Quantum vortices in superfluid Helium droplets: An Introduction

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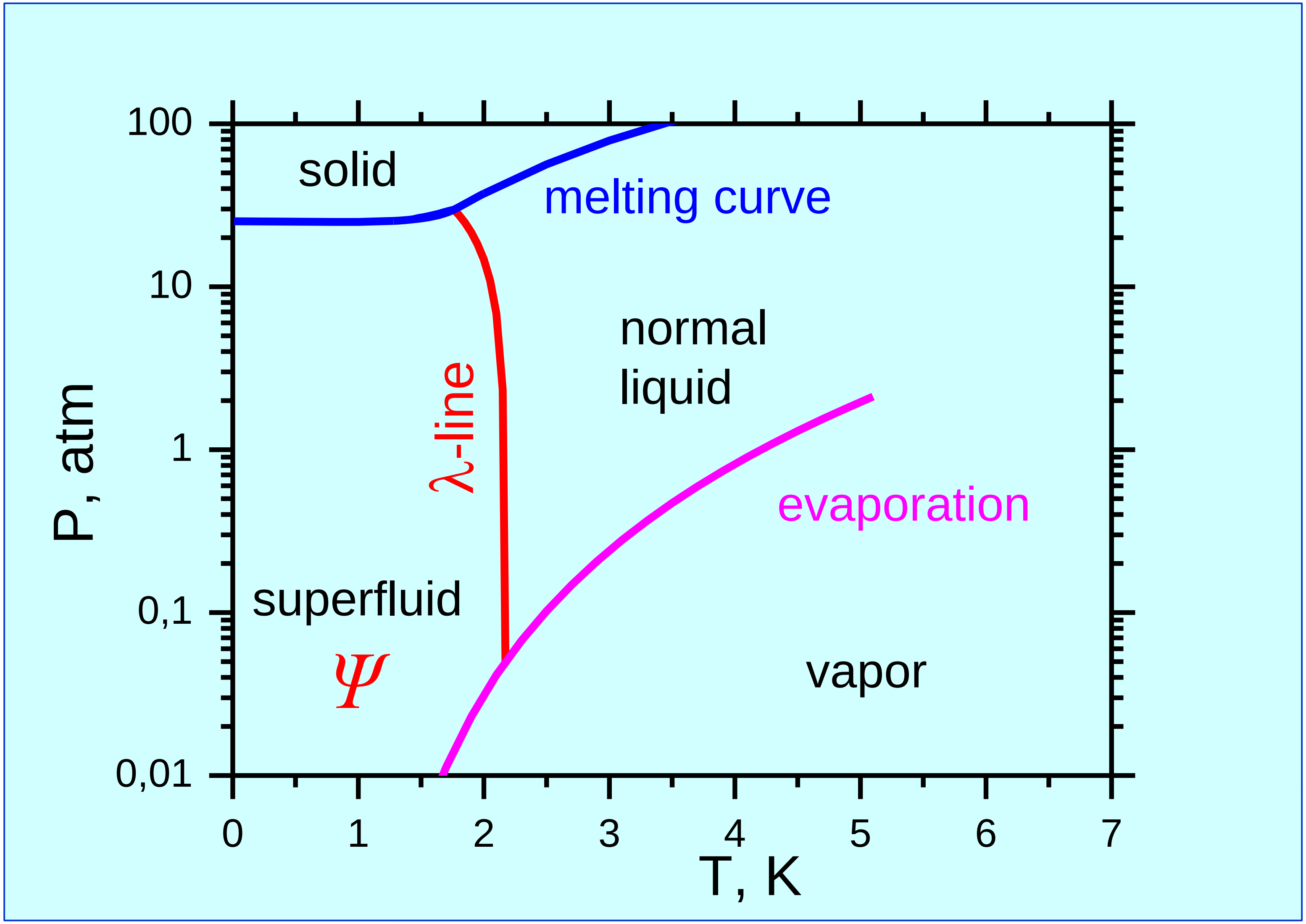
R. J. Donnelly, Quantized vortices in HeII, Cambridge University Press, 1991

O.Gessner, A.Vilesov, Imaging quantum vortices in superfluid helium droplets, Annu. Rev. Phys. Chem. 2019. 70:203–28

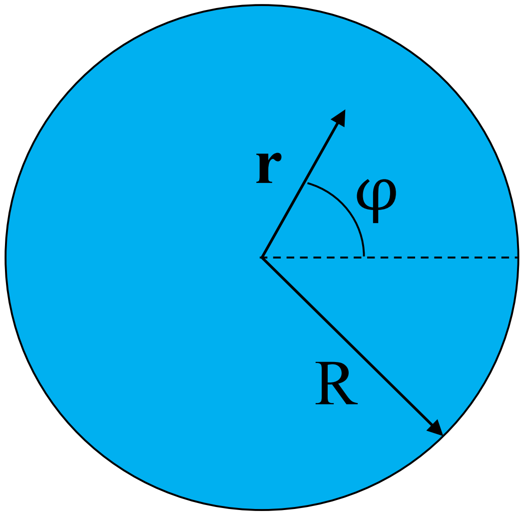
^{4}He at low temperature



Superfluidity, macroscopic wave function and vortex.

At constant SHe density: $\rho=|\Psi_0|^2$

$$\Psi(\boldsymbol{r}) = \Psi_0 \exp(i \cdot S(\boldsymbol{r}))$$



SHe in a cylinder of radius R, vortex wavefunction:

$$\Psi(\boldsymbol{r}) = \Psi_0 \exp(i \cdot \varphi \cdot n)$$

n=0,1,2….

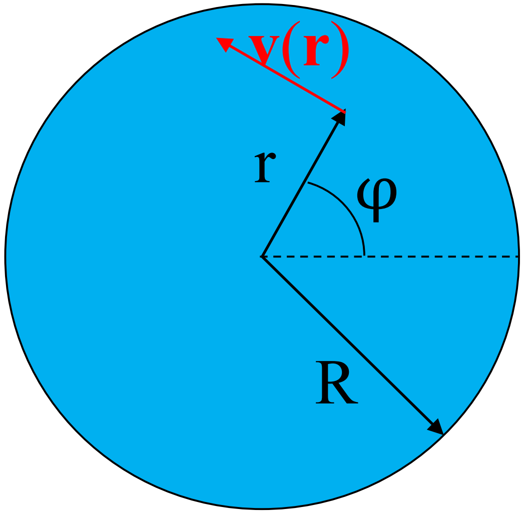
bolds are vectors

Current density and velocity in vortex

Current density: $$\boldsymbol{j}(\boldsymbol{r}) = \rho \cdot \boldsymbol{v}(\boldsymbol{r}) = \frac{\hbar}{2 \cdot m \cdot i}(\Psi^* \boldsymbol{\nabla} \Psi - \Psi \boldsymbol{\nabla} \Psi^*)$$

$$\Psi(\boldsymbol{r}) = \Psi_0 \exp(i \cdot \varphi \cdot n)$$

In polar coordinates: $$\nabla = \frac{\partial}{\partial r}\boldsymbol{e}_r + \frac{\partial}{r\partial \varphi}\boldsymbol{e}_\varphi$$



$$\boldsymbol{j}(\boldsymbol{r}) = \frac{\hbar}{2 \cdot m \cdot i}\Psi_0^* \cdot \Psi_0 \frac{2 \cdot i \cdot n}{r} \cdot \boldsymbol{e}_\varphi = \frac{\hbar \cdot n}{m \cdot r}\rho \cdot \boldsymbol{e}_\varphi$$

$$\mathbf{v}(\boldsymbol{r}) = \frac{\kappa \cdot n}{2\pi \cdot r} \cdot \boldsymbol{e}_\varphi \,, \quad \kappa = \frac{h}{m}$$

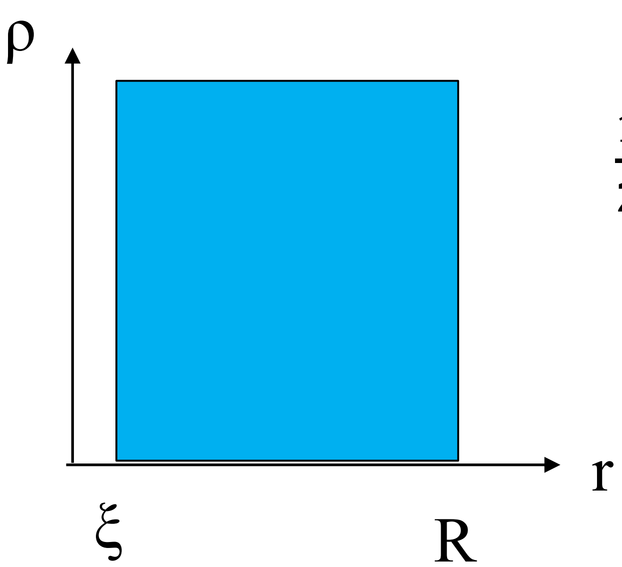
κ– quantum of circulation

$$\mathbf{v}(\boldsymbol{r}) = \frac{\kappa}{2\pi} \cdot \boldsymbol{\nabla} S$$ potential flow

v(r) ≈ 15/r(nm) [m/s]

Kinetic energy of a vortex per unit cylinder length

$$\mathrm{v}(r) = \frac{\kappa \cdot n}{2\pi \cdot r}$$



$$K_n = \frac{1}{2} \cdot \int_{\xi}^{R} \rho \cdot m \cdot v^2 \cdot 2\pi r \cdot dr =$$

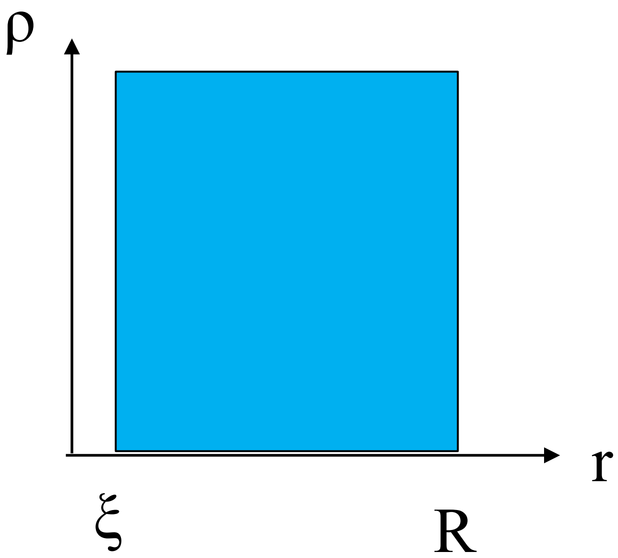
$$\frac{1}{2} \cdot \int_{\xi}^{R} \rho \cdot m \cdot \frac{\kappa^2 \cdot n^2}{2\pi r} \cdot dr = \frac{\rho \cdot m \cdot \kappa^2 \cdot n^2}{4\pi} \cdot \ln(\frac{R}{\xi})$$

$K_2 = 4K_1$ ⇨ *only vortices with* $n = 1$ *are stable*

$\xi \approx 0.1$ nm – vortex core radius

Angular momentum of a vortex per unit cylinder length

$$\mathrm{v}(r) = \frac{\kappa}{2\pi \cdot r}$$



$$L = \int_{\xi}^{R} \rho \cdot m \cdot v \cdot r \cdot 2\pi r \cdot dr =$$

$$\rho \cdot m \cdot \kappa \int_{\xi}^{R} r \cdot dr \approx \rho \cdot m \cdot \frac{h}{m} \cdot \frac{R^2}{2} =$$

$$L = N_{He} \cdot \hbar \qquad L = \hbar \; per\ atom!!!$$

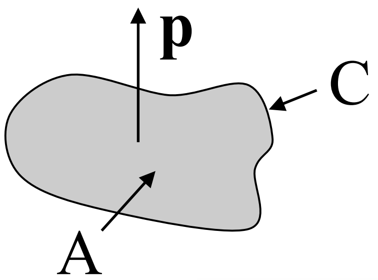
$\xi \approx 0.1$ nm – vortex core radius

$\omega = \frac{v}{R} = \frac{2\pi}{T}$

Angular velocity and vorticity

Vorticity: $\boldsymbol{\Theta} = \nabla \times \boldsymbol{v} = curl(\boldsymbol{v}),\ curl(\boldsymbol{v})_Z = (\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y})$

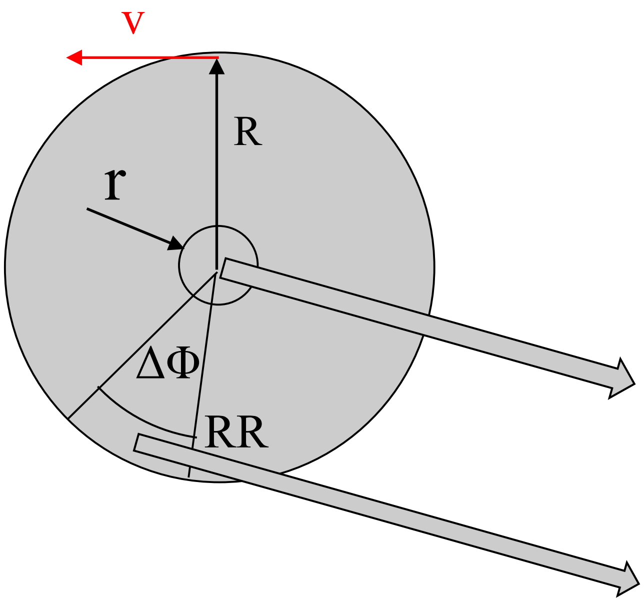
$$\boldsymbol{\Theta} = curl(\boldsymbol{v}) = \lim_{A \to 0} \frac{\boldsymbol{p}}{|A|} \oint_C \boldsymbol{v} \cdot \boldsymbol{dl}$$



C – contour in plane

A- area

Vorticity for rigid body rotation (RBR)



$$v(r) = \omega \cdot r \qquad \boldsymbol{\Theta} = \lim_{A\to 0} \frac{\boldsymbol{p}}{|A|} \oint_C \boldsymbol{v} \cdot \boldsymbol{dl}$$

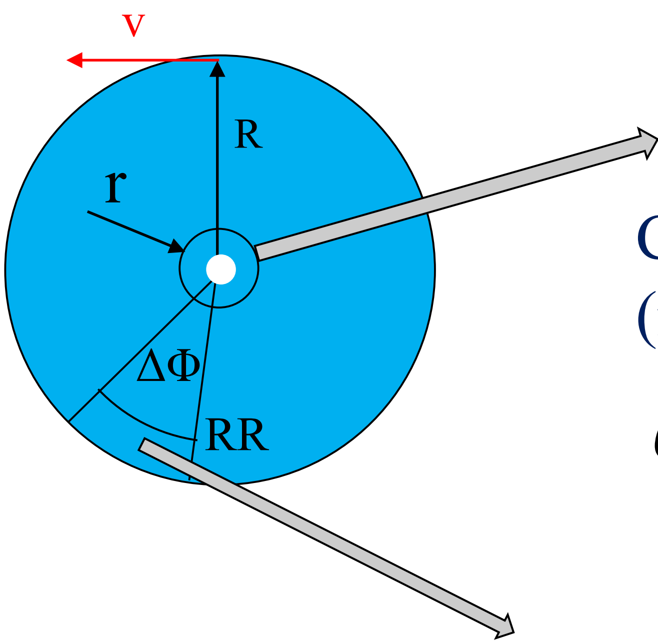
$$\Theta = \lim_{r\to 0} \frac{\omega \cdot r \cdot 2\pi r}{\pi r^2} = 2\omega$$

$$\Theta = \lim_{A\to 0} \frac{\omega \cdot (R^2 - RR^2) \cdot \Delta\Phi}{(R^2 - RR^2) \cdot \Delta\Phi/2} = 2\omega$$

For RBR vorticity is constant: $\boldsymbol{\Theta} = \frac{\boldsymbol{p}}{|A|} \oint_C \boldsymbol{v} \cdot \boldsymbol{dl}$

Vorticity in a vortex

$$\boldsymbol{\Theta} = \lim_{A\to 0} \frac{p}{|A|} \oint_C \boldsymbol{v} \cdot \boldsymbol{dl} \qquad \mathrm{v}(r) = \frac{\kappa}{2\pi \cdot r}$$



$$\oint_C v \cdot dl = \frac{\kappa \cdot 2\pi r}{2\pi r} = \kappa$$

Circulation=κ, for paths including core (multiply connected region)

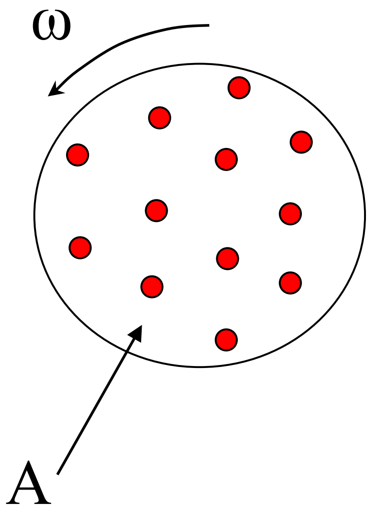
$$\Theta = \lim_{r\to 0} \frac{\kappa}{\pi r^2} = \kappa \cdot \delta(0)$$

$$\Theta = \lim_{r\to 0} \frac{\kappa \cdot \left(\frac{R}{R} - \frac{RR}{RR}\right) \cdot \Delta\Phi}{2\pi (R^2 - RR^2) \cdot \Delta\Phi/2} = 0$$

Θ=0 for paths excluding core as curl(∇S(r))=0

(single connected region) potential flow is irrotational

Average vorticity and Feynman's formulae



At a given L: lowest energy state has constant vorticity

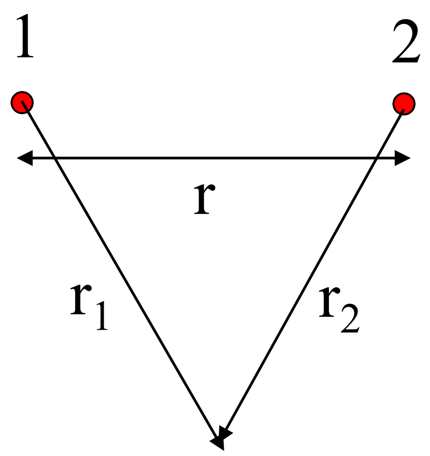
$$< \Theta > = \frac{1}{A}\oint_C v \cdot dl = \frac{N_V}{A} \cdot \kappa = n_V \cdot \kappa = 2 \cdot \omega$$

n_V = areal number density of vortices

$$\omega = \frac{n_V \cdot \kappa}{2}$$

$$L \approx \omega \cdot I_{RBR} = \frac{n_V \cdot \kappa}{2} \cdot \frac{N_{He} \cdot m \cdot R^2}{2} = \frac{N_V \cdot \hbar}{2} N_{He}$$

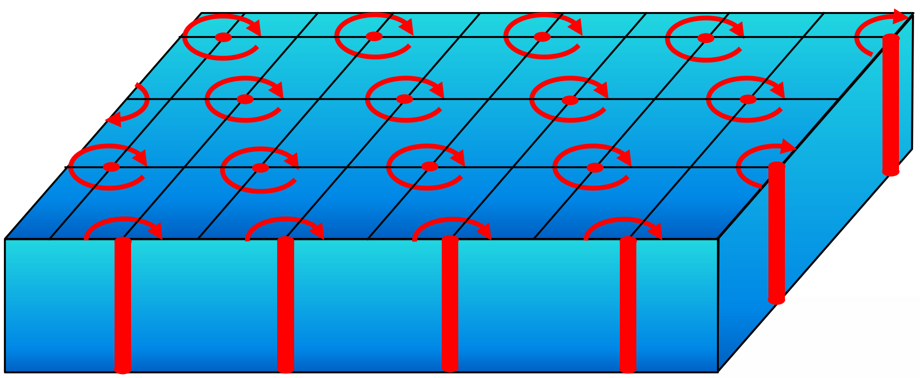
Vortex-vortex repulsion



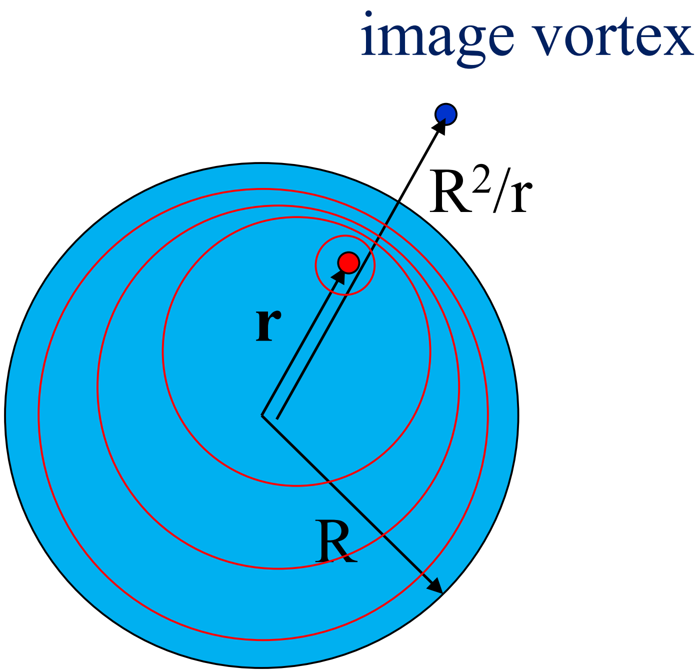
$$v = v_1 + v_2$$

$$K(r) = \frac{1}{2}\int_{space} \rho \cdot m \cdot v^2 \cdot d\tau =$$

$$K_1 + K_2 + \frac{\rho \cdot m \cdot \kappa^2}{2\pi} \cdot \ln\left(\frac{R}{r}\right)$$



Off center vortex

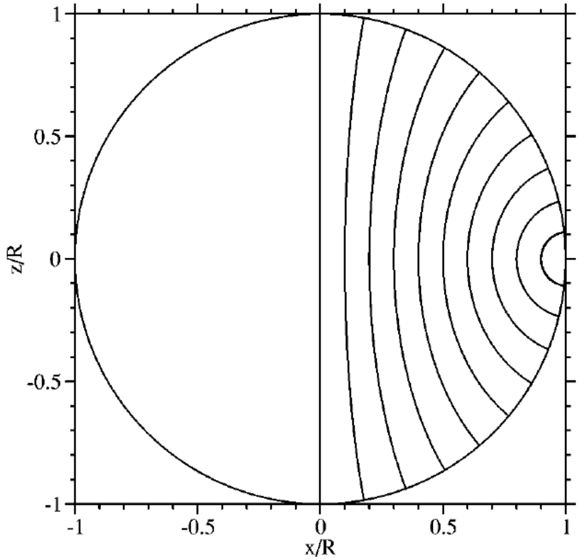


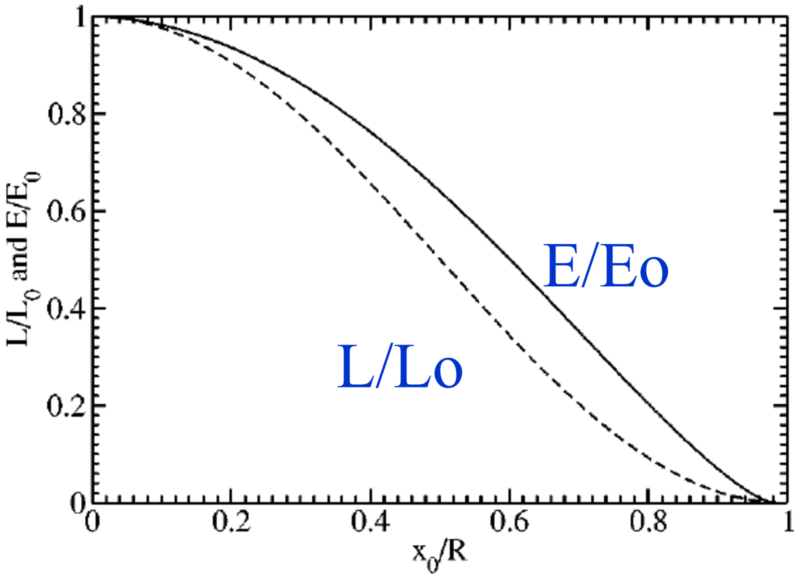
$$K(r) == \frac{\rho \cdot m \cdot \kappa^2}{4\pi} \cdot \ln\left(\frac{R^2 - r^2}{R \cdot \xi}\right)$$

$$L = \rho \cdot m \cdot \kappa \cdot \frac{R^2 - r^2}{2}$$

$$L = \hbar \frac{R^2 - r^2}{R^2}$$ per atom

Real 3D Droplets

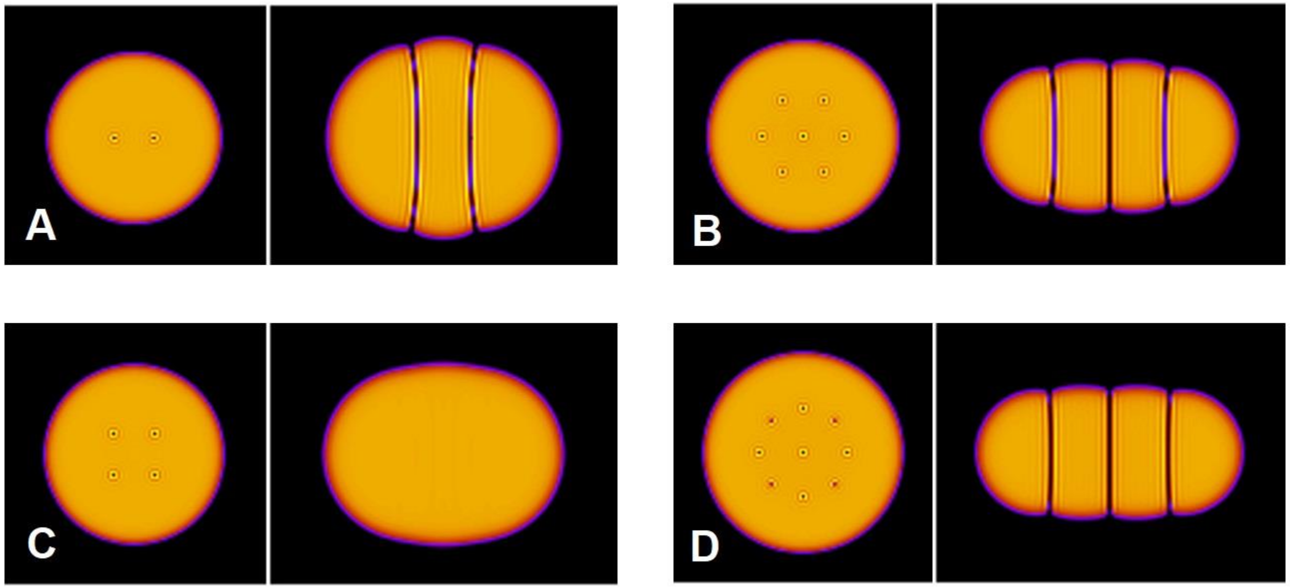




No analytic solution in 3D for sphere or other shapes

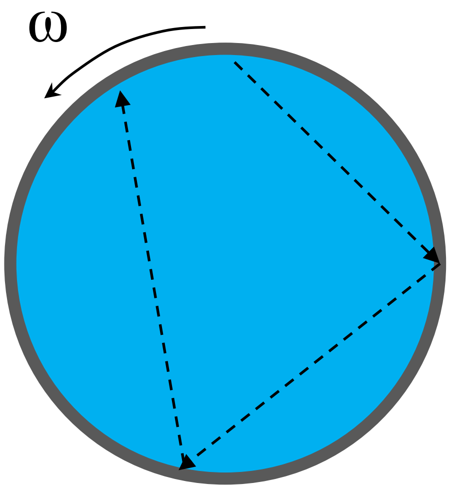
Lehmann, Schmied, Phys. Rev. B, 68 (2003) 224520
See also:
Bauer, Donnelly, Vinen, JLTP 98 (1995) 47

DFT calculations of vortices in deformed droplets, $N_{He} = 15000$



Ancilotto F, Pi M, Barranco M. 2015. Vortex arrays in nanoscopic superfluid helium droplets. Phys. Rev. B 91:100503(R)

Angular momentum in phonons and ripplons is small



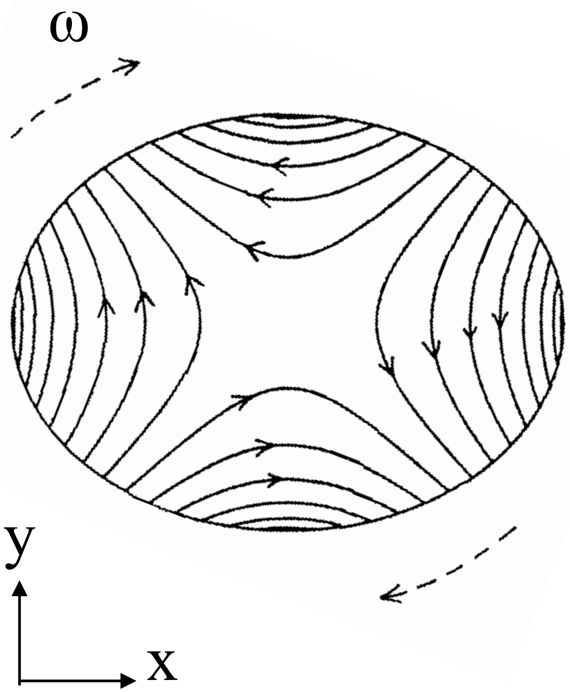
Phonons and ripplons have <v> =0 in the rotating frame

$$L_{ph} = I_{ph} \cdot \omega$$

$$\frac{I_{ph}}{I_{RBR}} = \frac{\rho_n}{\rho} \approx 10^{-5}$$

$$\frac{I_{rippl}}{I_{RBR}} = 0.3 \cdot T^{5/3} \cdot N_{He}^{-1/3} \approx 10^{-4}$$

Angular momentum due to capillary waves in prolate droplets is substantial



Seidel, G. M.; Maris, H. J., Morphology of Superfluid Drops with Angular-Momentum. *Physica B-Condensed Matter* **1994,** *194*, 577-578.

Potential flow: curl(**v**) = 0

$$L_{cap} = I_{cap} \cdot \omega$$

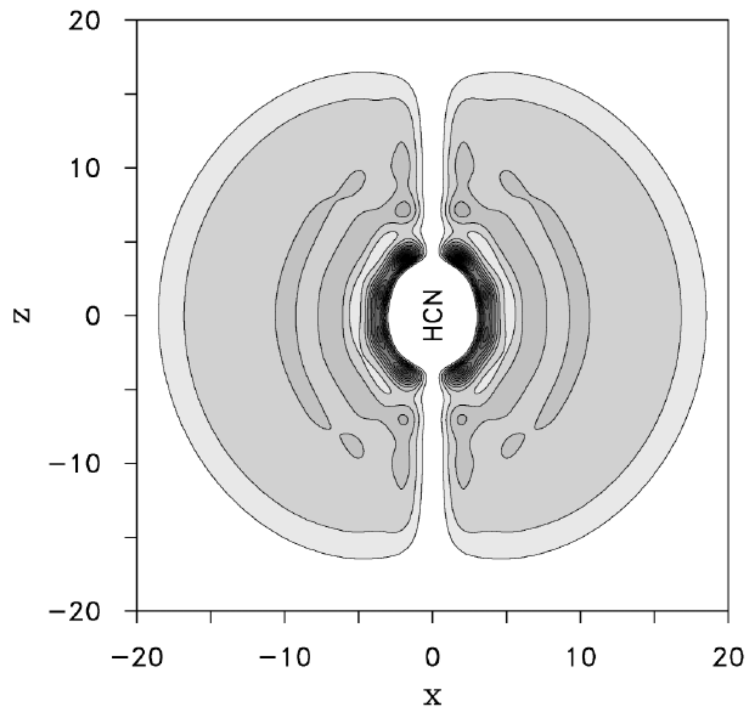
In ellipsoid:

$$\frac{I_{cap}}{I_{RBR}} = \frac{< x^2 - y^2 >}{< x^2 > + < y^2 >} \approx 1$$

In prolate droplets:

$$\boldsymbol{v} = \boldsymbol{v}_{cap} + \boldsymbol{v}_{vort} \qquad L = L_{cap} + L_{vort}$$

Vortex cores are too narrow to be observed

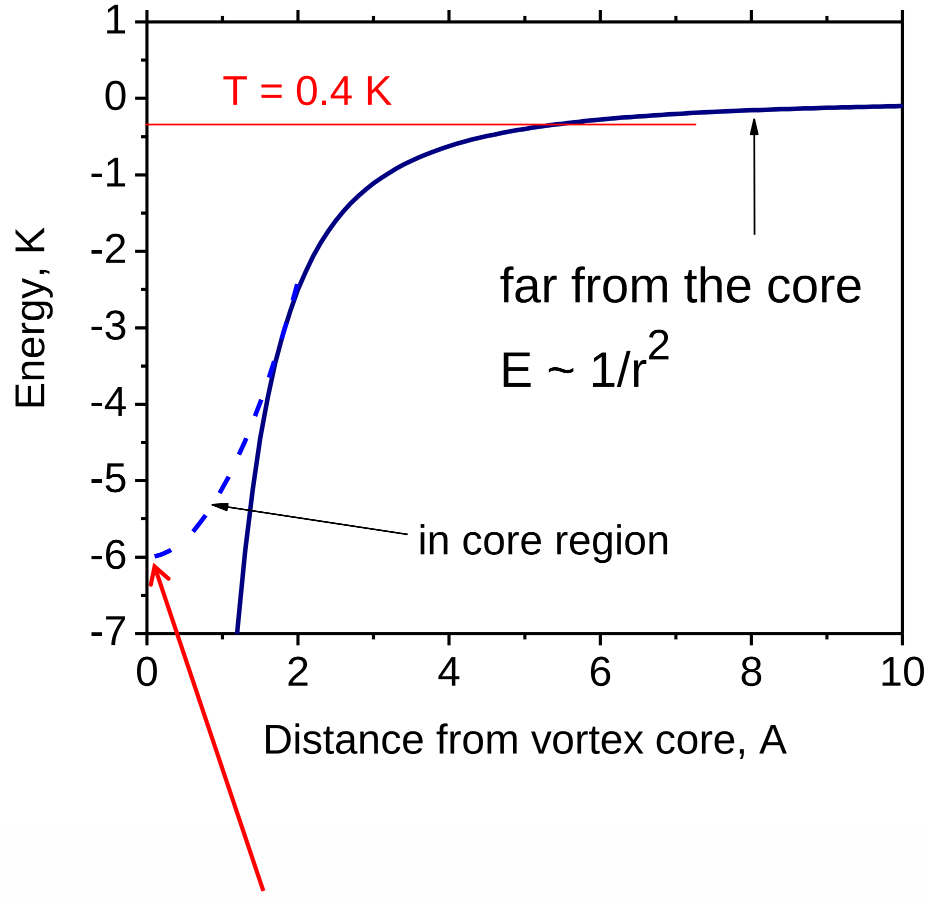


Density from: Dalfovo, Mayol, Pi, Barranco, PRL 85 (2000) 1028

Vortex – impurity interaction

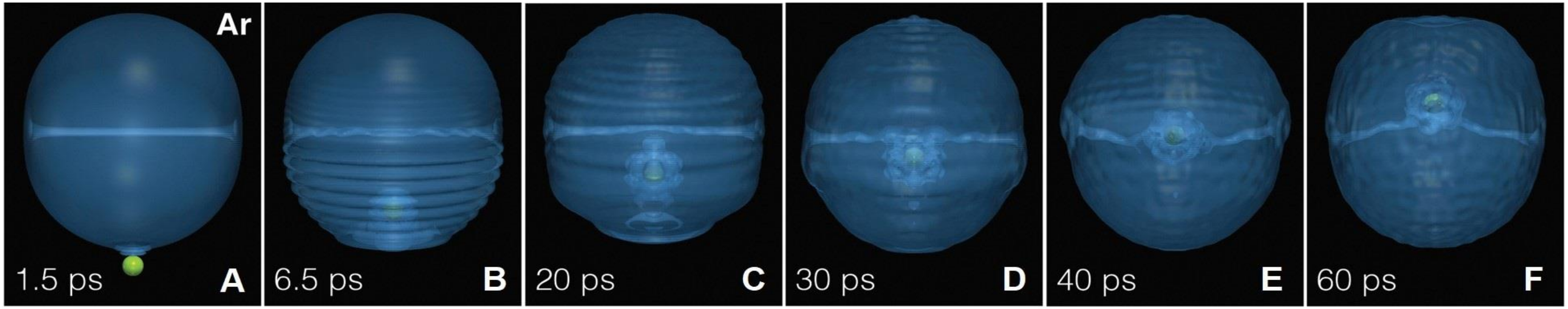
Kinetic energy of displaced superfluid:

$$T = \int_{\substack{impurity \\ volume}} \frac{\rho \cdot m \cdot v(r)^2}{2} d\tau$$



Dalfovo, Mayol, Pi, Barranco, PRL 85 (2000) 1028

The dynamics of the vortex doping

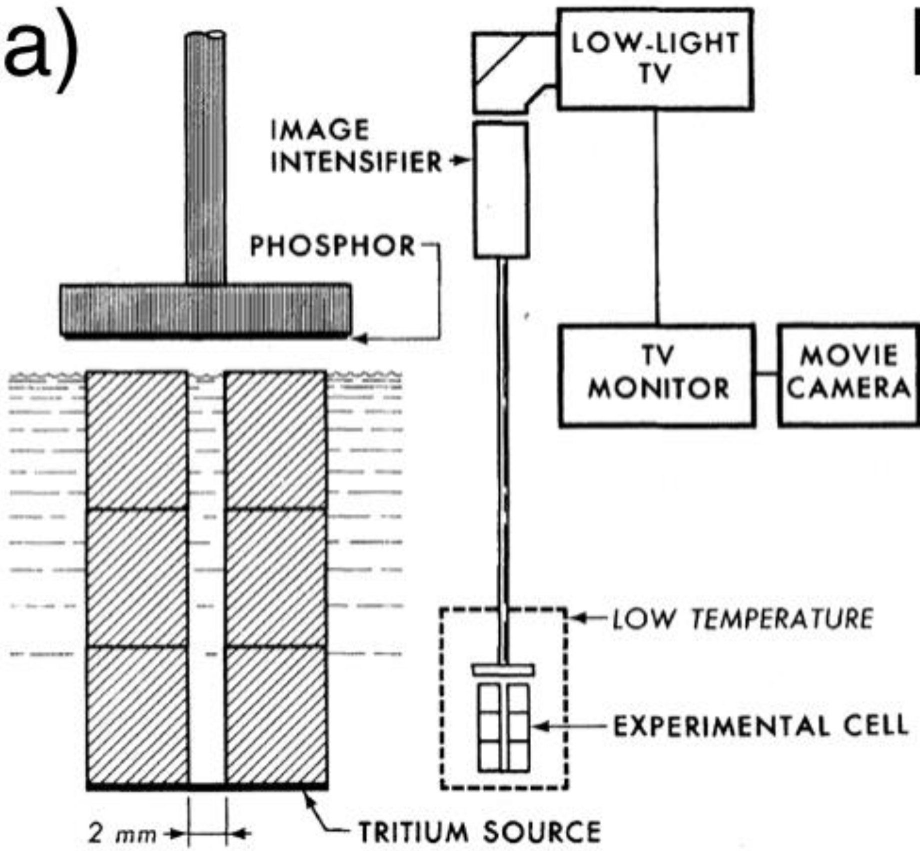


Coppens F, Ancilotto F, Barranco M, Halberstadt N, Pi M. 2017. Capture of Xe and Ar atoms by quantized vortices in ^{4}He nanodroplets. Phys. Chem. Chem. Phys. 19:24805–18

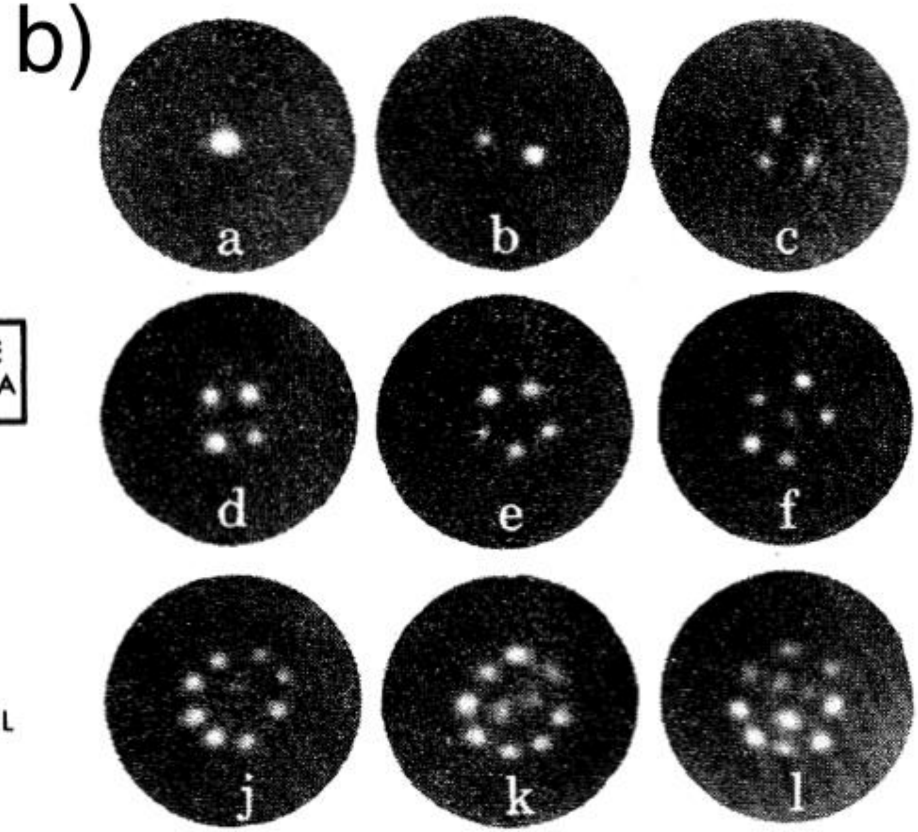
Capture impact parameter for Xe ≈ 0.5 nm

Pshenichnyuk, I. A.; Berloff, N. G., Inelastic scattering of xenon atoms by quantized vortices in superfluids. Physical Review B 2016, 94 (18), 184505-1-8.

Imaging vortices with electrons



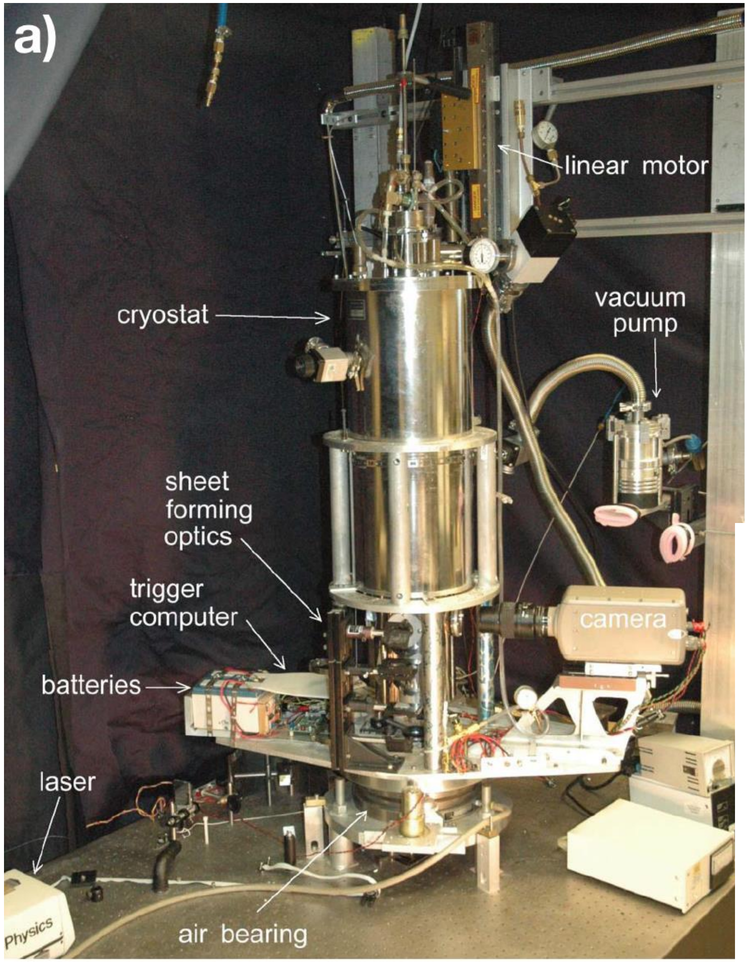
ω=0.3 rad/s

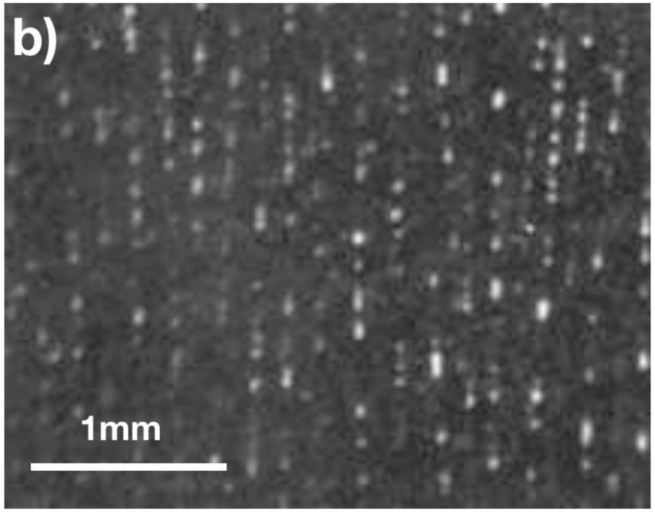


ω=0.59 rad/s

Yarmchuk EJ, Gordon MJV, Packard RE. 1979. Observation of stationary vortex arrays in rotating superfluid-helium. Phys. Rev. Lett. 43:214–17

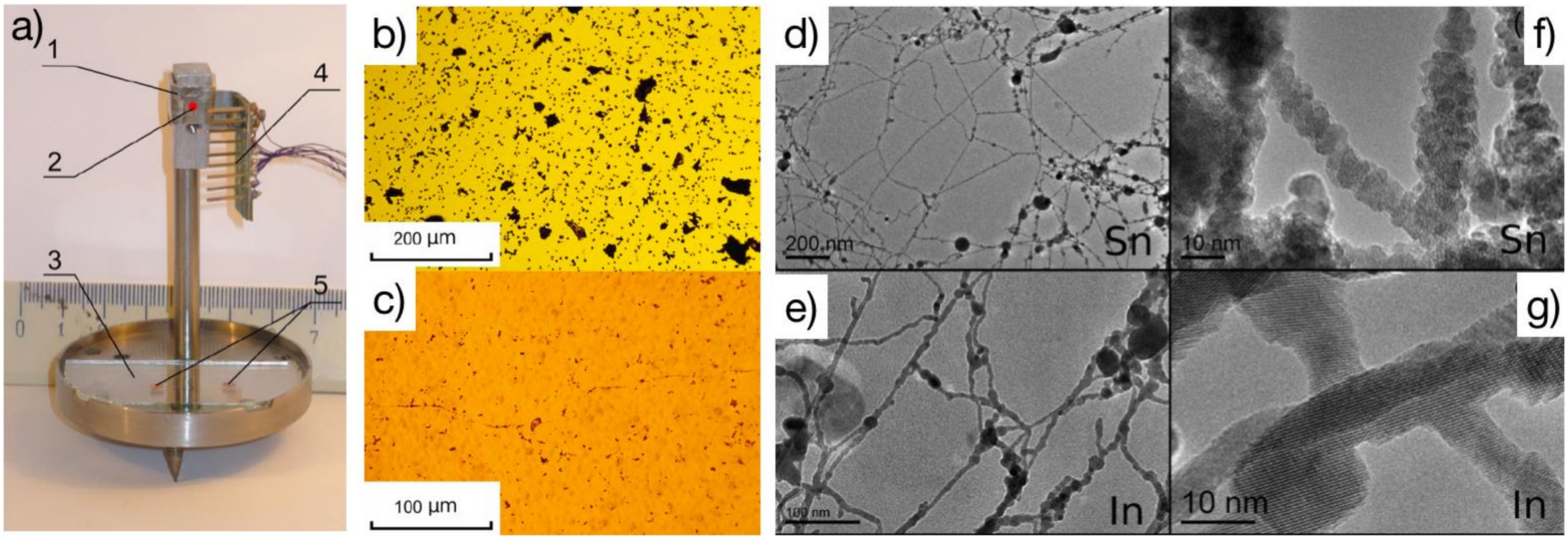
Imaging vortices with μm H_2 clusters





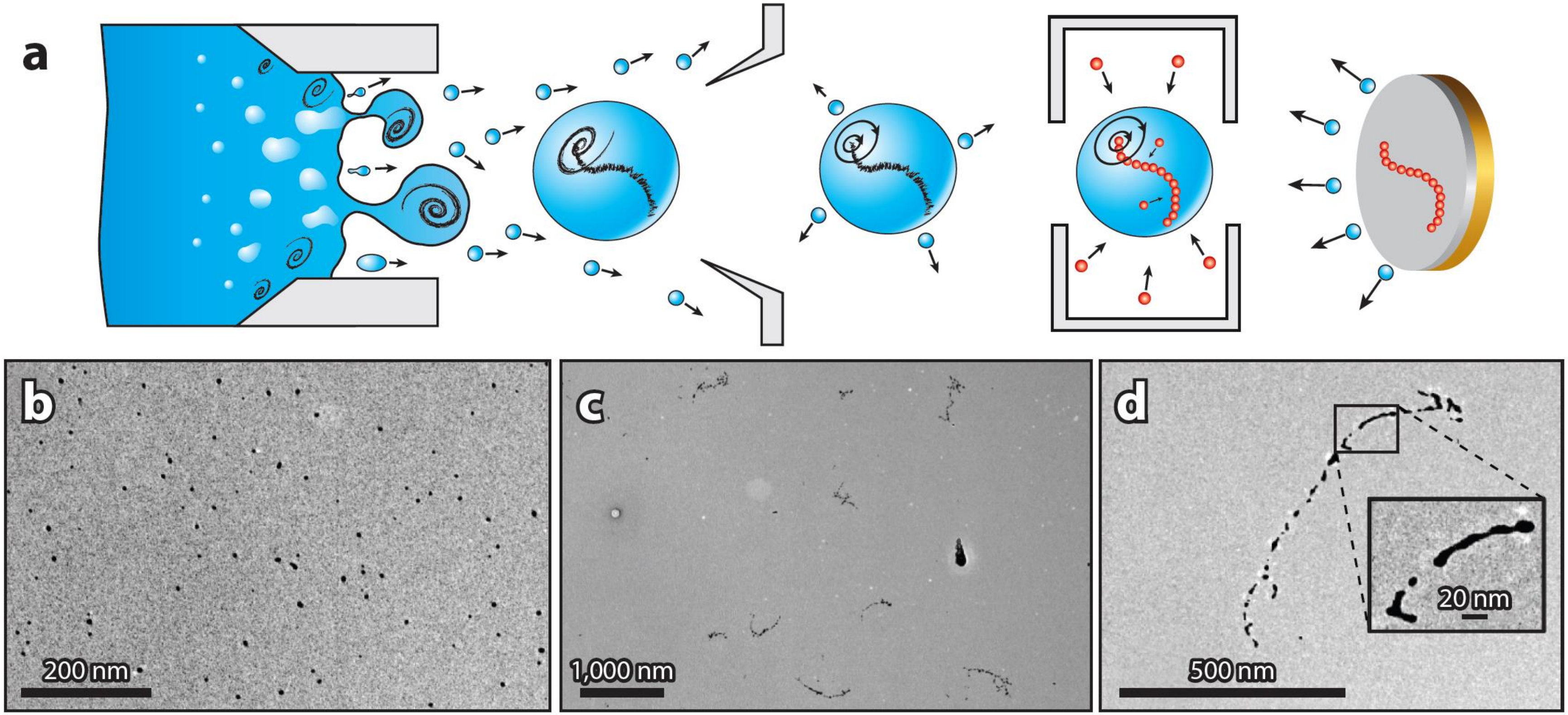
Bewley GP, Lathrop DP, Sreenivasan KR. 2006. Superfluid helium—visualization of quantized vortices.
Nature 441:588

Aggregation of metals on vortices in bulk SHe



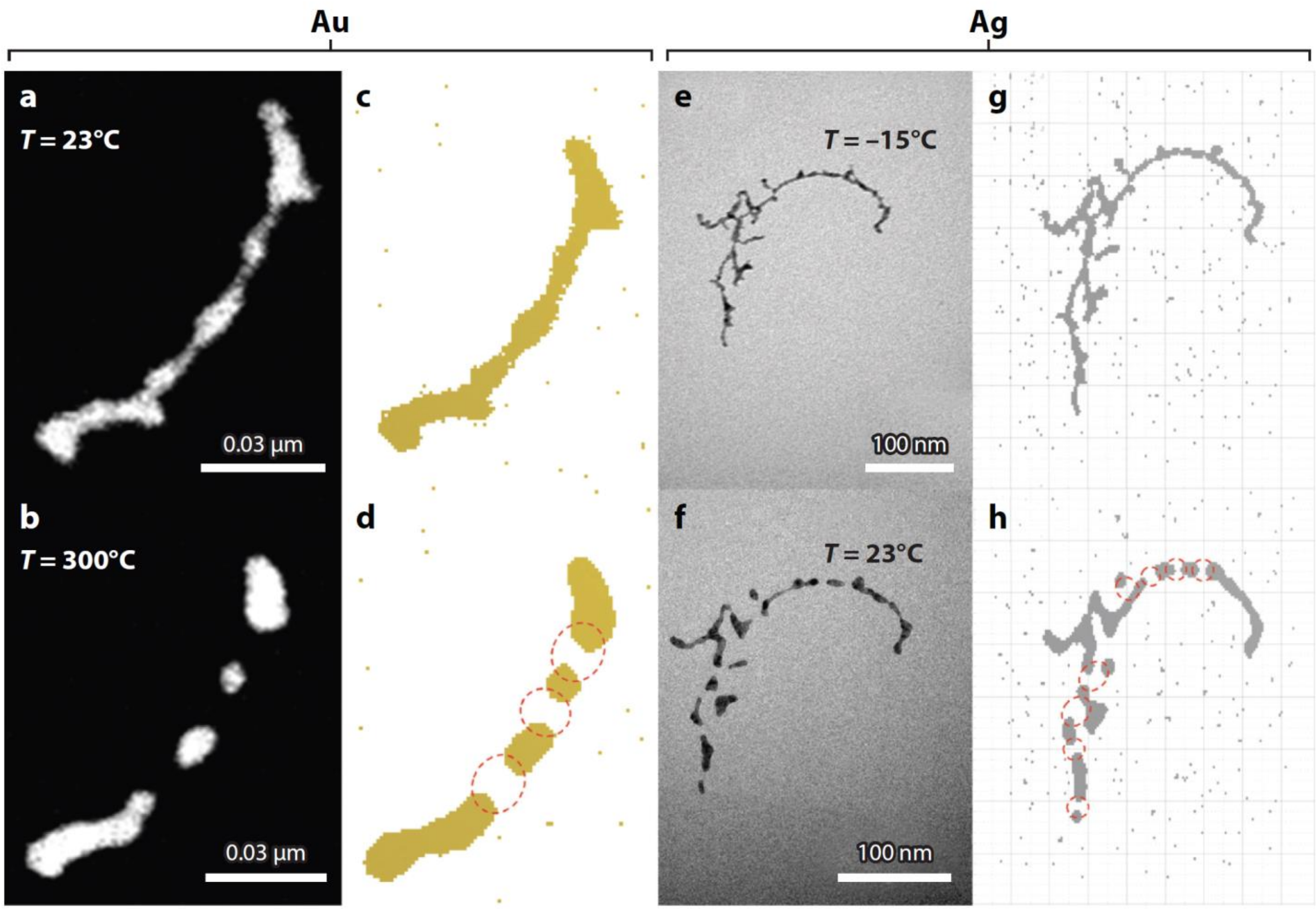
Gordon EB, Karabulin AV, Matyushenko VI, Sizov VD, Khodos II. 2012. The role of vortices in the process of impurity nanoparticles coalescence in liquid helium. Chem. Phys. Lett. 519–20:64–68
Lebedev V, Moroshkin P, Grobety B, Gordon E, Weis A. 2011. Formation of metallic nanowires by laser ablation in liquid helium. J. Low Temp. Phys. 165:166–76
Gordon EB, Karabulin AV, Matyushenko VI, Sizov VD, Khodos II. 2012. The electrical conductivity of bundles of superconducting nanowires produced by laser ablation of metals in superfluid helium. Appl. Phys. Lett. 101:052605

Formation of metallic filaments in He droplets



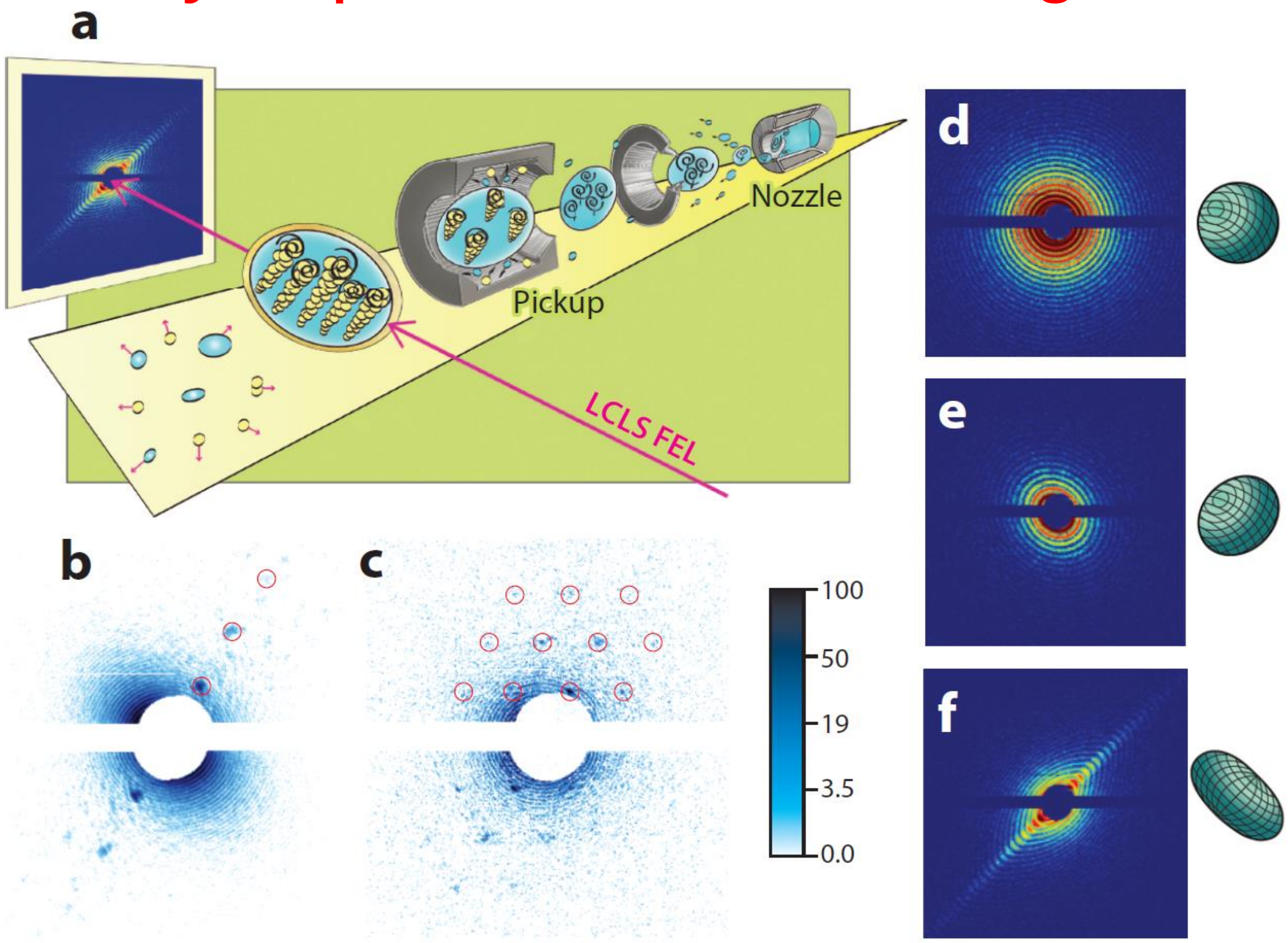
Gomez LF, Loginov E, Vilesov AF. 2012. Traces of vortices in superfluid helium droplets. Phys. Rev. Lett. 108:155302

Deposition and reconstruction of the filaments



SchnedlitzM, LasserusM,Knez D,Hauser AW, Hofer F, Ernst WE. 2017. Thermally induced breakup of metallic nanowires: experiment and theory. Phys. Chem. Chem. Phys. 19:9402–8

X-ray experiments: small angle diffraction



E=1.5 keV

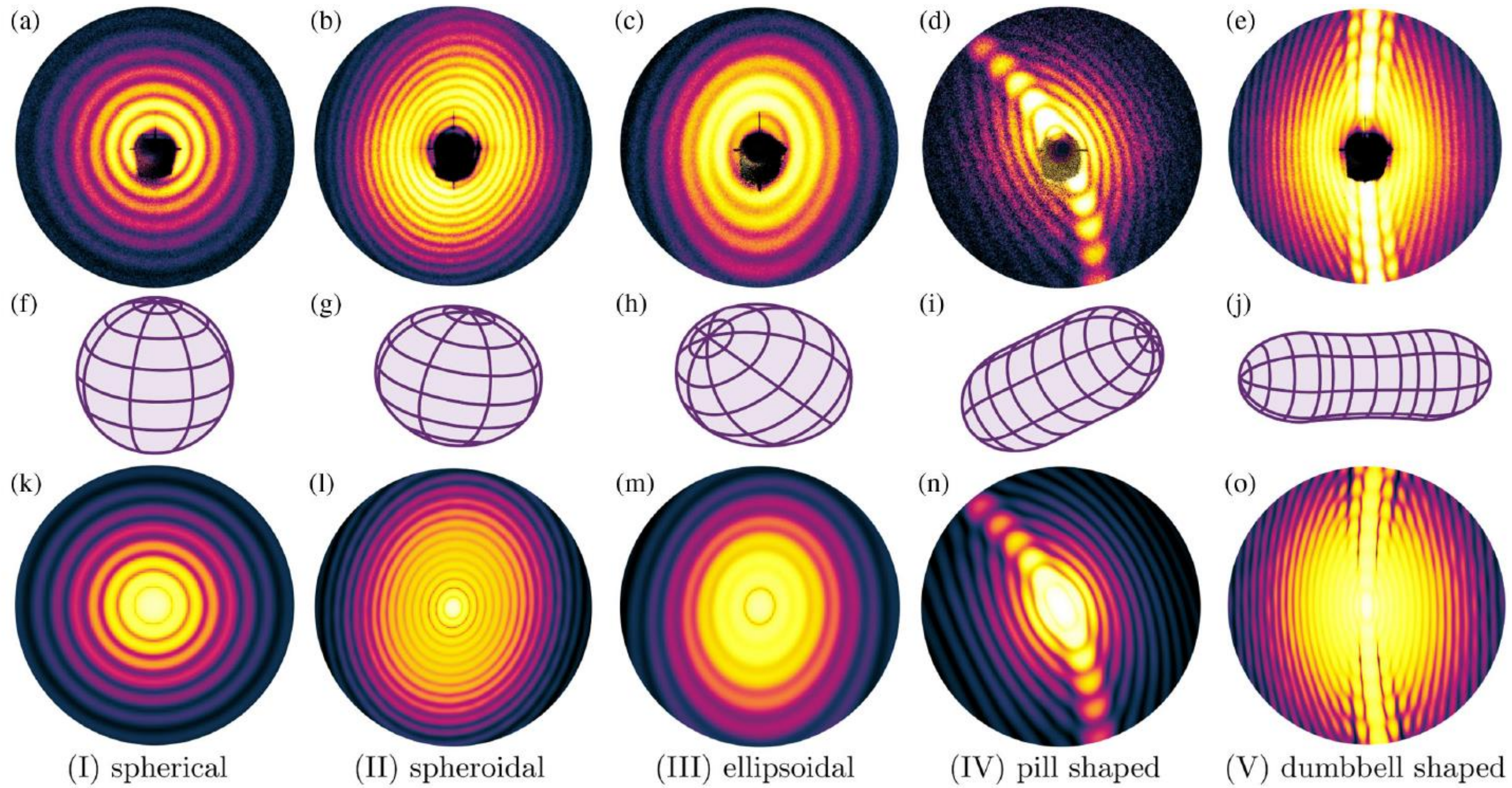
λ = 0.82 nm

$n_{refr} \approx 1.000$

Gomez LF, Ferguson KR, Cryan JP, Bacellar C, Tanyag RMP, et al. 2014. Shapes and vorticities of superfluid helium nanodroplets. Science 345:906–9

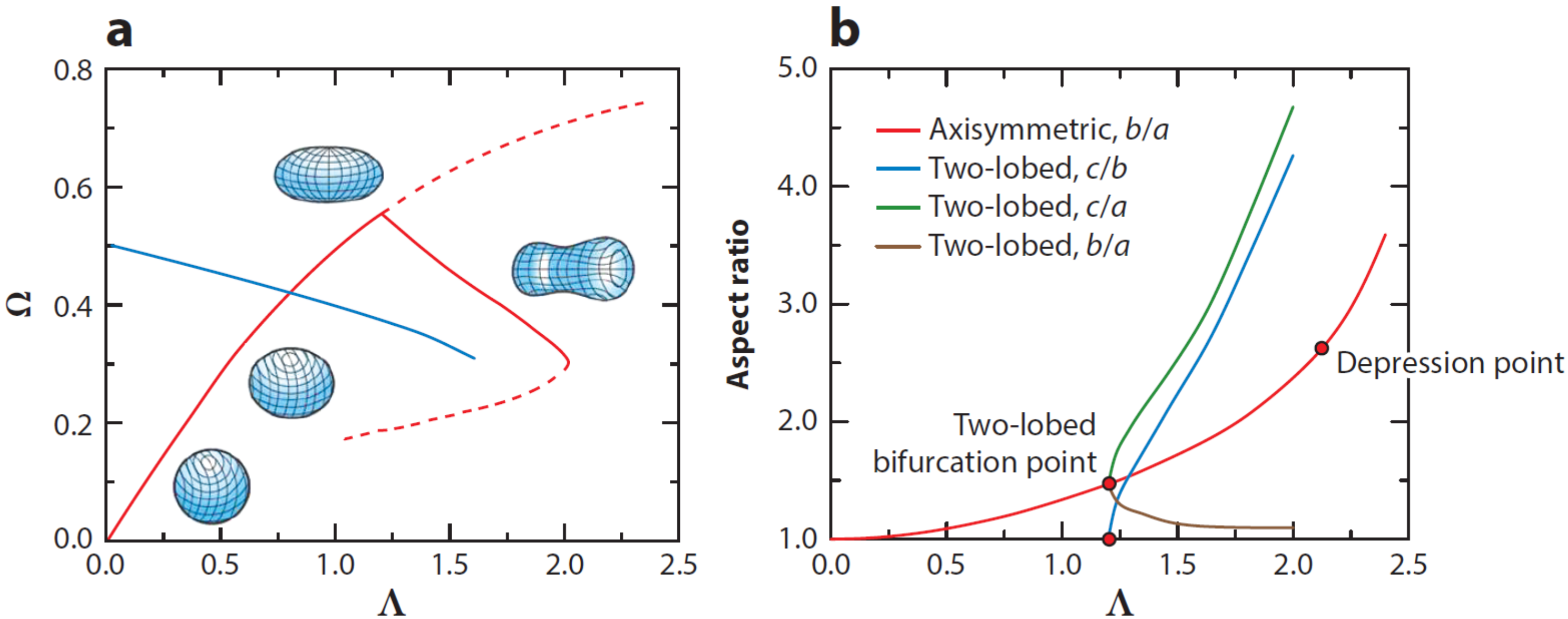
Shape from wide angle diffraction: Oblate vs Prolate droplets

E≈ 20 eV
λ ≈ 60 nm
n_{REFR}≈1.15+i·0.03



B. Langbehn, K. Sander, Y. Ovcharenko, C. Peltz, A. Clark,…… F. Stienkemeier, C. Callegari, T. Fennel, D. Rupp, and T. M□oller. 2018. Three-dimensional shapes of spinning helium nanodroplets. Phys. Rev. Lett. 121(25):255301

Shapes of rotating classical droplets



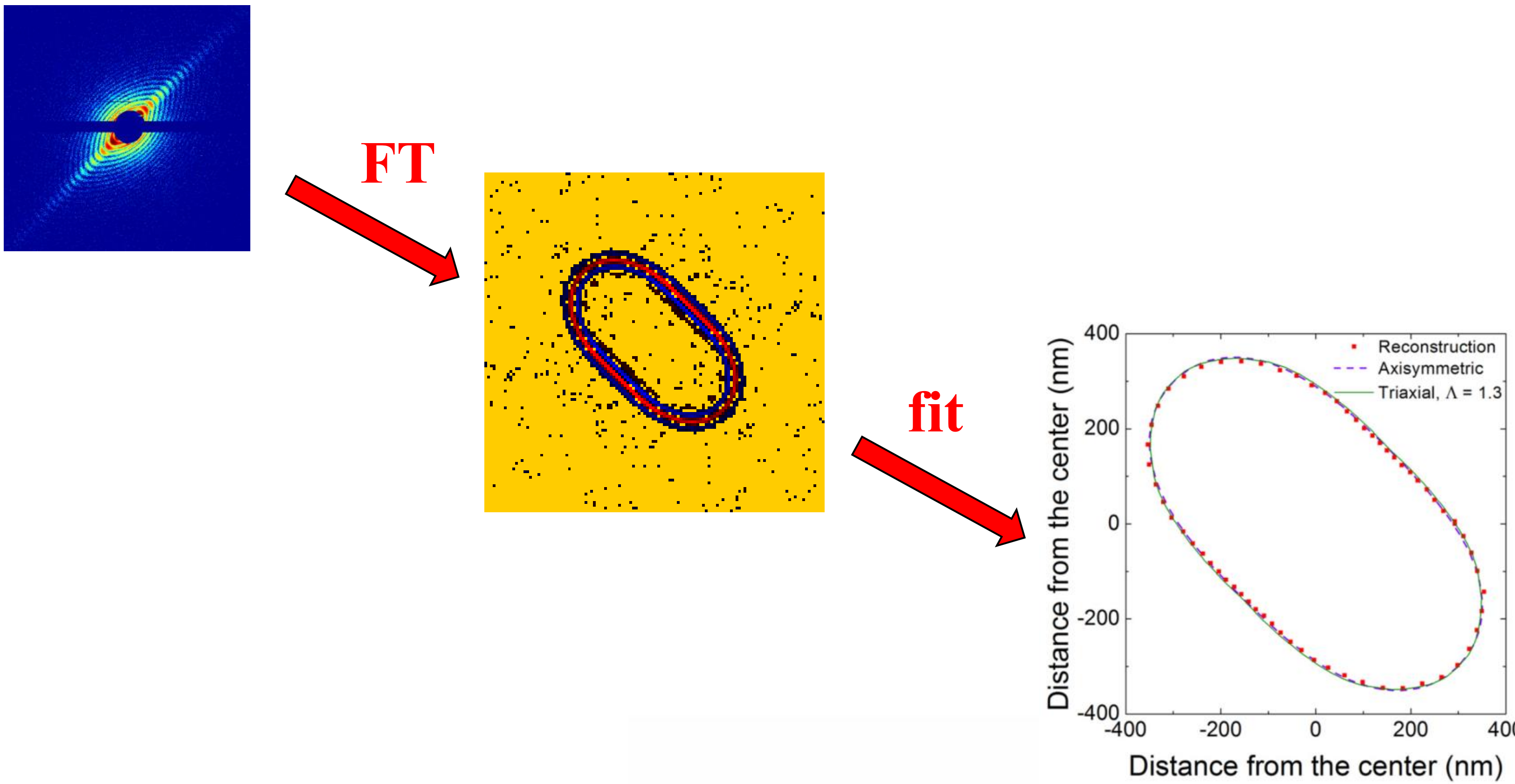
$$\Omega = \sqrt{\frac{\rho \cdot R^3}{8 \cdot \sigma}} \cdot \omega \qquad \Lambda = \frac{1}{\sigma \cdot R^2} \cdot \sqrt{\frac{8 \cdot \sigma}{\rho \cdot R^3}} \cdot l$$

Bernando C, Tanyag RM, Jones C, Bacellar C, Bucher M, et al. 2017. Shapes of rotating superfluid helium nano-droplets. Phys. Rev. B 95:064510
Ancilotto F, Barranco M, Pi M. 2018. Spinning superfluid 4He droplets. Phys. Rev. B 97:184515

Learning about sf droplets

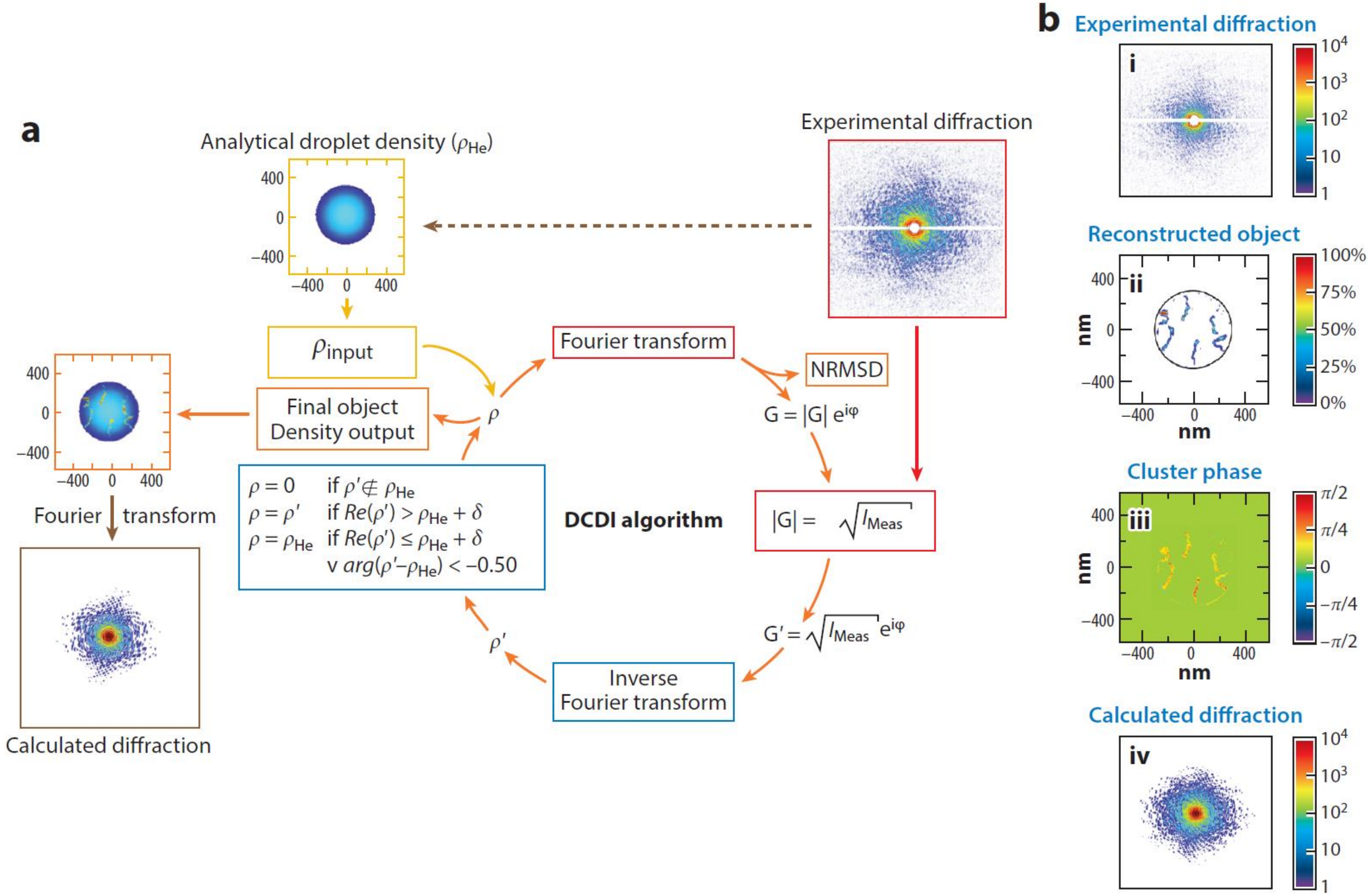
- Do they have same shape at same Ω, Λ as classical droplets?
- How the vortices are organize inside the droplets?
- If presence of vortices influences shape?
- Interplay of capillary waves and vortices?

Small angle scattering: much more simple analysis, but 2D information – projection of the droplet's contour



Bernando C, Tanyag RM, Jones C, Bacellar C, Bucher M, et al. 2017. Shapes of rotating superfluid helium nano-droplets. Phys. Rev. B 95:064510

DCDI: getting column density from diffraction



Tanyag RMP, Bernando C, Jones CF, Bacellar C, Ferguson KR, et al. 2015. Communication: X-ray coherent diffractive imaging by immersion in nanodroplets. Struct. Dyn. 2:051102

Unpublished material is removed

Conclusions

- Analytic calculations for vortices in 2D
- Vortex doping and imaging
- Vortex structures in droplets of different shape
- Some future XFEL experiments are discussed in: O.Gessner, A.Vilesov, Annu. Rev. Phys. Chem. 2019. 70:203–28

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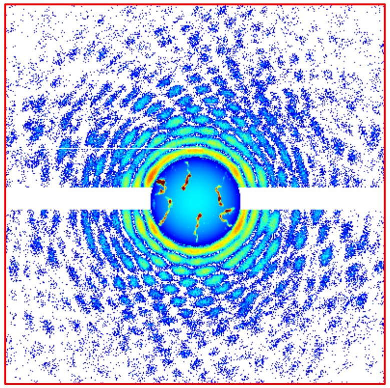

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