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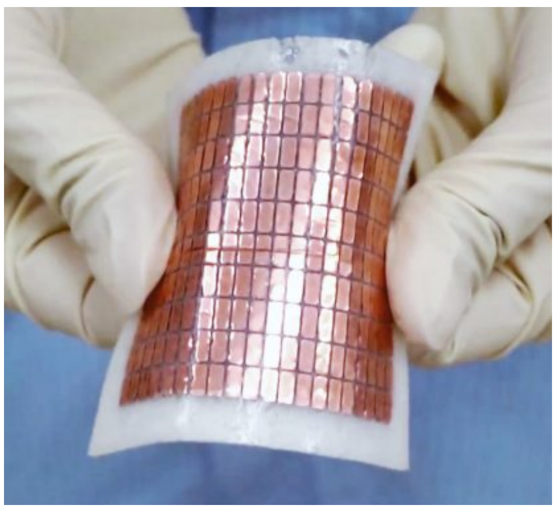
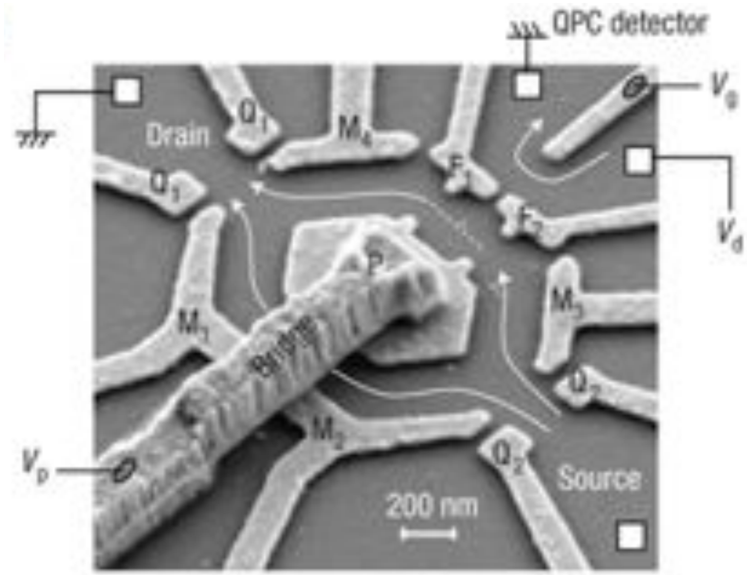
UNIVERSITÉ DE GENÈVE

An introduction to thermoelectricity
& a recent proposal: a Aharonov-Bohm heat engine

Géraldine Haack
University of Geneva



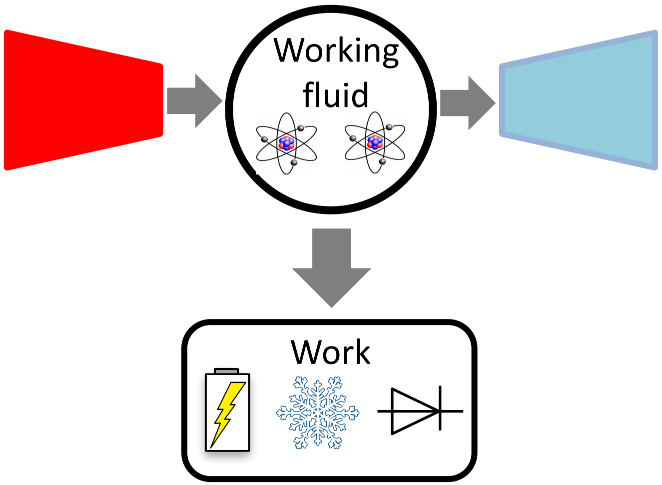
Flexible TE device (2018)



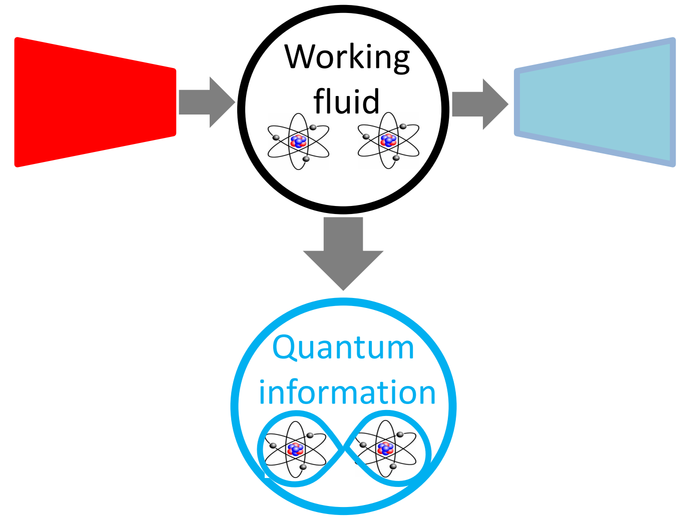
Weizmann institute Israel
Chang et al., Nat. Phys. 4 (2008)

Conference “Quantum Thermodynamics for Young Scientists”
Bad-Honnef, Germany
03.02.2020

Quantum thermal machines



versus



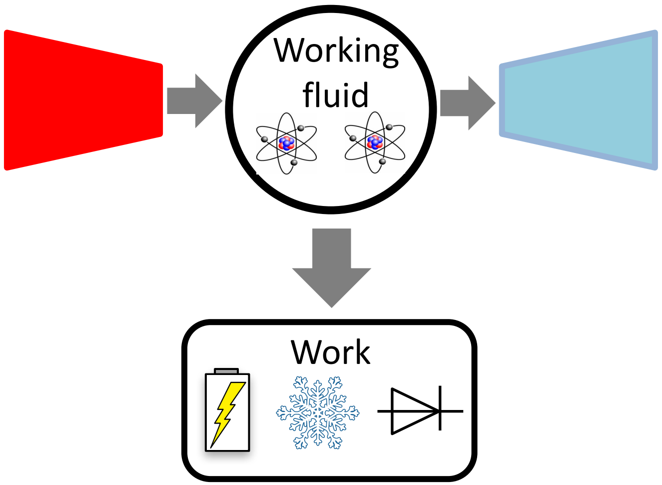
“Classical” output

Power (heat engine)
Cooling ratio (fridge)

“Quantum” output

Quantum correlations
Entanglement, multipartite ent.

Quantum thermal machines



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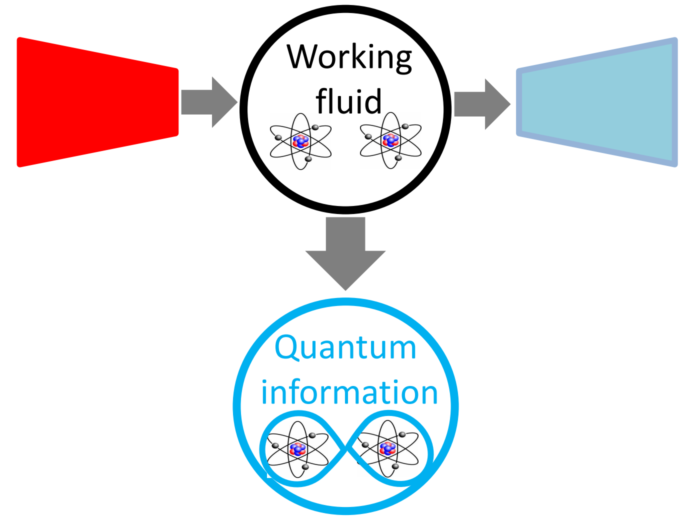

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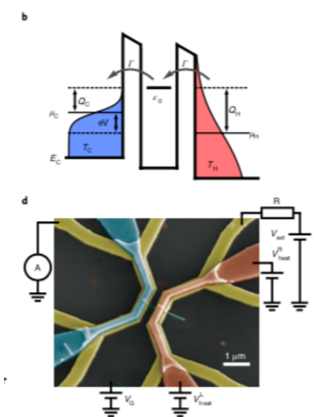


Josefsson et al., Nat. Nano. 13 (2018)
Samuelsson et al., PRL 118 (2017)
Elouard, Jordan, PRL 120 (2018)
Buffoni et al., PRL 122 (2019)
Haack, Giazotto, PRB 100 (2019)
Chiaracane et al., PRResearch 2 (2020)
Verteletsky, Mölmer, arXiv:1907.01039

Brask, Haack, Brunner, Huber, NJP 17 (2015)
Tavakoli et al., Quantum 2 (2018)
Hegwill et al., PRA 98 (2018)
Tavakoli et al., PRA 101 (2020)

Talks of C. Chiaracane, A. Tavakoli, M. Mitchison, N. Poovakkattil & posters!

Outline

- What is thermoelectricity?
- The transport coefficients and the Onsager matrix
- Thermodynamics of thermoelectricity
- The Aharonov-Bohm heat engine

K. Behnia, Fundamentals of thermoelectricity, Oxford University Press
Benenti, Casati, Saito, Whitney, Phys. Rep. 69, 1 (2017)
Van Houten, Molenkamp, Beenakker, Foxon, Semi. Sc. & Tech. (1992)

Thermoelectricity

- Thermocouple

You generate an electrical current from a thermal gradient ΔT

You generate a heat current from a voltage bias V

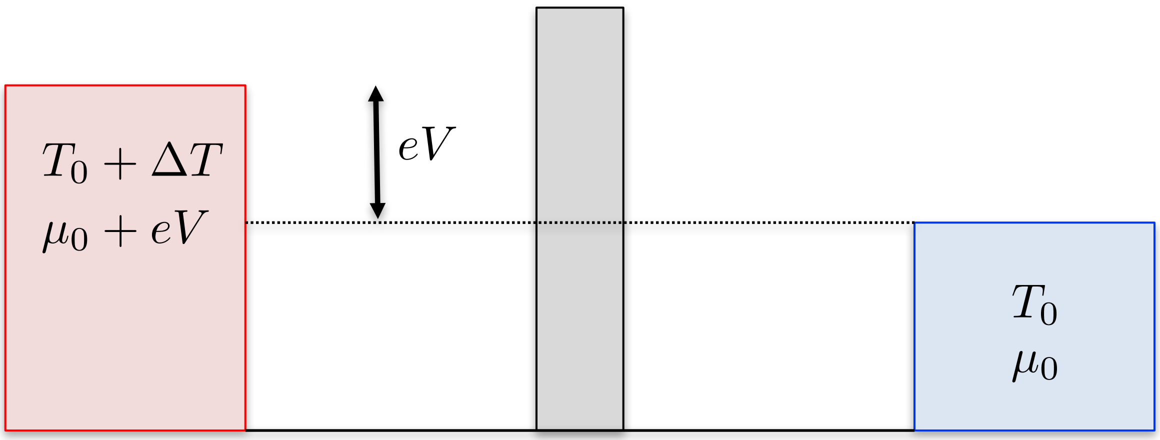
1821: Thomas Johann Seebeck

1834: Jean-Charles Peltier

1850 : Lord Kelvin conjectured both effects are related

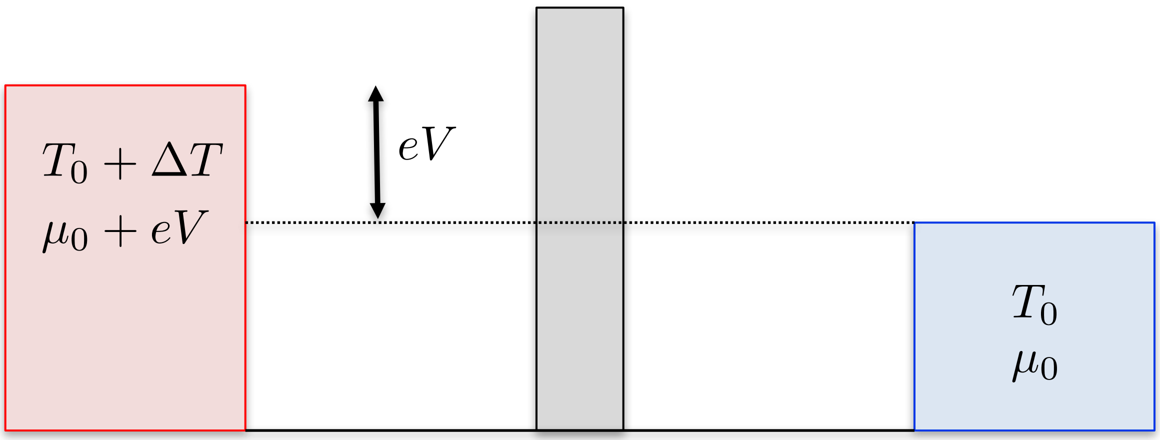
1931 : Reciprocal relations by Lars Onsager

Quantum transport



- Linear response regime $\Delta T \ll T_0 \quad eV \ll \mu_0$

Quantum transport



- Linear response regime $\quad \Delta T \ll T_0 \quad eV \ll \mu_0$
- Fermionic reservoirs

$$f(\mu, T, E) \sim f(\mu_0, T_0, E) + \left.\frac{\partial f}{\partial \mu}\right|_{\mu=\mu_0} (\mu - \mu_0) + \left.\frac{\partial f}{\partial T}\right|_{T=T_0} (T - T_0)$$

$$\frac{\partial f}{\partial \mu} = \left(-\frac{\partial f}{\partial E}\right) = \frac{1}{4k_B T} \frac{1}{\cosh^2[E - \mu/(k_B T)]} \qquad \frac{\partial f}{\partial T} = \frac{E - \mu}{T} \left(-\frac{\partial f}{\partial E}\right)$$

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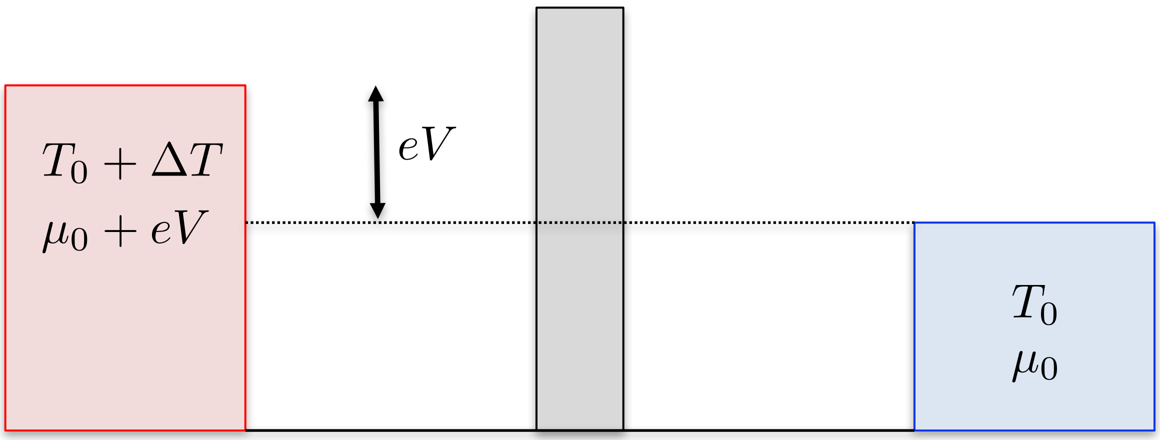

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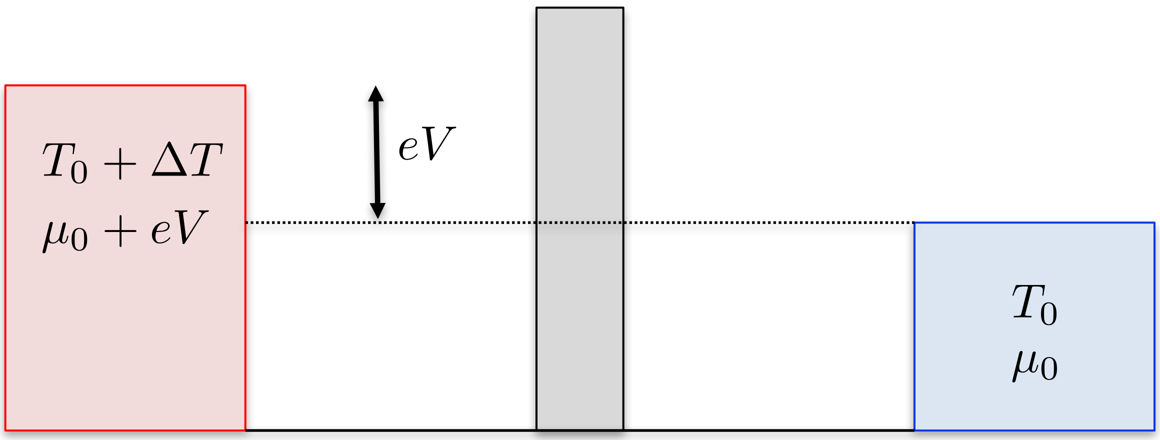
- Difference of Fermi distributions

$$f_L(\mu, T) - f_R(\mu_0, T_0) \sim \left(-\frac{\partial f}{\partial E}\right) eV + \frac{E - \mu_0}{T_0}\left(-\frac{\partial f}{\partial E}\right) \Delta T$$

Quantum transport



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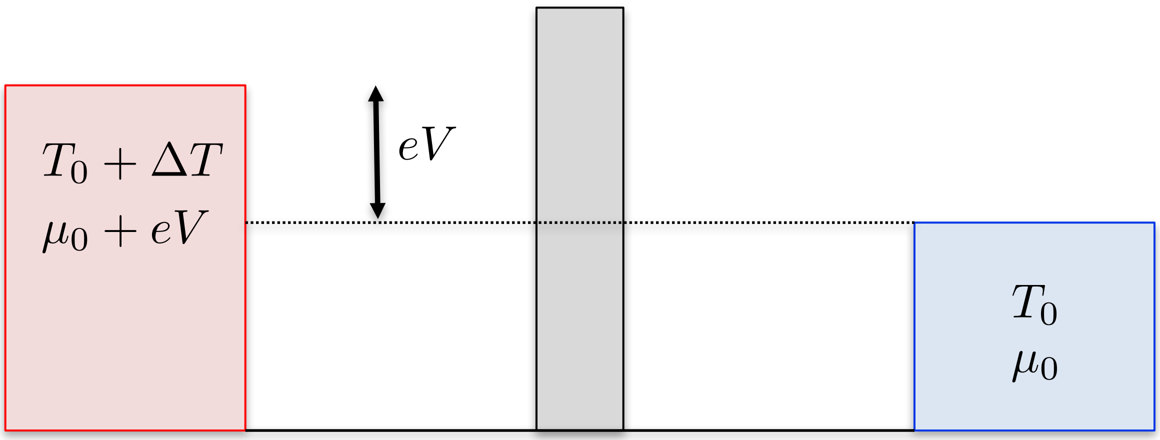

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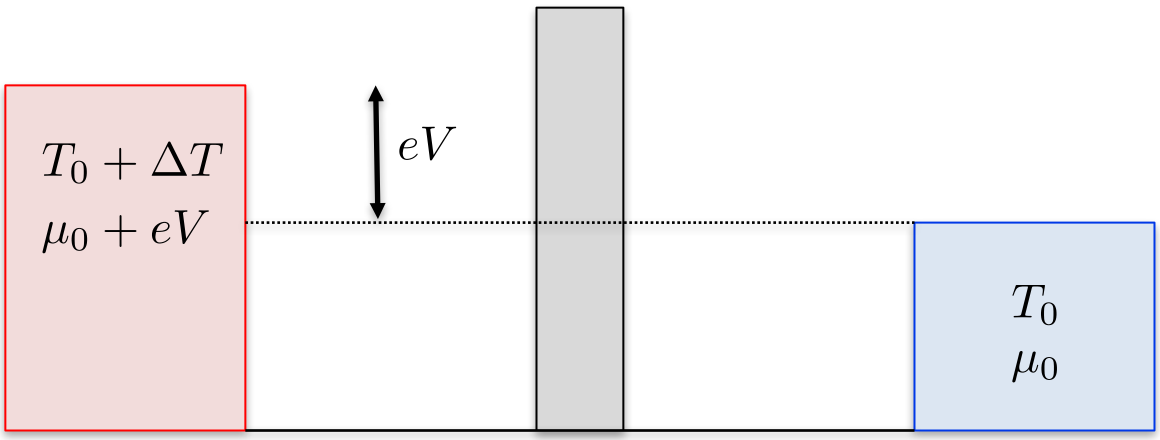
- Currents

$$I = \frac{2e}{h}\int dE\ T(E)\,(f_L - f_R)$$

$$J = \frac{2}{h}\int dE\ (E-\mu)T(E)\,(f_L - f_R)$$

Quantum transport



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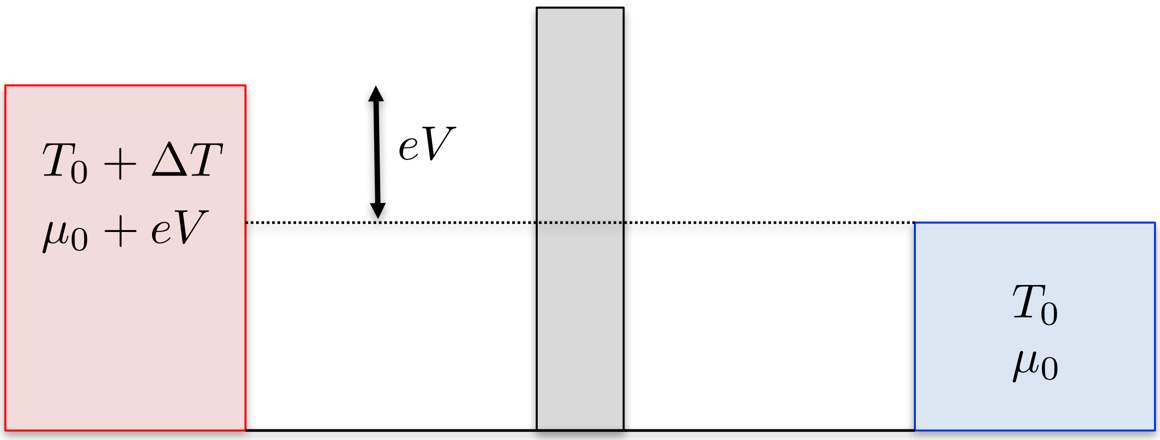
Onsager matrix

$$\begin{pmatrix} I \\ J \end{pmatrix} = \begin{pmatrix} G & \alpha \\ \alpha T & K' \end{pmatrix}\begin{pmatrix} \Delta V \\ \Delta T \end{pmatrix}$$

Quantum transport



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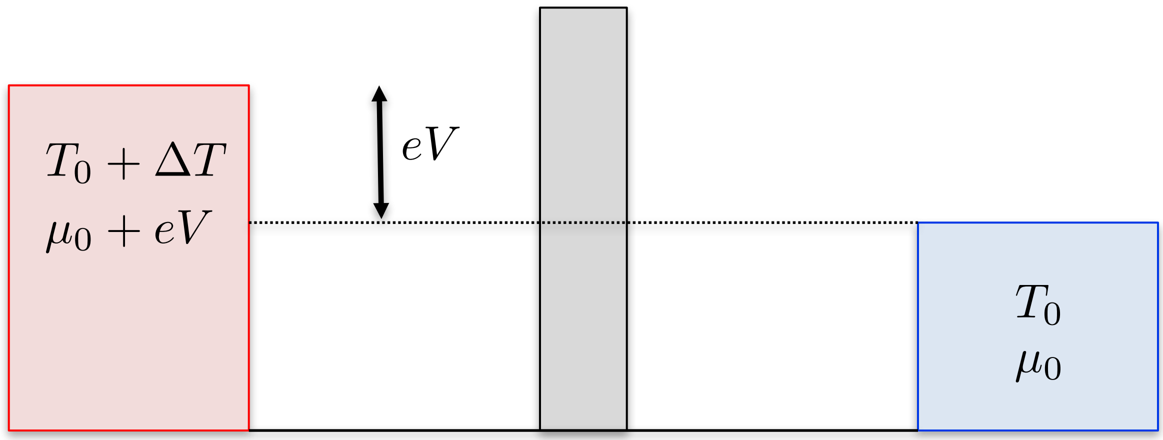
Time-reversal symmetric

Quantum transport



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Electrical conductance

Thermal conductance

Thermo-electric coefficients

$$G = \frac{2e^2}{h} \int dE\ T(E) \left(-\frac{\partial f}{\partial E}\right)$$

$$K' = \frac{2}{hT_0} \int dE\ (E-\mu_0)^2\ T(E) \left(-\frac{\partial f}{\partial E}\right)$$

$$\alpha = \frac{2e}{hT_0} \int dE\ (E-\mu_0)\ T(E) \left(-\frac{\partial f}{\partial E}\right)$$

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Quantum transport

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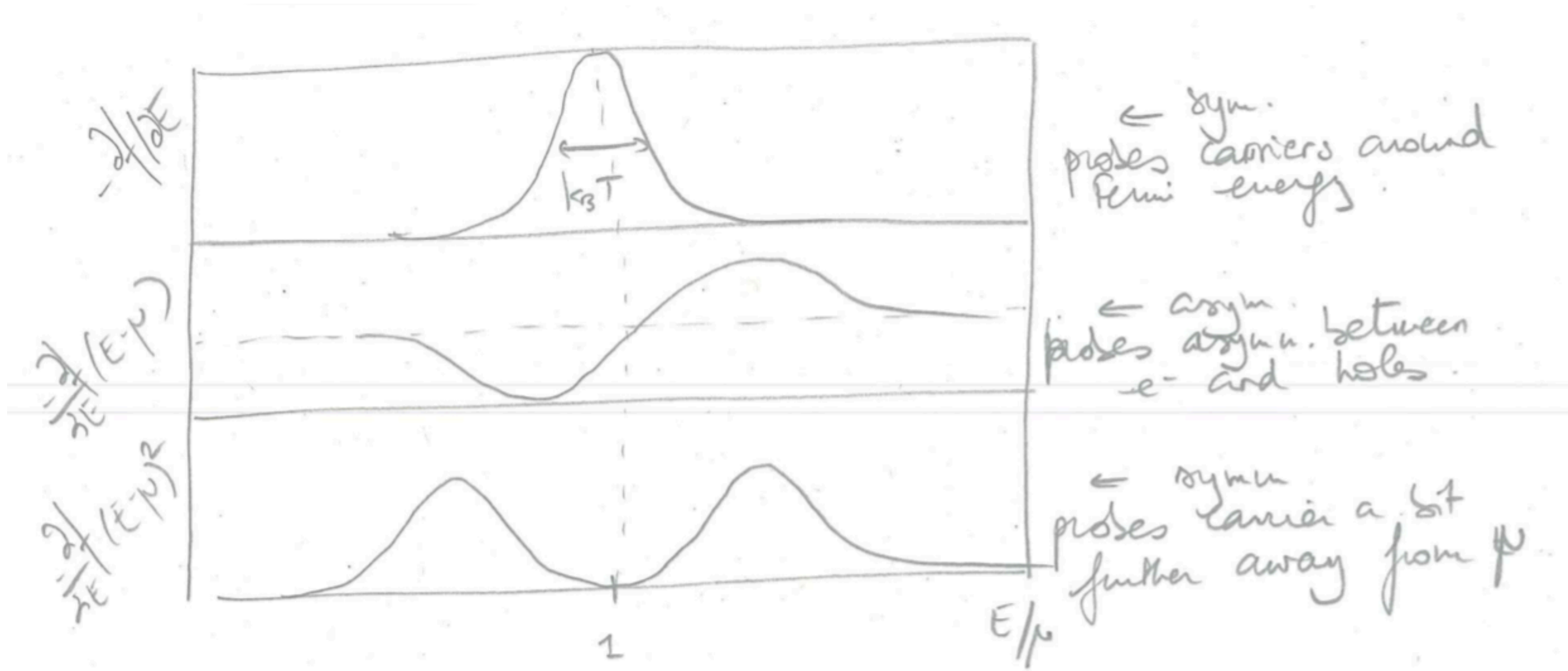

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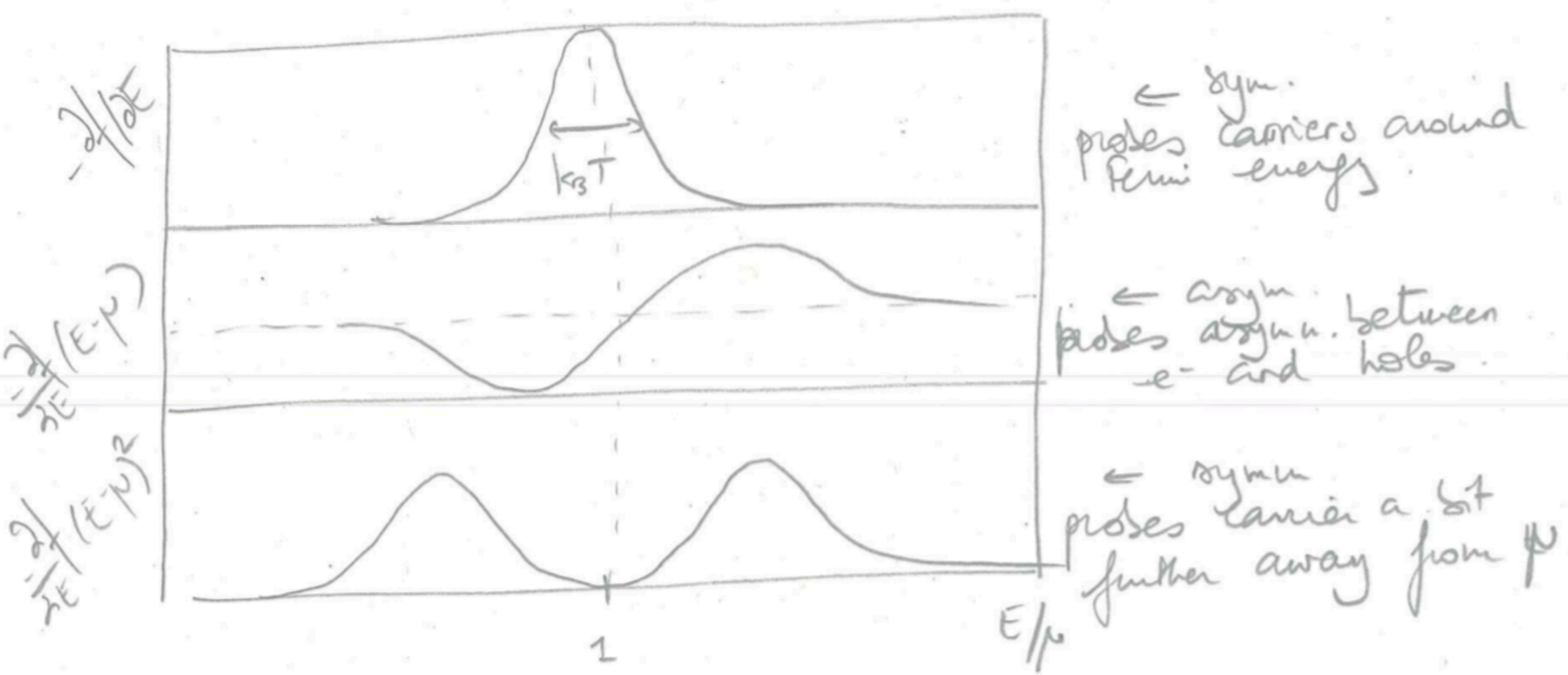

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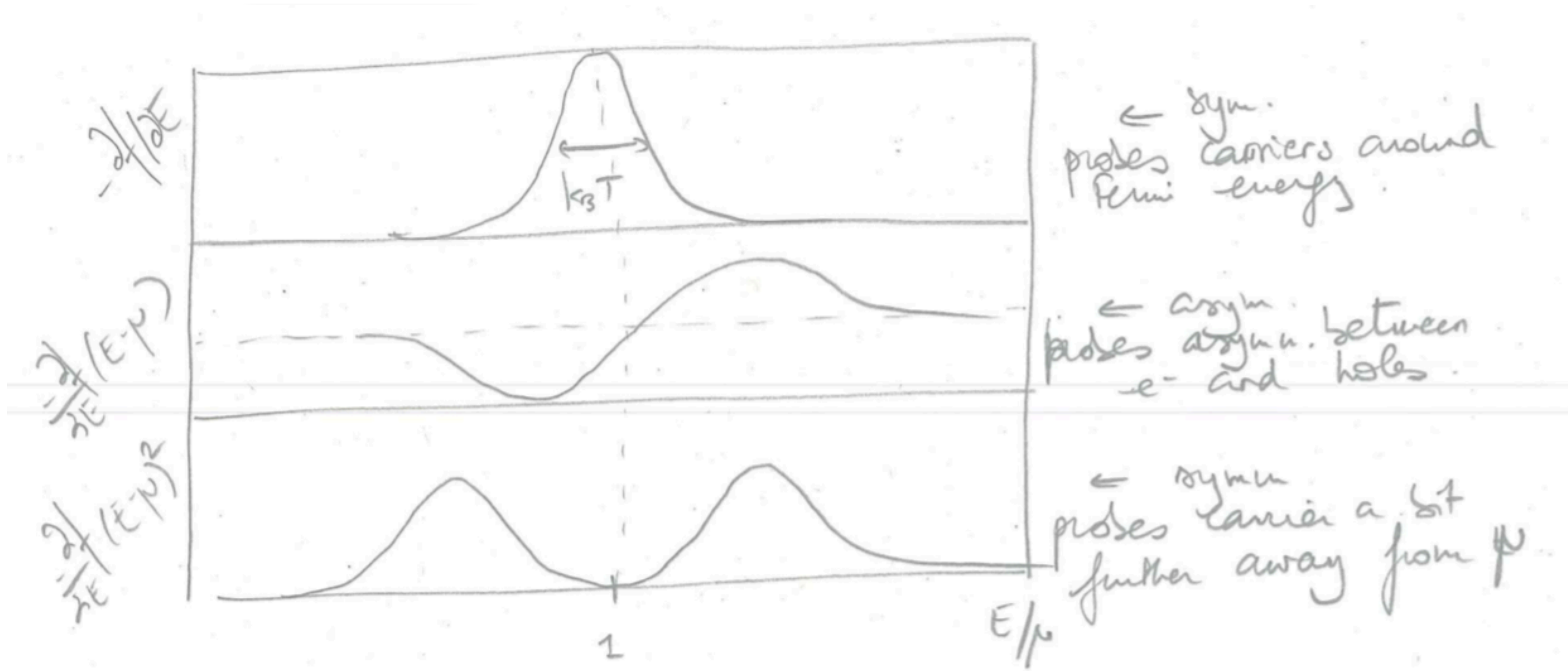
If T(E) smooth enough

$$\frac{K'}{GT_0} = \frac{k_B^2 \Pi^2}{3e^2} \equiv L_o$$

Wiedemann-Franz law

When does it break?

Quantum transport



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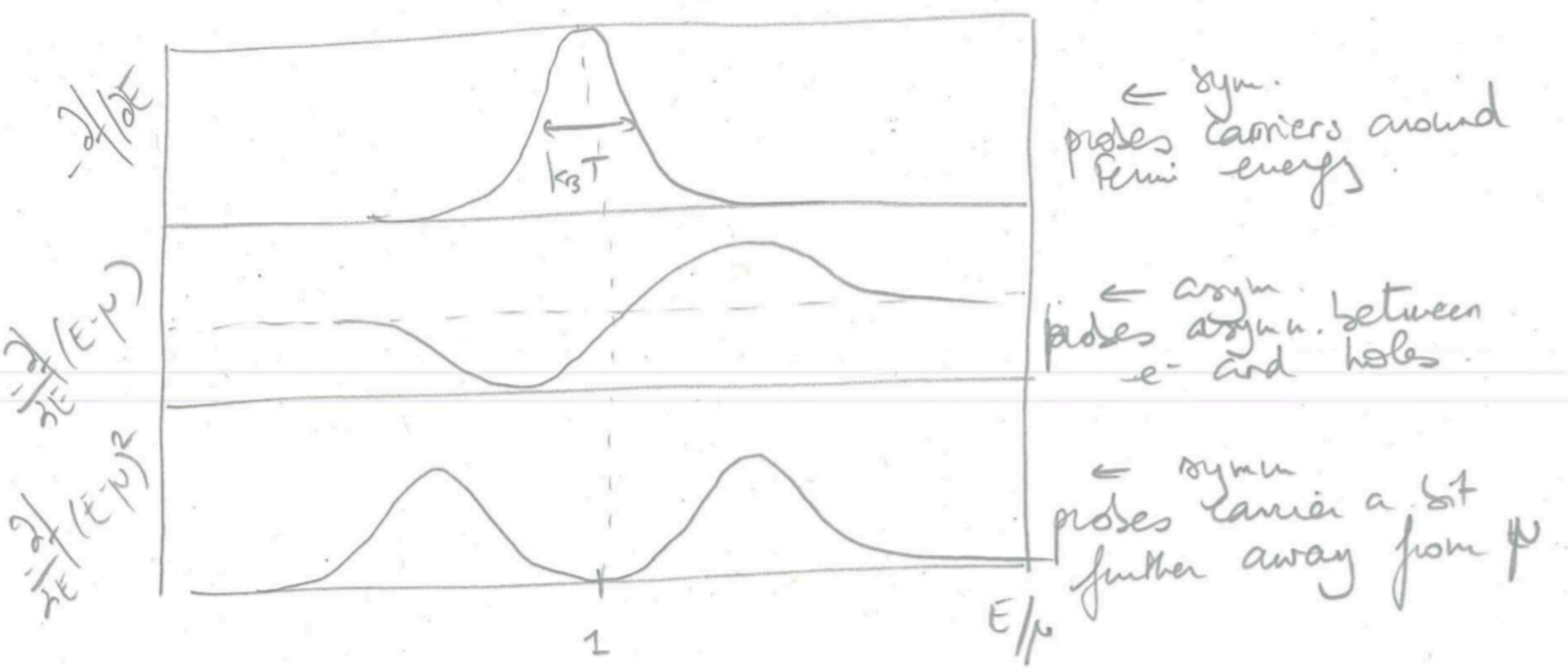

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Non-zero only if T asymmetric in energy

How to design efficient thermoelectric devices?

Thermodynamics of thermoelectricity

Equilibrium thermodynamics

inf. change in free energy:

$$\delta U = T\,\delta S + \mu\, dN + p dV$$

extensive quantities: S, N, V → scale with syst size

intensive quantities: T, μ, p.

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→ ways to enhance free energy:

↑S or ↑ # of particles or ↑V

→ T, μ, P measures of these changes

Let us focus on the pair S/T and μ/N as p/V does not play any role in thermoelectricity.

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Energy flux: $J_u = T J_s + \mu J_N$

E/(s·A)

(current density).

⇒ How does the syst. respond to a spatial variation of temperature or chemical potential?

$$\nabla \cdot J_u = (\nabla T)\, J_s + T(\nabla \cdot J_s) + (\nabla \mu)\, J_N + \mu(\nabla J_N).$$

Thermodynamics of thermoelectricity

- 1st law of thermodynamics:
  energy is conserved: $\nabla \cdot J_u = 0$.
- # of particles is conserved: $\nabla \cdot J_N = 0$.
- entropy flow is not conserved: $\nabla \cdot J_S = \dot{S}$.

$$0 = J_S \cdot \nabla T + T \cdot \dot{S} + \nabla\mu \, J_N$$

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- if $\dot{S} = 0$:

$$\nabla\mu \, J_N = - J_S \nabla T$$

$$\Leftrightarrow \quad J_e \cdot V = - J_h \cdot T \cdot \Delta T$$

$$\Leftrightarrow \quad I V = - J \cdot T \cdot \Delta T .$$

→ each change of potential associated with a particle flux (charge current density) generates a temperature gradient associated with entropy flux to conserve energy.

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Thermopower:
Entropy flow per charge carrier

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Each system that conserves number of particles will give rise to thermoelectric effect

Thermodynamics and Onsager matrix

- Entropy production in terms of currents and thermodynamic forces

$$\dot{s} = J_Q \mathcal{F}_Q + J_e \mathcal{F}_e$$
$$= K' T^2 \mathcal{F}_Q^2 + G T \mathcal{F}_e^2 + 2\alpha T^2 \mathcal{F}_Q \mathcal{F}_e$$

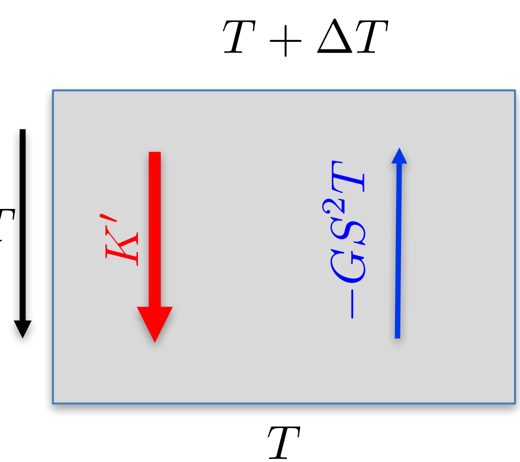
- Second law: valid for all forces

$$G, K' \geq 0$$
$$K' \geq G S^2 T$$

- Figure of merit

$$ZT \equiv \frac{G S^2 T}{K}$$

$$J\Big|_{I=0} = K \Delta T$$



$$I = 0 \Leftrightarrow G V_{th} = -\alpha \Delta T$$
$$V_{th} = S \Delta T$$
$$J = \alpha T V_{th} + K' \Delta T$$
$$= (K' - G S^2 T) \Delta T$$

K. Behnia, Fundamentals of thermoelectricity, Oxford University Press

Good thermoelectric devices

- Energy-asymmetric transmission probability
- Electrons and holes contribute differently
- Low thermal conductance but high electrical conductance
- Tunable

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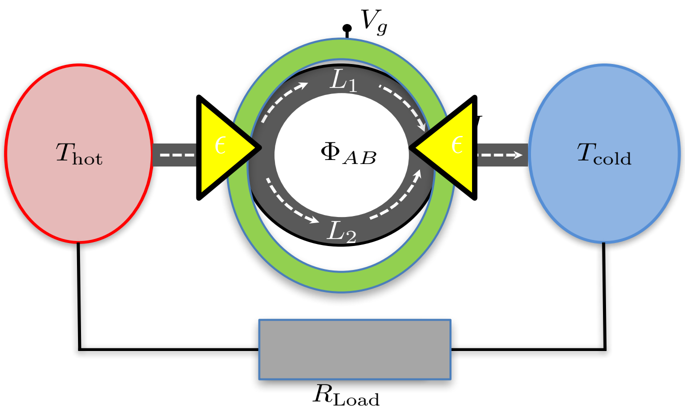
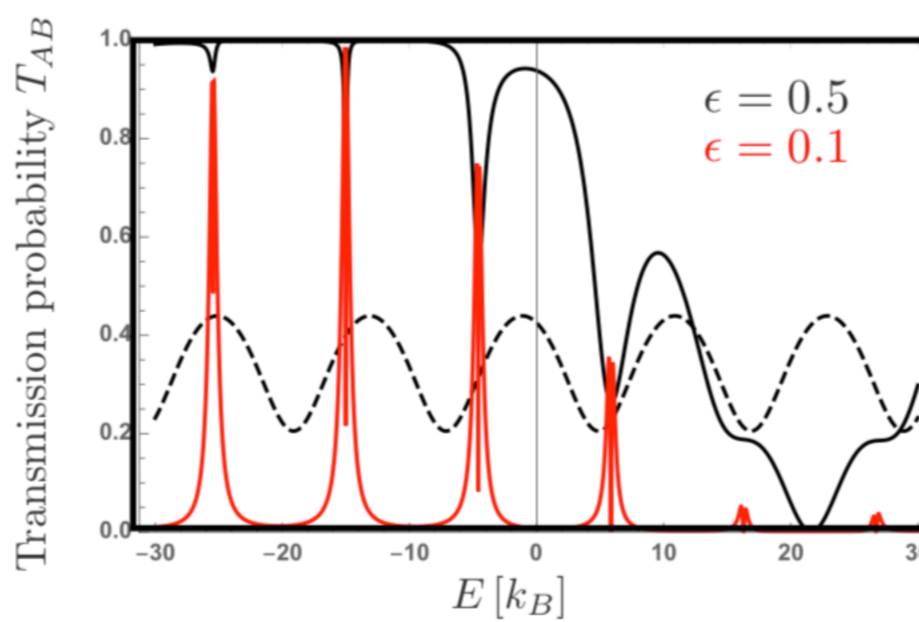
Aharonov-Bohm quantum heat engine

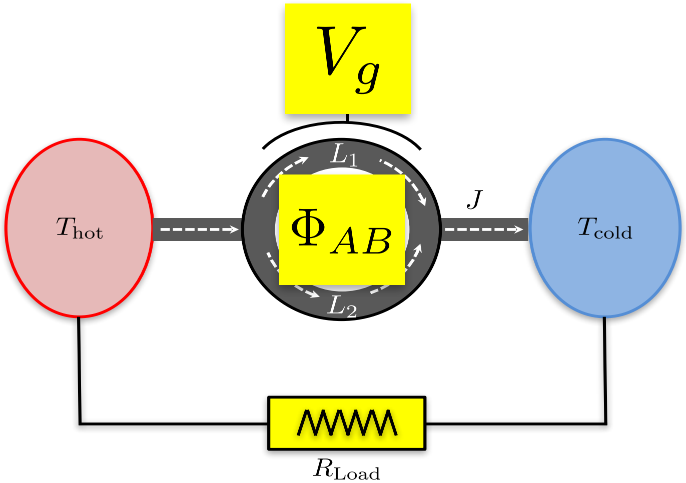
With Francesco Giazotto (Experimentalist, Pisa)

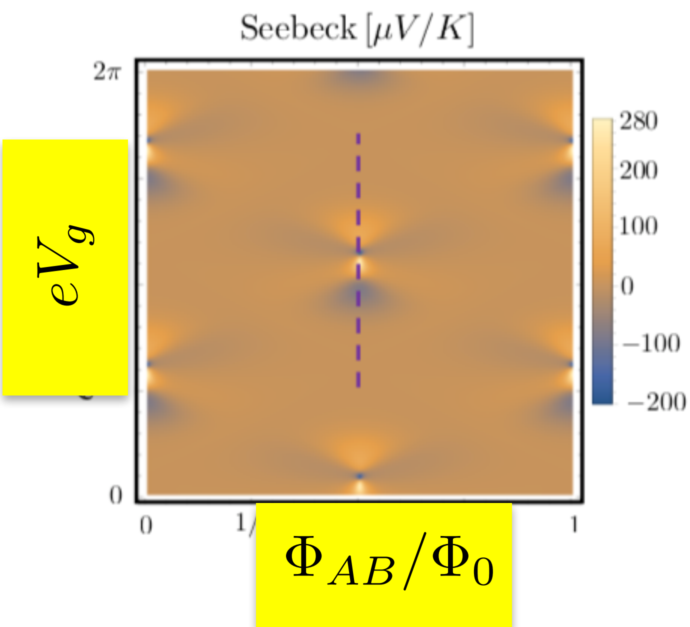




Haack, Giazotto, PRB 100 (2019)

Aharonov-Bohm quantum heat engine





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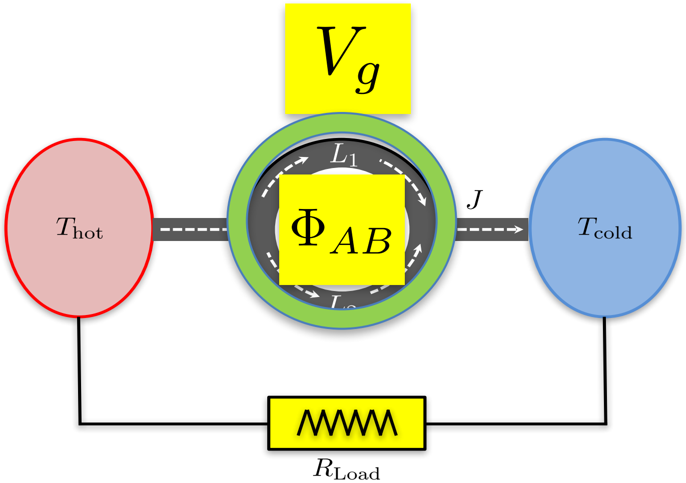
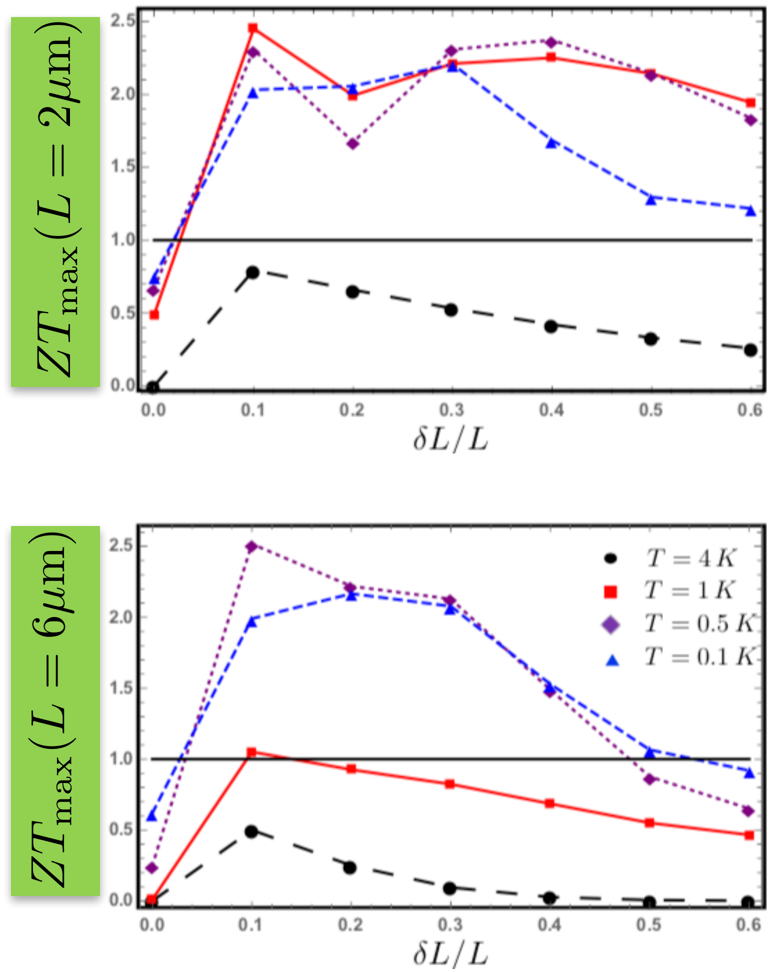
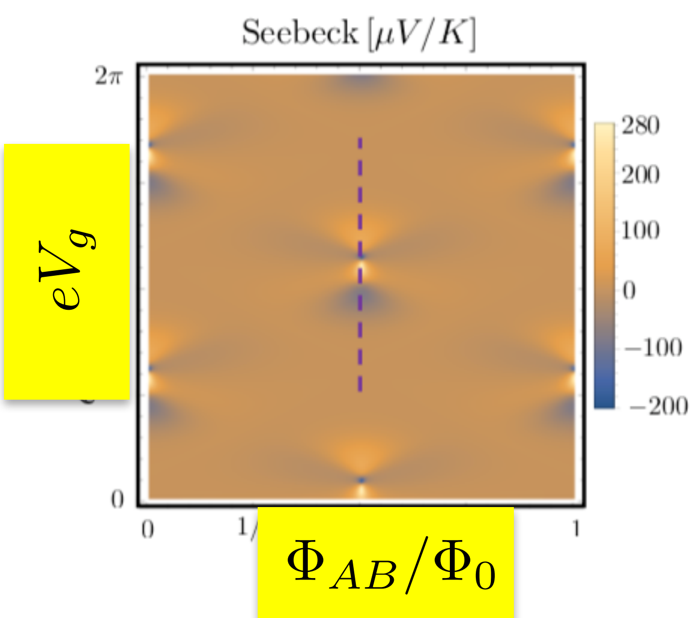


$$ZT = GS^2T/\kappa_{th}$$

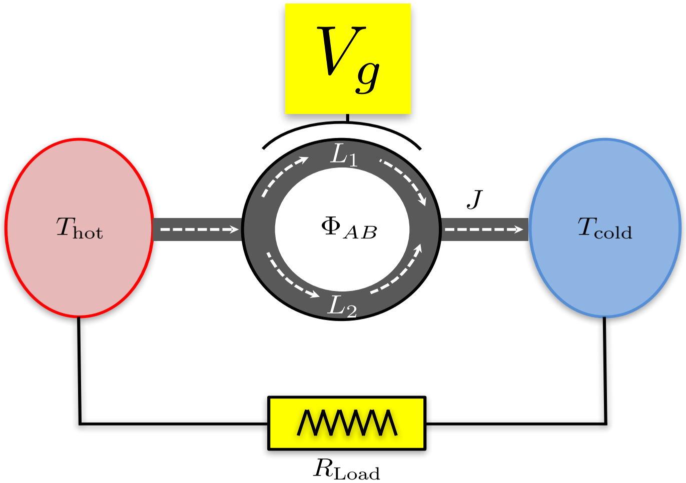




Thermoelectric effects become important: ZT > 1

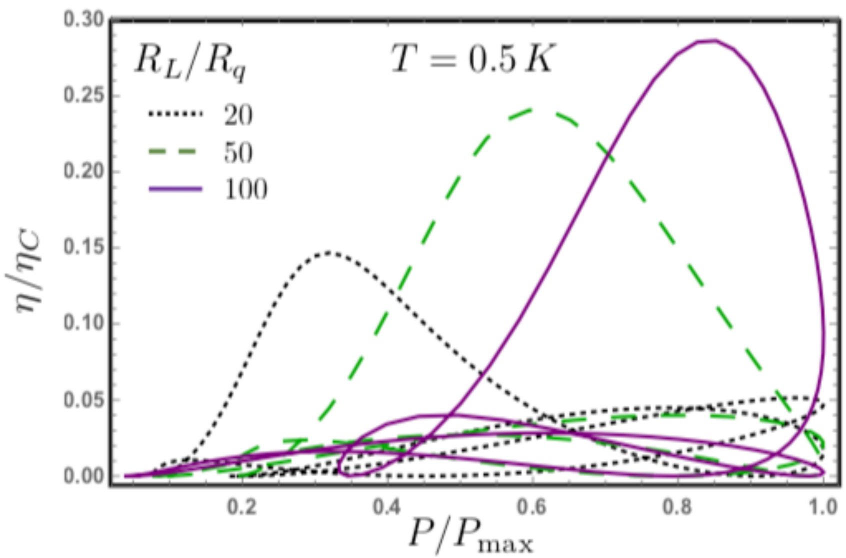
Haack, Giazotto, PRB 100 (2019)

Aharonov-Bohm quantum heat engine

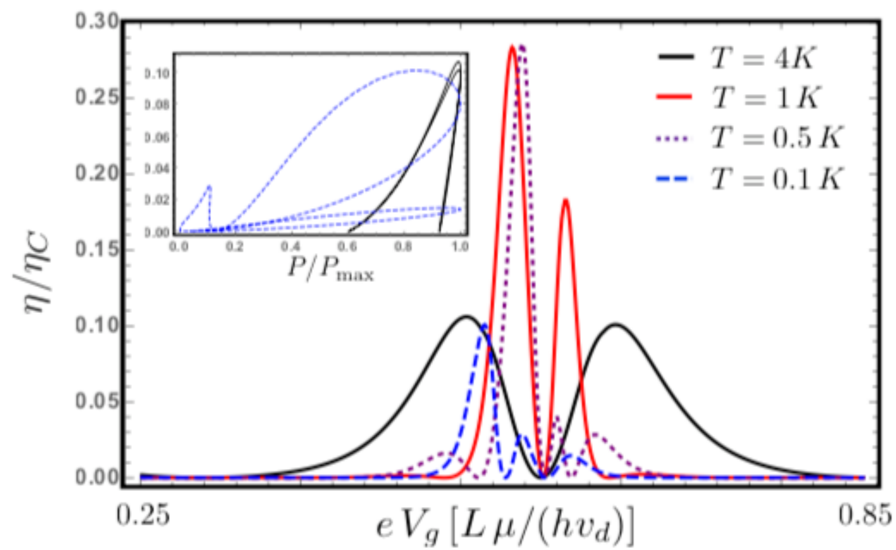


Emblematic phase-coherent mesoscopic device

Promising for experiments



$\eta_{\text{max}} \neq \eta(P_{\text{max}})$



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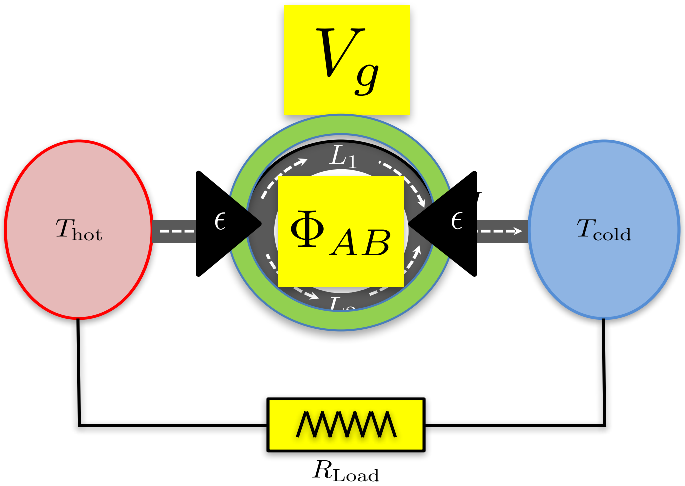
Aharonov-Bohm quantum heat engine

Effect of non-perfectly transmitting T-junctions -> Resonant tunneling



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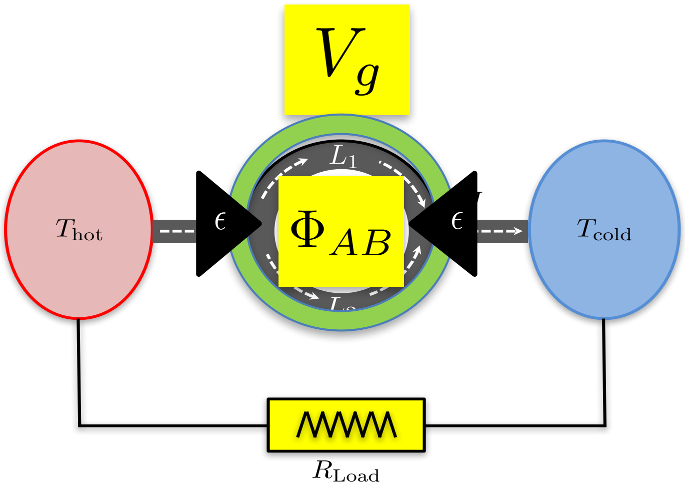
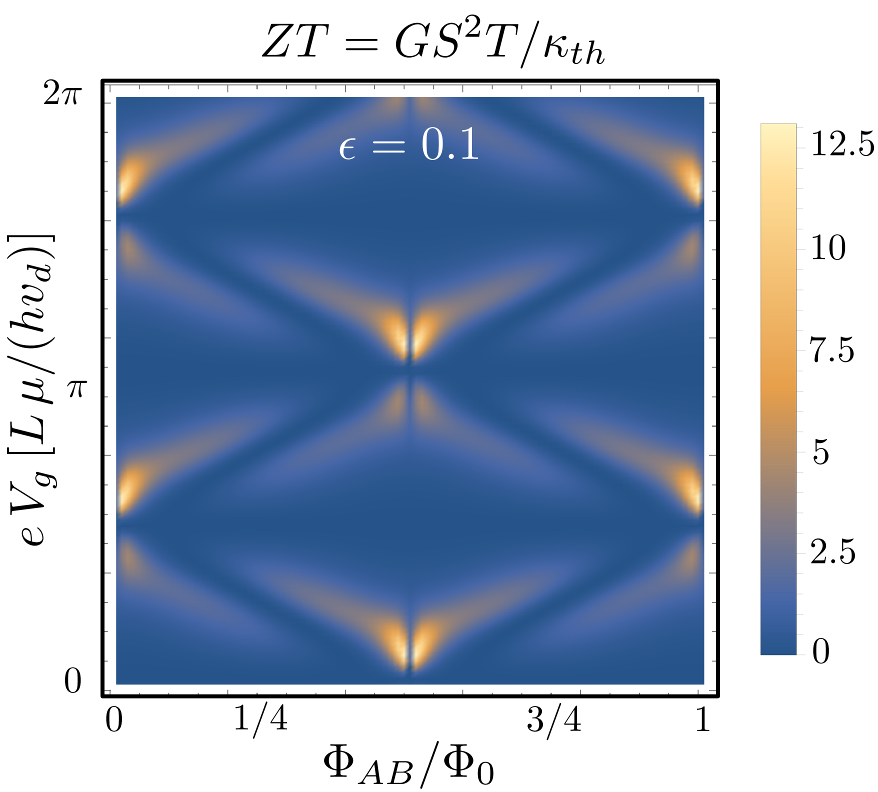
Aharonov-Bohm quantum heat engine

Effect of non-perfectly transmitting T-junctions -> Resonant tunneling





Haack, Giazotto, PRB 100 (2019)

Aharonov-Bohm quantum heat engine

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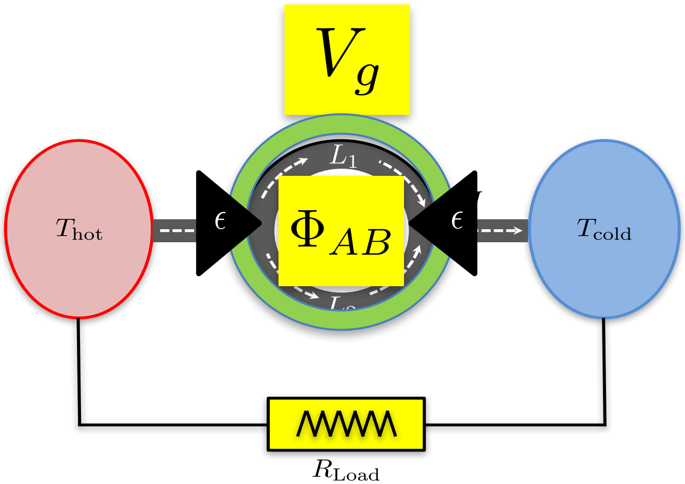
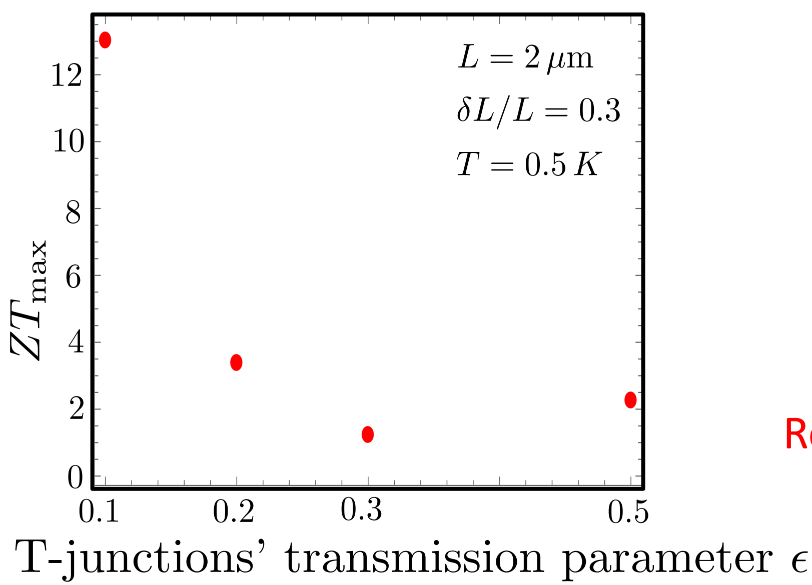
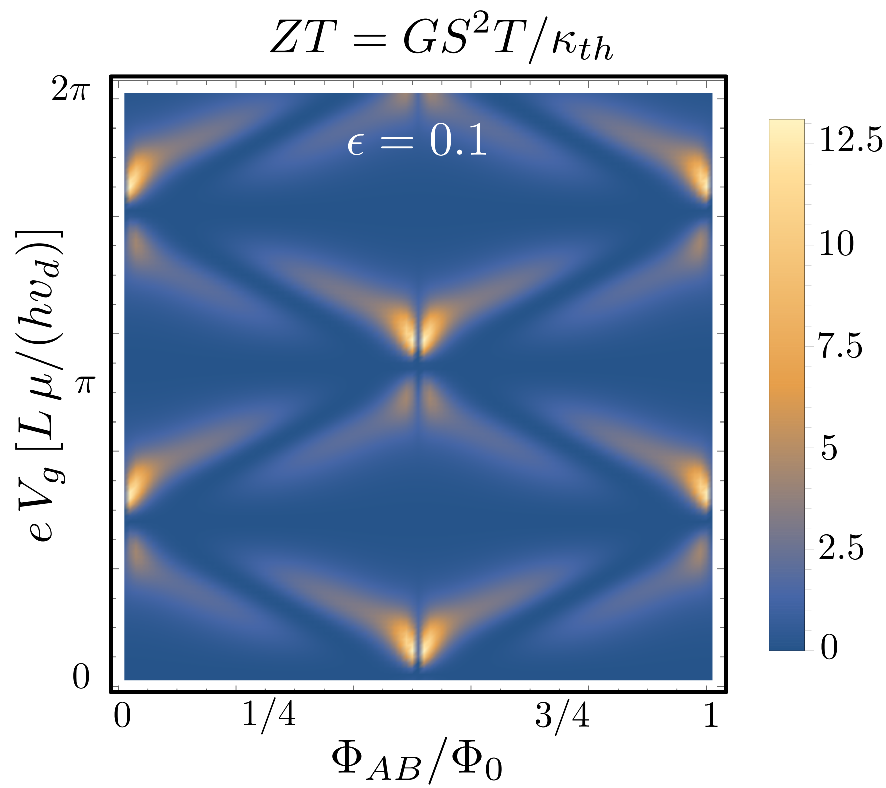


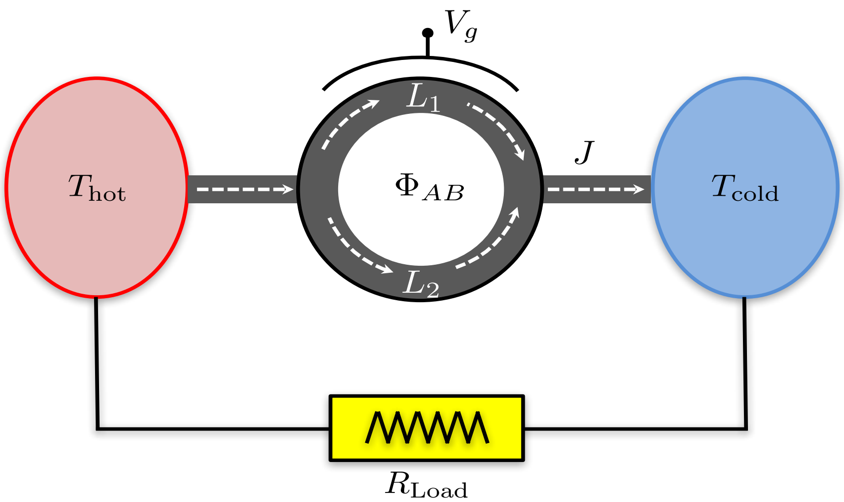


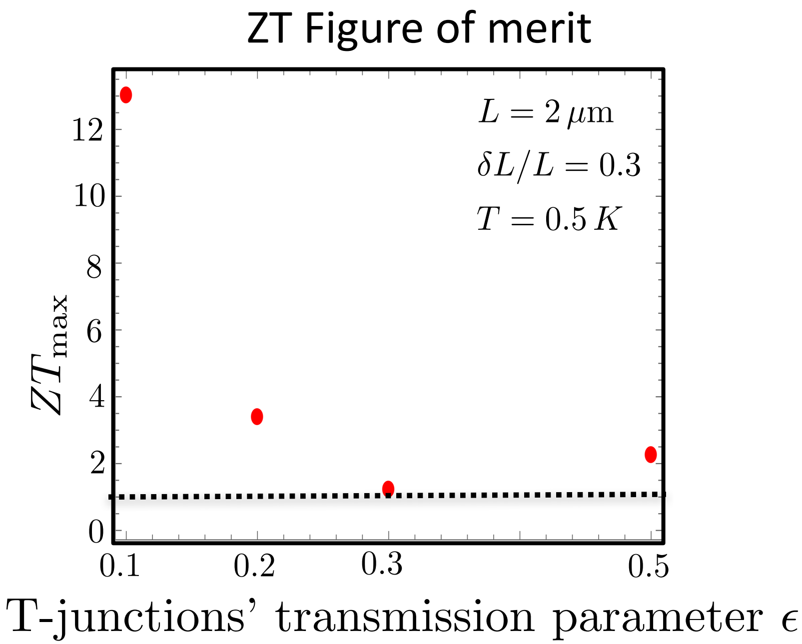


Relies only on single-particle phase-coherent effects

Haack, Giazotto, PRB 100 (2019)

Outlook

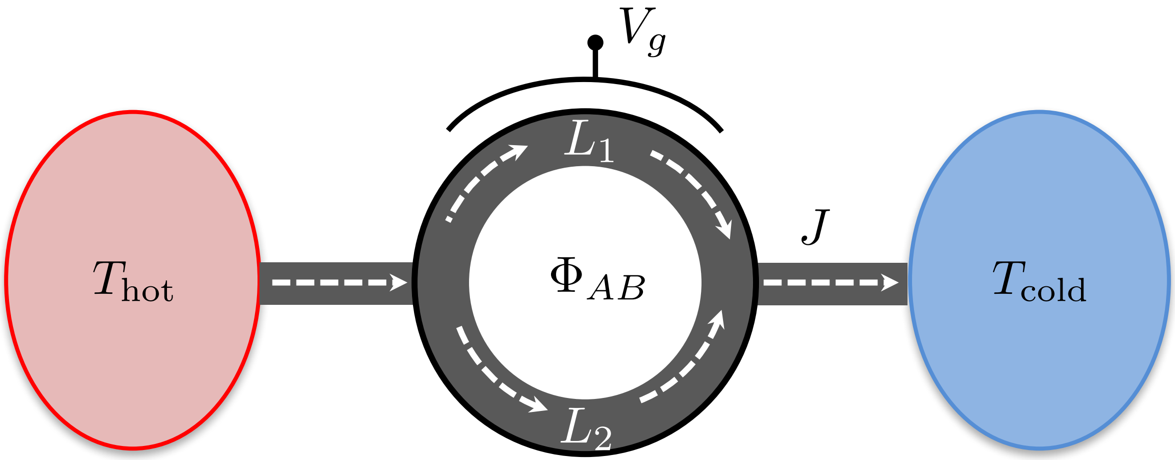




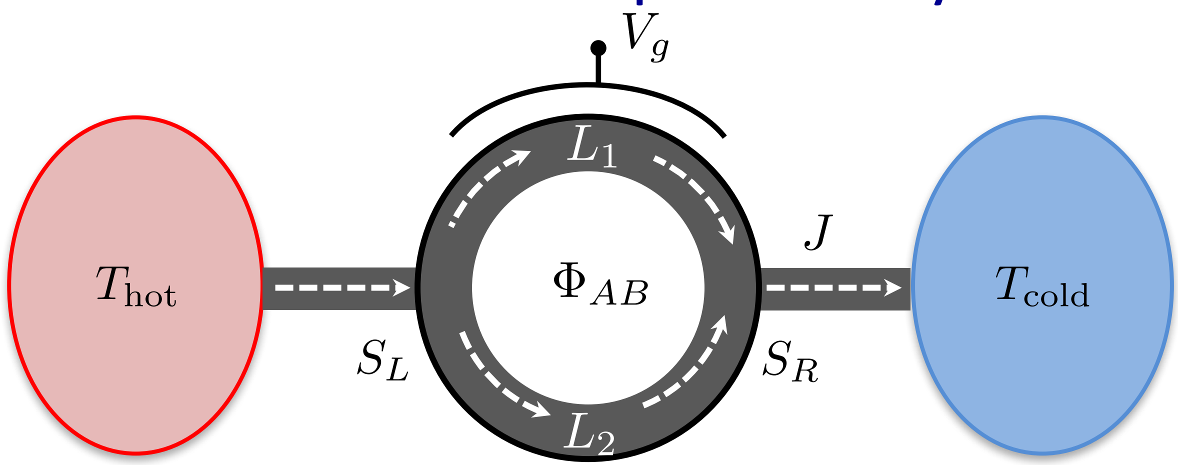
- Non-linear regime
- Autonomous machines
- Versatile platform - > rectifier, thermal diode
- Other applications

Thank you!

Transmission probability



Transmission probability

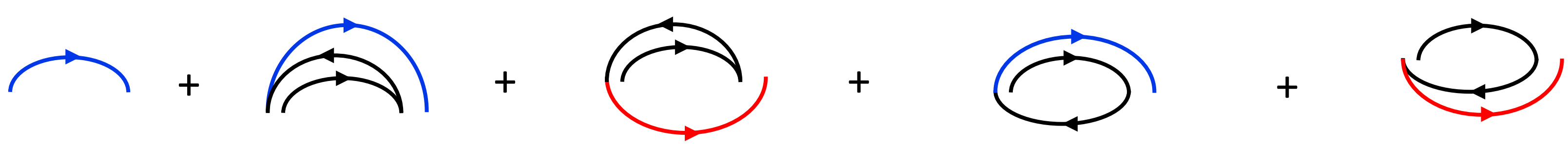


- T-junctions: fully transmitting

$$S_L = \begin{pmatrix} 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/2 & 1/2 \\ 1/\sqrt{2} & 1/2 & -1/2 \end{pmatrix} \quad S_R = \begin{pmatrix} -1/2 & 1/2 & 1/\sqrt{2} \\ 1/2 & -1/2 & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \end{pmatrix}$$

- Different types of trajectories

$t_u =$

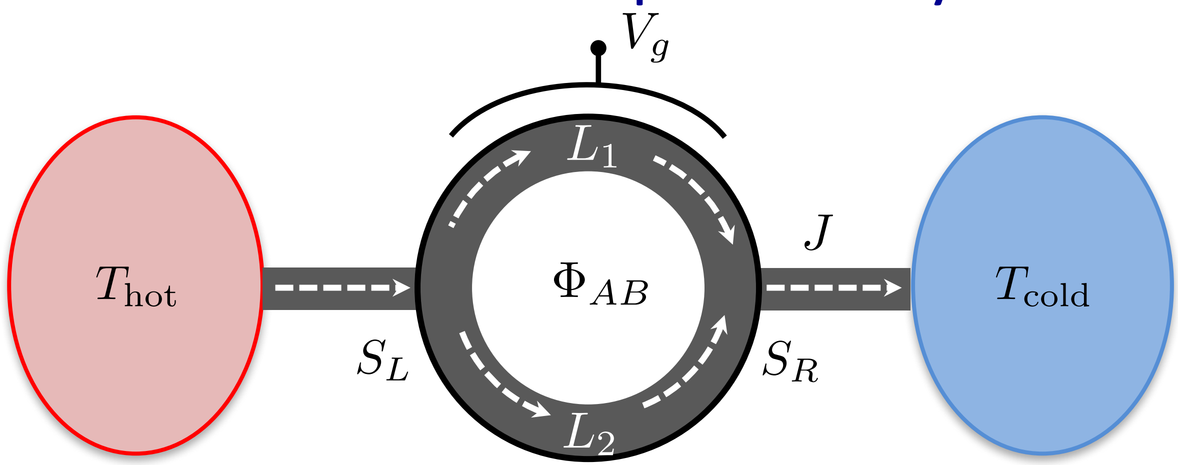


$$t_{AB} = (t_u + t_d)/2 \qquad T_{AB} = |t_{AB}|^2$$

Büttiker et al., PRA 30 (1984)

Nazarov, Blanter "Quantum Transport: Introduction to Nanoscience"

Transmission probability



$$T_{AB} = \frac{1 - \cos\chi \cos\delta\chi + \cos(2\pi\,\Phi_{AB}/\Phi_0)\,(\cos\delta\chi - \cos\chi\,)}{\sin^2\chi + \left(\frac{2(1-\epsilon)\cos\chi - (1-\epsilon-\sqrt{1-2\epsilon})\cos\delta\chi - (1-\epsilon+\sqrt{1-2\epsilon})\cos(2\pi\,\Phi_{AB}/\Phi_0)}{2\epsilon}\right)^2}.$$

- Linearization around the Fermi energy μ

$$\chi = (2L + \delta L)\left(\tilde{k}_\mu + \frac{E-\mu}{\hbar v_d}\right) - \frac{eV_gL}{\hbar v_d}$$

$$\delta\chi = \delta L\left(\tilde{k}_\mu + \frac{E-\mu}{\hbar v_d}\right) + \frac{eV_gL}{\hbar v_d}$$

- Depends on energy E , on arms' imbalance δL , gate voltage V_g and magnetic flux Φ_{AB}

Haack, Giazotto, arXiv:1905.12672