Thermodynamics of Quantum Information Flows

Phys. Rev. Lett. 122, 150603 (2019)

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Quantum Thermodynamics for Young Scientists

Bad Honnef, 2-6 February 2020

Outline

- Topic: relation between the second law of thermodynamics and the information theory
- More specifically: relation between the entropy balance of a subsystem of a Markovian open quantum system and the information flow between the subsystems
- Introduction: second law of thermodynamics for open quantum systems out of equilibrium
- Main result: local Clausius inequality
- Thermodynamics of information flow
- Derivation of the local Clausius inequality
- Application: autonomous quantum Maxwell demon

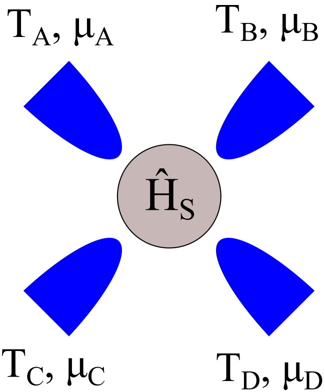
Second law of thermodynamics for open systems

- Second law of thermodynamics → thermodynamics irreversibility
- For open systems out of equilibrium [M. Esposito *et al.*, New J. Phys. 12, 013013 (2010)]

$$\sigma = \Delta S - \sum_{\alpha} \beta_\alpha Q_\alpha \geq 0$$

where

- σ – entropy production
- $S = -\mathrm{Tr}(\rho \ln \rho)$ – von Neumann entropy of the system
- Q_α – heat delivered from the reservoir α



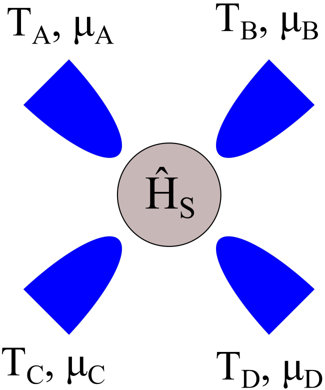
Markovian systems

- Markovian system – no memory effects due to correlation with environment – for weak coupling
- Second law in differential form

$$\dot{\sigma} = d_t S - \sum_{\alpha} \beta_{\alpha} \dot{Q}_{\alpha} \geq 0$$

where

  - $\dot{\sigma}$ – entropy production rate
  - $S = -\mathrm{Tr}(\rho \ln \rho)$ – von Neumann entropy of the system
  - $\dot{Q}_{\alpha}$ – heat flow from the reservoir α



Main result: local Clausius inequality

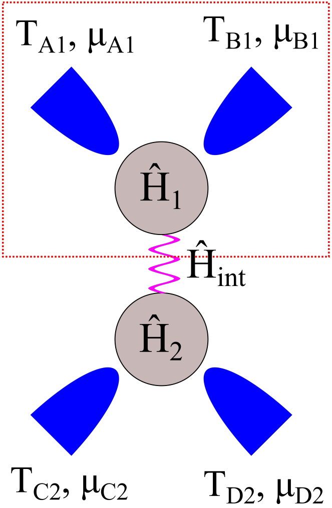
$$\hat{H}_S = \sum_i \hat{H}_i + \hat{H}_{\text{int}}$$

- Can we define 2nd law for a single subsystem? <u>Yes, we can!</u>
- **Local Clausius inequality**

$$\dot{\sigma}_i = d_t S_i - \sum_{\alpha_i} \beta_{\alpha_i} \dot{Q}_{\alpha_i} \boxed{-\dot{I}_i} \geq 0$$

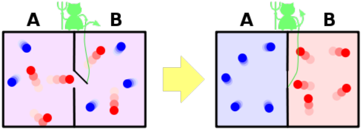
where

  - $\dot{\sigma}_i$ – local entropy production rate
  - $S_i = -\mathrm{Tr}(\rho_i \ln \rho_i)$ – von Neumann entropy of the subsystem i
  - $\dot{Q}_{\alpha_i}$ – heat flow from reservoir α_i
  - $\dot{I}_i$ – information flow between the subsystems (defined later)



Maxwell demons

- Maxwell demon – entropy of a stochastic system can be reduced by a feedback control by an intelligent being



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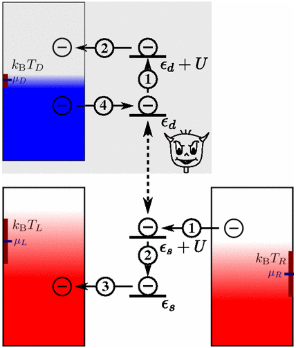
- Experimental realizations
  - Colloidal particles [S. Toyabe *et al.*, Nat. Phys. 6, 988–992 (2010)]
  - Single-electron boxes [J. V. Koski *et al.*, Phys. Rev. Lett. 113, 030601 (2014)]
  - Superconducting circuits [Y. Masuyama *et al.*, Nat. Commmun. 9, 1291 (2018)]
- Generalized second laws [e.g. T. Sagawa, M. Ueda, Phys. Rev. Lett. 100, 080403 (2008)]

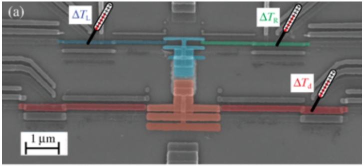
$$\Delta S - \Delta I > 0$$

where ΔI – information collected by the feedback device

Autonomous Maxwell demons

- Autonomous Maxwell demon – entropy of a stochastic system can be reduced by a feedback control by another stochastic system
- Theoretical proposal – Coulomb coupled quantum dots – P. Strasberg *et al.*, Phys. Rev. Lett. 110, 040601 (2013)
- Experimental realization – J. V. Koski *et al.*, Phys. Rev. Lett. 115, 260602 (2015)
- Theoretical framework for classical systems based on stochastic thermodynamics – J. M. Horowitz, M. Esposito, Phys. Rev. X 4, 031015 (2014)





Theoretical framework – bipartite dynamics

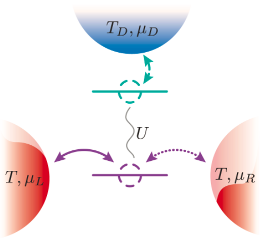
- Two subsystems: X and Y
- Classical rate equation for state probabilities

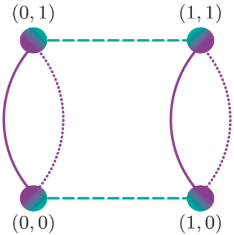
$$\dot{p}(x,y) = \sum_{x',y'} \left[W_{x,x'}^{y,y'} p(x',y') - W_{x',x}^{y',y} p(x,y) \right]$$

$W_{x,x'}^{y,y'}$ – rate of transition $(x',y') \to (x,y)$

- Transitions – either in X or Y, not simultaneous

$$W_{x,x'}^{y,y'} = \begin{cases} w_{x,x'}^{y} & x \neq x; y = y' \\ w_{x}^{y,y'} & x = x', y \neq y' \\ 0 & x \neq x', y \neq y' \end{cases}$$





J. M. Horowitz, M. Esposito, Phys. Rev. X **4**, 031015 (2014)

Local second law of thermodynamics

$$I = \sum_{x,y} p(x,y) \ln \frac{p(x,y)}{p(x)p(y)}$$

- Mutual information – measure of correlation between subsystems
- Decomposition: $d_t I = \dot{I}_X + \dot{I}_Y$

$$\dot{I}_X = \sum_{x \geq x';y} \left[w^y_{x,x'} p(x',y) - w^y_{x',x} p(x,y) \right] \ln \frac{p(y|x)}{p(y|x')}$$

$$p(x|y) = p(x,y) / \sum_{y'} p(x,y')$$

$$\boxed{\dot{\sigma}_i = \dot{H}_i - \beta_i \dot{Q}_i - \dot{I}_i \geq 0}$$

- All already done? **Not yet**

Limitations of the previous approach

- Previous approach – limited to restricted class of systems
- Only for non-coherent systems (without unitary dynamics)
- Even in classical limit – not applicable to systems with $[\hat{H}_{\text{int}}, \hat{H}_i] \neq 0$
  - Rate equations describe transtions between eigenstates of the total Hamiltonian $\hat{H}_S$
  - For non-commuting Hamiltonians – eigenstates of $\hat{H}_S$ are not products of eigenstates of subsystem Hamiltonians $\hat{H}_i$
  - As a result, rate equations do not have a bipartite structure
- **Our result**: generalizes the concept of autonomous information flow to a generic Markovian open quantum system

Derivation: Assumptions

- Dynamics described by Lindblad equation (internal unitary dynamics + dissipation)

$$d_t\rho = -i\left[\hat{H}^{\text{eff}}, \rho\right] + \mathcal{D}\rho$$

- Additivity of dissipation – interaction with each reservoir gives an independent contribution to the dissipation – valid for Markovian systems

$$\mathcal{D} = \sum \mathcal{D}_\alpha$$

- Local equilibration

$$\mathcal{D}_\alpha \rho_\alpha^{\text{eq}} = 0$$
$$\rho_\alpha^{\text{eq}} = Z_\alpha^{-1} e^{-\beta_\alpha \left(\hat{H}_S - \mu_\alpha \hat{N}\right)}$$

where ρ_α^{eq} – Gibbs state with respect to α

Partial Clausius inequality

- Applying Spohn's inequality [H. Spohn, J. Math. Phys. (N.Y.) 19, 1227 (1978)]

$$-\text{Tr}\left[(\mathcal{D}^\alpha \rho)\left(\ln \rho - \ln \rho_{\text{eq}}^\alpha\right)\right] \geq 0$$

  one obtains the **partial Clausius inequality** [G. B. Cuetara, M. Esposito, G. Schaller, Entropy 18, 447 (2016)]

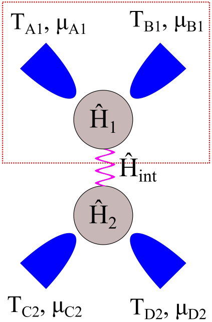
$$\boxed{\dot{\sigma}_\alpha = \dot{S}^\alpha - \beta_\alpha \dot{Q}_\alpha \geq 0}$$

  where
    - $\dot{\sigma}_\alpha$ – partial entropy production rate
    - $\dot{S}^\alpha = -\text{Tr}\left[(\mathcal{D}^\alpha \rho) \ln \rho\right]$ – rate of change of the von Neumann entropy due to interaction with the reservoir α
    - $\dot{Q}_\alpha = \text{Tr}\left[(\mathcal{D}^\alpha \rho)\left(\hat{H}_S - \mu_\alpha \hat{N}\right)\right]$ – heat flow from the reservoir α
- **Meaning**: interaction with each reservoir gives a non-negative contribution to the entropy production

Local Clausius inequality

$$\dot{\sigma}_\alpha = \dot{S}^\alpha - \beta_\alpha \dot{Q}_\alpha \geq 0$$

- Local entropy production rate – sum of $\dot{\sigma}_{\alpha_i}$ associated with reservoirs α_i coupled to subsystem i



$$\dot{\sigma}_i = \sum_{\alpha_i} \dot{\sigma}_{\alpha_i} = \sum_{\alpha_i} \dot{S}^{\alpha_i} - \sum_{\alpha_i} \beta_{\alpha_i} \dot{Q}_{\alpha_i} =$$

$$d_t S_i \underbrace{\boxed{-d_t S_i + \sum_{\alpha_i} \dot{S}^{\alpha_i}}}_{-\dot{I}_i} - \sum_{\alpha_i} \beta_{\alpha_i} \dot{Q}_{\alpha_i} \geq 0$$

- We obtain inequality analogous to the result of Horowitz and Esposito

$$\boxed{\dot{\sigma}_i = d_t S_i - \sum_{\alpha_i} \beta_{\alpha_i} \dot{Q}_{\alpha_i} - \dot{I}_i \geq 0}$$

What is $\dot{I}_i$? – Information flow

- What is a physical meaning of $\dot{I}_i$? Is it related to information flow?
- Answer: yes, it is!

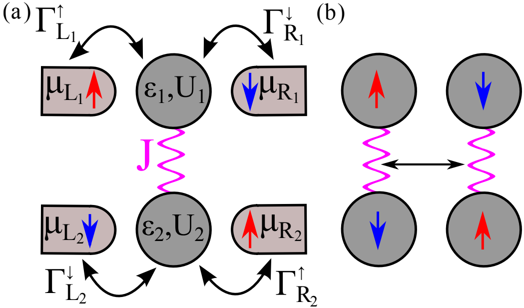
$$\sum_i \dot{I}_i = d_t I$$

where $I = \sum_i S_i - S$ – (multipartite) mutual information between subsystems

- In the classical limit (secular approximation) with $[\hat{H}_{\text{int}}, \hat{H}_i] = 0$ our inequality is equivalent to the result of Horowitz and Esposito

Autonomous quantum Maxwell's demon

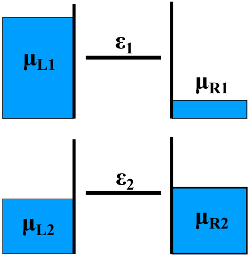
$$\hat{H}_S = \sum_{i\in\{1,2\}} \sum_{\sigma\in\{\uparrow,\downarrow\}} \epsilon_i c^\dagger_{i\sigma} c_{i\sigma} + \sum_{i\in\{1,2\}} U_i n_{i\uparrow} n_{i\downarrow} + J(\hat{S}^x_1 \hat{S}^x_2 + \hat{S}^y_1 \hat{S}^y_2)$$

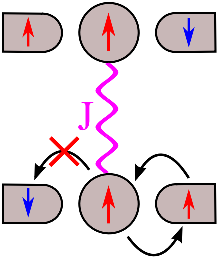


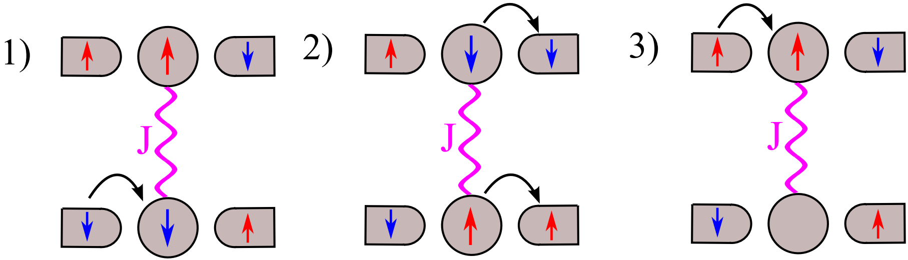
K. Ptaszyński, Phys. Rev. E **97**, 012116 (2018)

- Operation based on coherent spin exchange + spin selective dissipative dynamics (polarized leads)
- Essentially non-bipartite dynamics: spin exchange simultaneously flip spins in both dots; $[\hat{H}_{\text{int}}, \hat{H}_i] \neq 0$
- Could not be described by previously existing approaches

Demon: operation

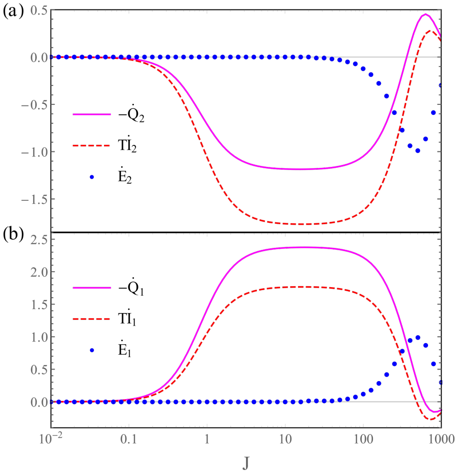






Results

- For $J \lessapprox 100$ "pure" Maxwell demon:
  - subsystem 2 cooled $(-\dot{Q}_2 < 0)$...
  - ...with a negligible energy flow $\dot{E}_i \approx 0$...
  - ...due to information flow $(-\dot{Q}_2 > T\dot{I}_2)$ – conversion of heat into work due to feedback control
  - This is compensated by heat dissipation in the first dot $(-Q_1 > T\dot{I}_1 > 0)$



$T = 100,\ V_1 = 60,\ V_2 = -30$

Conclusions

- We derived an inequality describing local entropy and information balance of a subsystem of a Markovian open quantum systems coupled to several reservoirs
- This provides a consistent mathematical description of thermodynamics of autonomous information flow in quantum systems
- Applicability of our approach was demonstrated on the example of an autonomous quantum Maxwell demon

Acknowledgments

- K. P. is supported by the National Science Centre, Poland, under the project Opus 11 (No. 2016/21/B/ST3/02160) and the doctoral scholarship Etiuda 6 (No. 2018/28/T/ST3/00154). M. E. is supported by the European Research Council project NanoThermo (ERC-2015-CoG Agreement No. 681456).

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