. In contrast the assumption of a constant force at the top table leads to fairly reduced stable drilling [3]. We will also present refined models wherein the flexibility of the hoisting mechanism will be taken into account. Two variations of the refined model viz. the top of the hoisting cable moving with a constant velocity or the top end of the cable under constant tension will be discussed [4]. We will demonstrate that the constant velocity of the hoisting cable, boundary condition at the top, leads to a much larger stable regime of operating parameters corresponding to steady cutting. We also find that the mean rate of progression is affected during unsteady drilling for the constant force boundary condition while it remains the same for the constant velocity boundary condition. These findings suggest that the control system for the hoisting mechanism should aim to achieve a constant rate of drilling rather than a constant weight on bit. In contrast to the case where the top mass was moving with a constant velocity wherein the largest stable regime was obtained for the case of 1:1 internal resonance between the axial and torsional modes, we find that incorporation of hoisting flexibility leads to a much larger stable regime under the untuned system. We will present the optimum system parameters leading to the largest regime of stable operation in the space of operating parameters.

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Tests for chaos and partial predictability with an application to time delay systems

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In the vicinity of a chaotic attractor two initially close-by trajectories diverge exponentially, which is the tell-tale sign of deterministic chaos [1] in dissipative dynamical systems. This initial process of decorrelation is characterized by a positive maximal Lyapunov exponent, with the corresponding time scale, the Lyapunov prediction time being the time up-to which there is still a certain amount of information about the starting state. After the initial divergence of trajectories the final decorrelation occurs diffusely on the chaotic attractor.

For the common type of *strong chaos* both exponential and diffusive decorrelation happen on the same time scale, which implies that the Lyapunov prediction time is the time scale for the complete loss of predictability. However, for another type, for *partially predictable chaos (PPC)*, the diffusion takes place on a significantly longer time scale, with the consequence that predictability stays finite for exceedingly long times. When the topology of the attractor is equivalent that of closed chaotic braids, the initial loss of correlation is caused by a divergence perpendicular to the braid, with the subsequent diffusion along the braid being much slower.

Standard tests for chaos, which rely generically on correlation measures, run often into difficulties when it comes to classify PPC, in particular because of circumstance that the maximal Lyapunov exponent of a PPC state may be exceedingly small. This difficulty is resolved by a novel test for chaos that is based on the cross-distance scaling of initially close-by trajectories [2]. This test is capable of robustly distinguishing chaos, including PPC, from regular flow in a binary fashion.

In combination with the finite-time cross-correlation of initially close-by trajectories, which acts as another effective binary test, we are also able to distinguish strong chaos and PPC due to the different time scales of the two decorrelation processes. With both tests we are capable of discriminating all three dynamical regimes robustly in an unambiguous $0-1$ fashion.

As both tests only rely on the computation of trajectories and straightforward data manipulation, it is robustly applicable even to computationally challenging systems. Therefore we present the result of binary tests for the infinite dimensional, time delayed Mackey-Glass system [3] and demonstrate that all three dynamical regimes – strong chaos, PPC, and regular flow – can be distinguished in a reliable manner.

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Robust synchronization in networks of spatially-distributed delay-coupled electronic clocks

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Increasing demands for computational power will lead to highly parallelized electronic architectures with many components in the near future. Currently, most of these architectures are operating on globally asynchronous locally synchronous clocking, impairing computational performance due to communication-delay induced latencies. Master-clock concepts with complicated tree-structures can achieve global synchrony in small systems but become inefficient as system size increases. In biological systems, mutually coupled periodic processes are found which show a remarkable degree of synchrony, even in noisy environments and without a master clock. Transferring concepts from these coupled biological oscillators to electronic networks, we present novel synchronization strategies for large, spatially distributed electronic clocking networks. We study systems of mutually delay-coupled phase-locked loops (PLLs) using a theory of phase oscillators and explore their synchronization properties. In particular, we study the effects of physically relevant factors such as transmission, feedback and processing delays, phase noise, and coupling topologies to find optimal synchronization scenarios. We also performed proof-of-concept experiments that validated our theoretical results qualitatively and quantitatively.

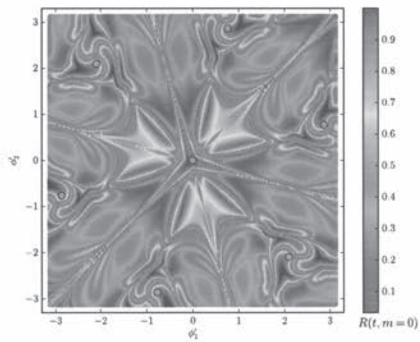

Figure 1: Basin of attraction plot of three mutually delay-coupled DPLLs in a ring topology. The parameters are the coupling strength $K = 0.25\,\text{Hz}$, the intrinsic frequency of the DPLLs $f = 1\,\text{Hz}$, the loop filter cut-off frequency $f_c = 0.1\,\text{Hz}$, and transmission delays $\tau = 0.4\,\text{s}$. The global frequency of the in-phase synchronized state is $\Omega = 1.25\,\text{Hz}$.