Dissipation close to equilibrium

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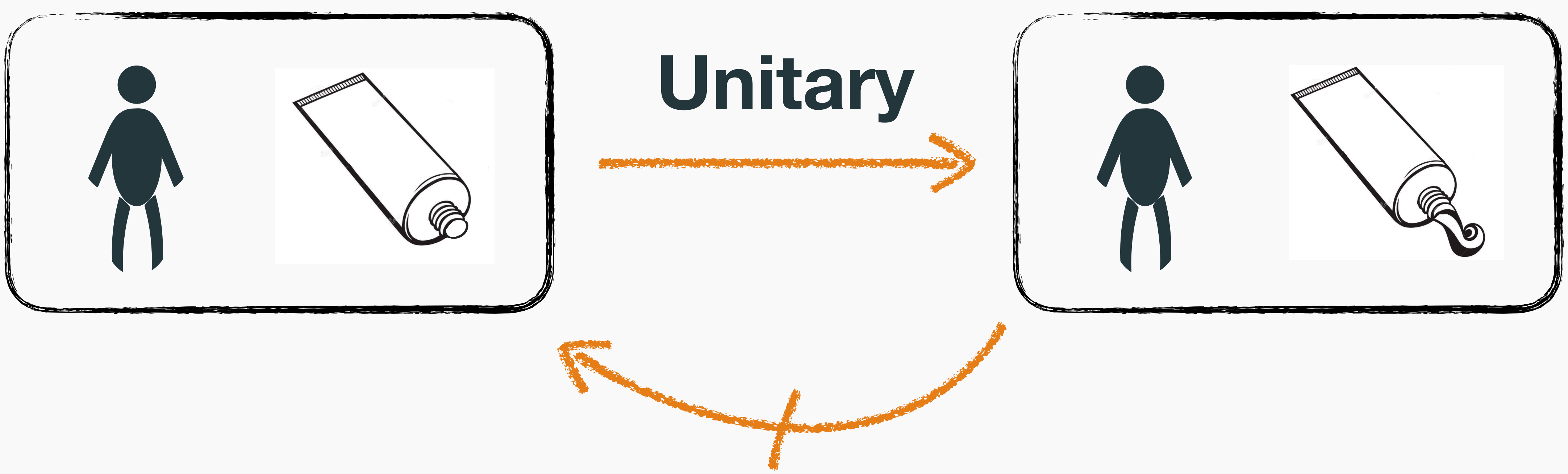
Irreversibility

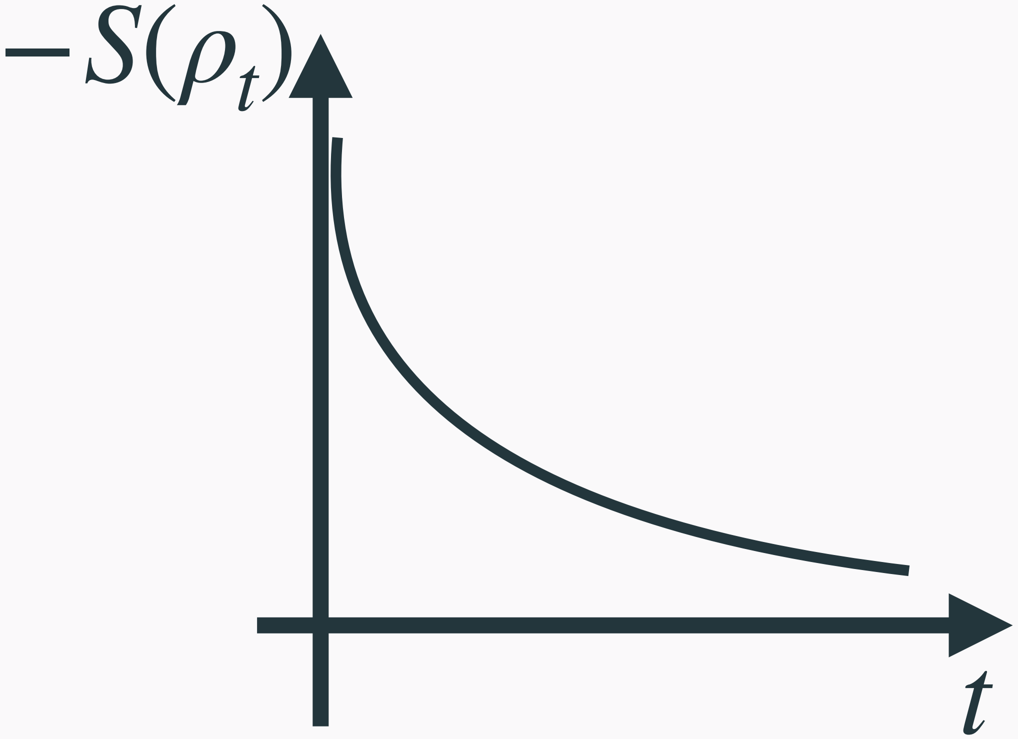
The arrow of time



Entropy: why you can't get the toothpaste back in the tube[1]

[1] Whatever works, W. Allen (2009)

Second law of thermodynamics



<u>H theorem:</u> $-S(\rho_t)$ **decreases monotonically**

Quantum second laws

$\pi :=$ **Gibbs state**

$\longrightarrow$ **measure of athermality**

H theorem $\longrightarrow$

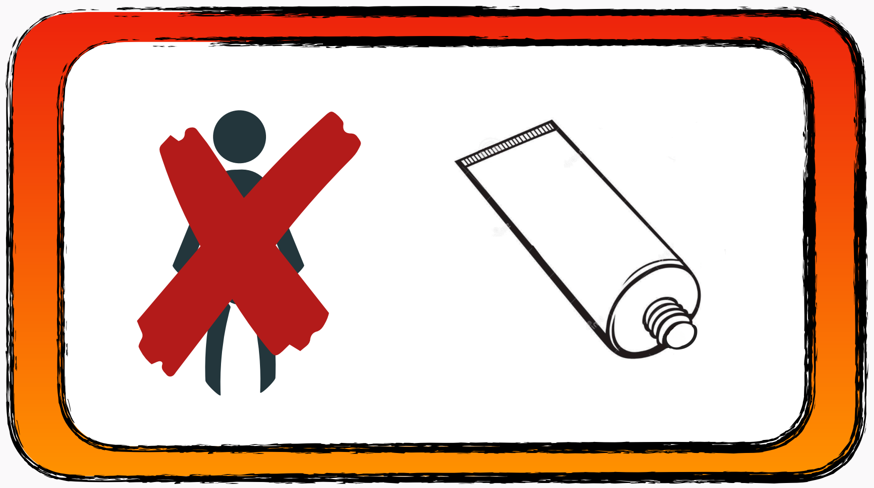
$S_\lambda(\rho \,||\, \pi)$ **decreases**

$S_\lambda(\rho \,||\, \mathscr{D}(\rho))$ **decreases**

$\mathscr{D}(\rho) :=$ **diagonal state**

$\longrightarrow$ **measure of coherence**

Framework



$$\rho_S = \mathrm{Tr}_{U \setminus S}[\rho]$$

$$H(t) = H_S(t) + \boldsymbol{gV} + \boldsymbol{H_B}$$

$$\beta Q = \Delta S - \Sigma$$

$$w = \Delta F + w_{diss}$$

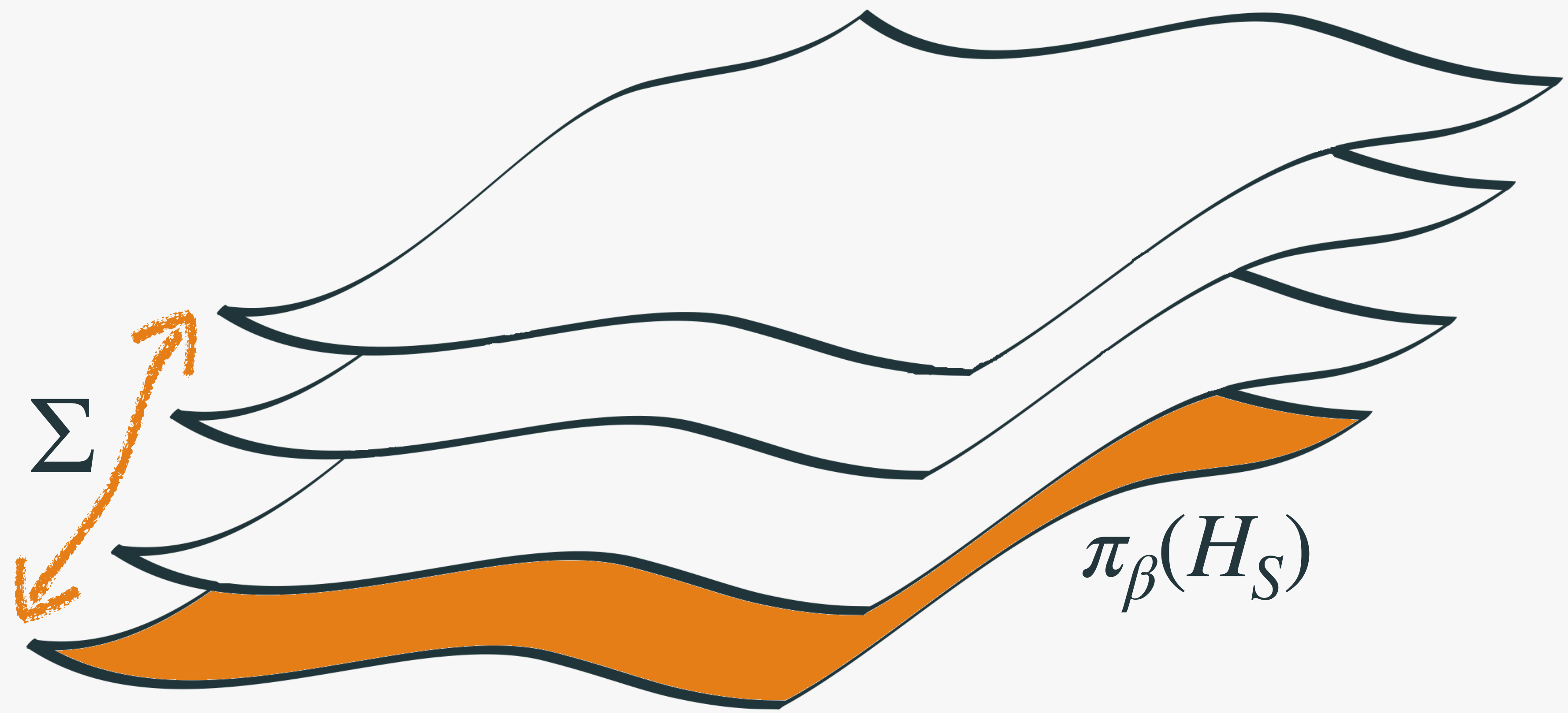
First Law

$$\Sigma = \beta w_{diss}$$

Thermodynamic space

$$H(t) = H_S(t) + gV + H_B$$

(H_S, ρ_S)



$$\rho_S \sim \pi_\beta(H_S)$$

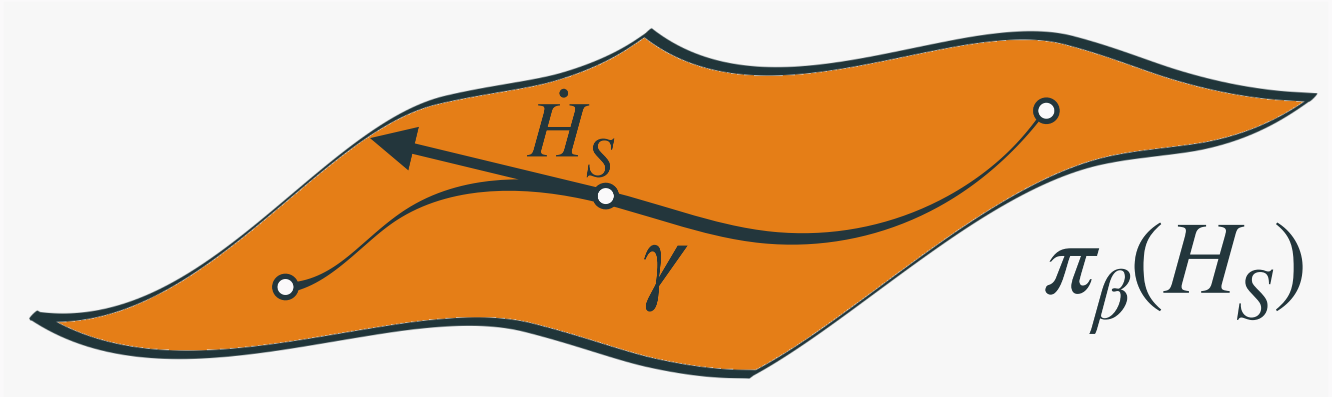
$$\Sigma \sim x^i \, (\partial_i \partial_j \Sigma) \, x^j$$

Close to equilibrium

Average dissipation[1]

$$\dot\rho_S = \mathscr{L}_t[\rho_S]$$

$$T \gg \tau$$



$$\langle w_{diss} \rangle = -\frac{1}{T}\int_\gamma \int_0^1 \mathrm{d}y\, \mathrm{cov}_t^y(\dot H_t, (\mathscr{L}_t^+)^\dagger[\dot H_t])$$

$$\mathrm{cov}_t^y(A,B) = \mathrm{Tr}[\pi_t^{1-y} A\, \pi_t^y\, B] - \mathrm{Tr}[\pi_t A]\mathrm{Tr}[\pi_t B]$$

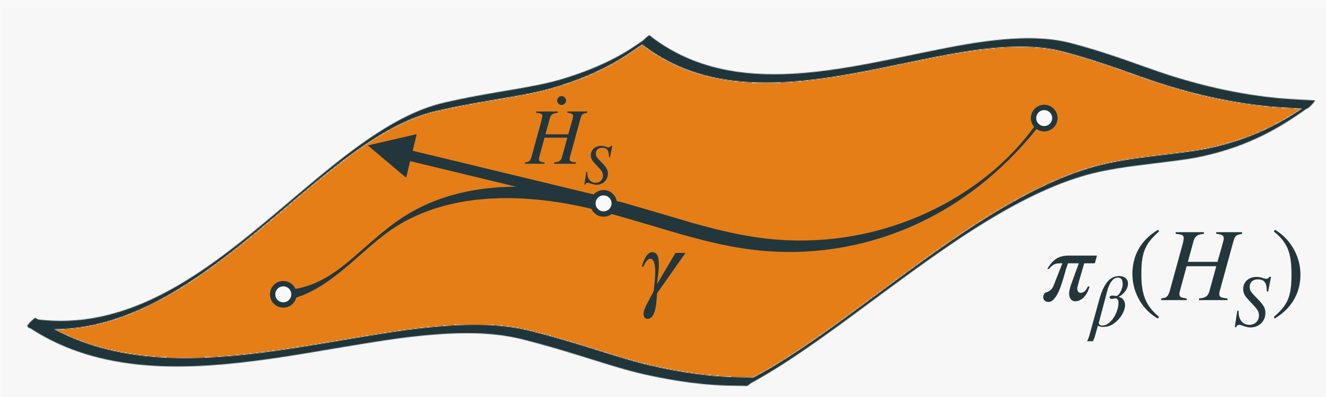
[1] M. Scandi, M. Perarnau-Llobet. arXiv:1810.05583

Thermodynamic metric[1]

$$\int_\gamma \int_0^1 \mathrm{d}y\, \mathrm{cov}_t^y(\dot{H}_t, (\mathscr{L}_t^+)^\dagger[\dot{H}_t])$$

- **Symmetric**
- ≥ 0
- **Smooth**

Metric

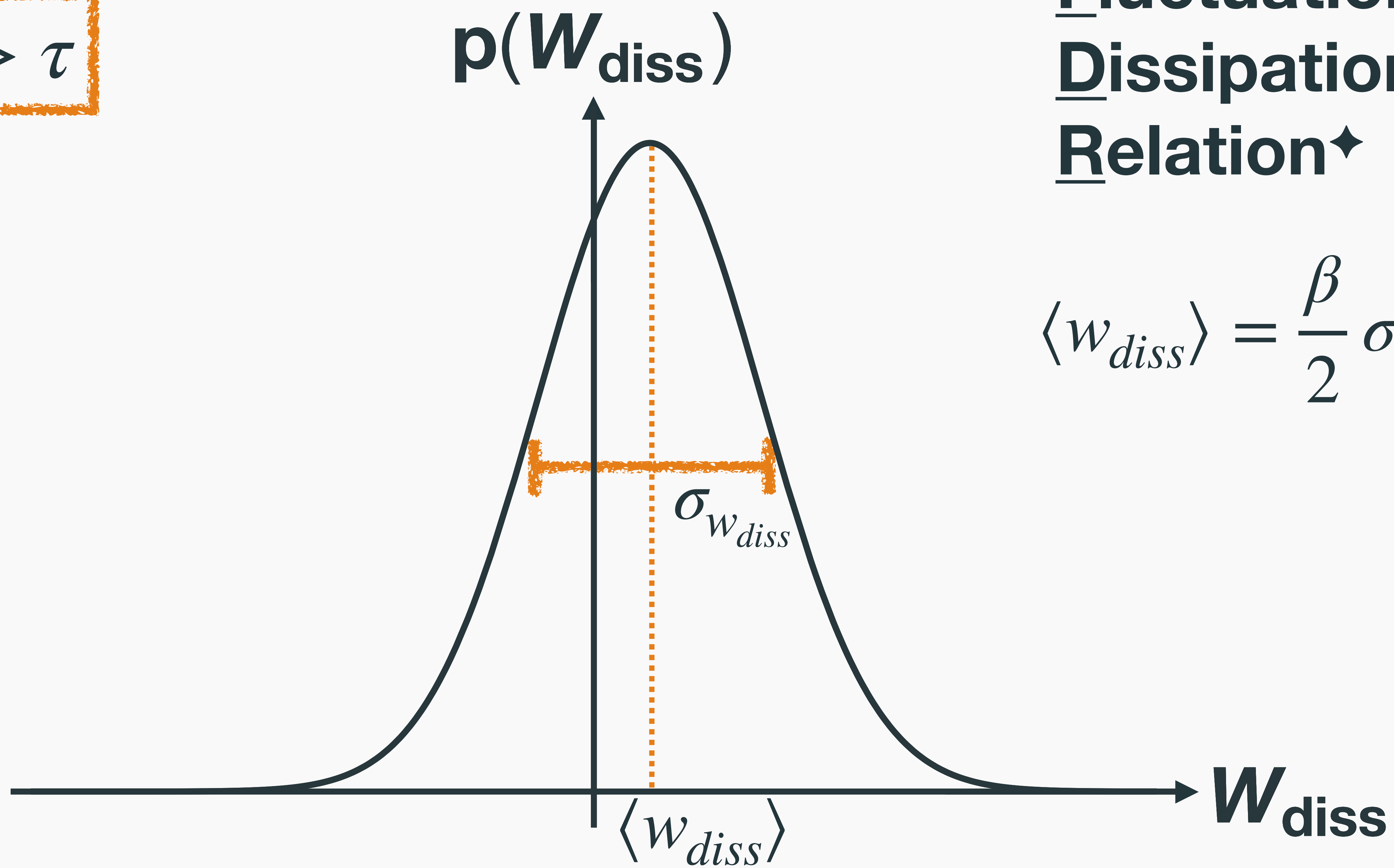


Geodesics minimise dissipation

[1] M. Scandi, M. Perarnau-Llobet. arXiv:1810.05583

Classical case

$T \gg \tau$



Gaussianity

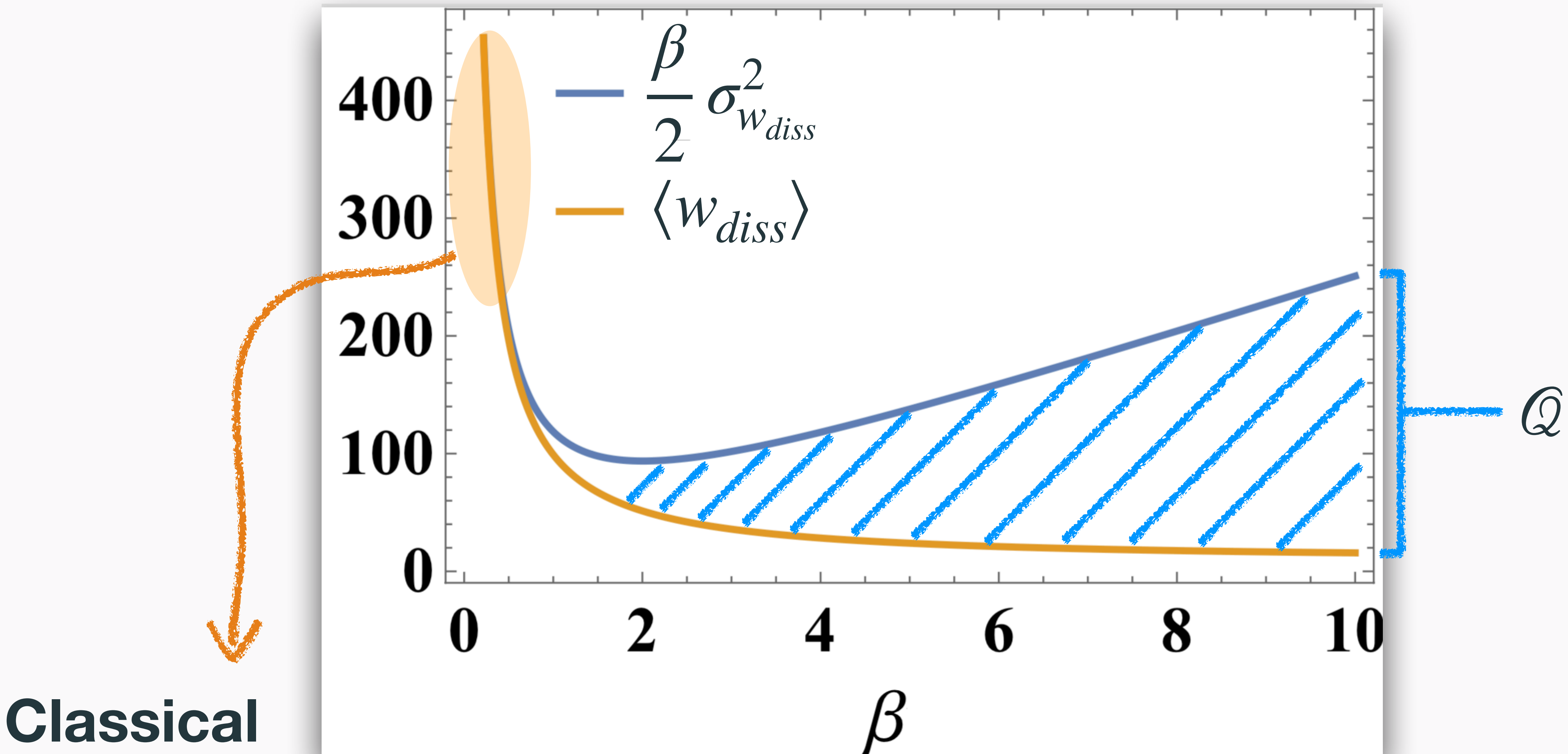
Fluctuations
Dissipation
Relation✦

$$\langle w_{diss} \rangle = \frac{\beta}{2}\, \sigma^2_{w_{diss}}$$

✦ C. Jarzynski, Phys. Rev. Lett. 78, 2690 (1997)

Quantum FDR[2]

$$\langle w_{diss} \rangle = \frac{\beta}{2}\sigma^2_{w_{diss}} + \mathcal{Q}$$



[2] M. Scandi, H. J. D. Miller, J. Anders, M. Perarnau-Llobet. arXiv:1905.07328

Cumulant Generating Function[3]

$$K(\lambda) = \log \int p(w_{diss}) e^{-\beta\lambda w_{diss}} \, \mathrm{d}w_{diss} = (\lambda - 1) S_\lambda(\pi_T || \rho_T)$$

$$= \sum_n \frac{\lambda^n}{n!} \kappa_n$$

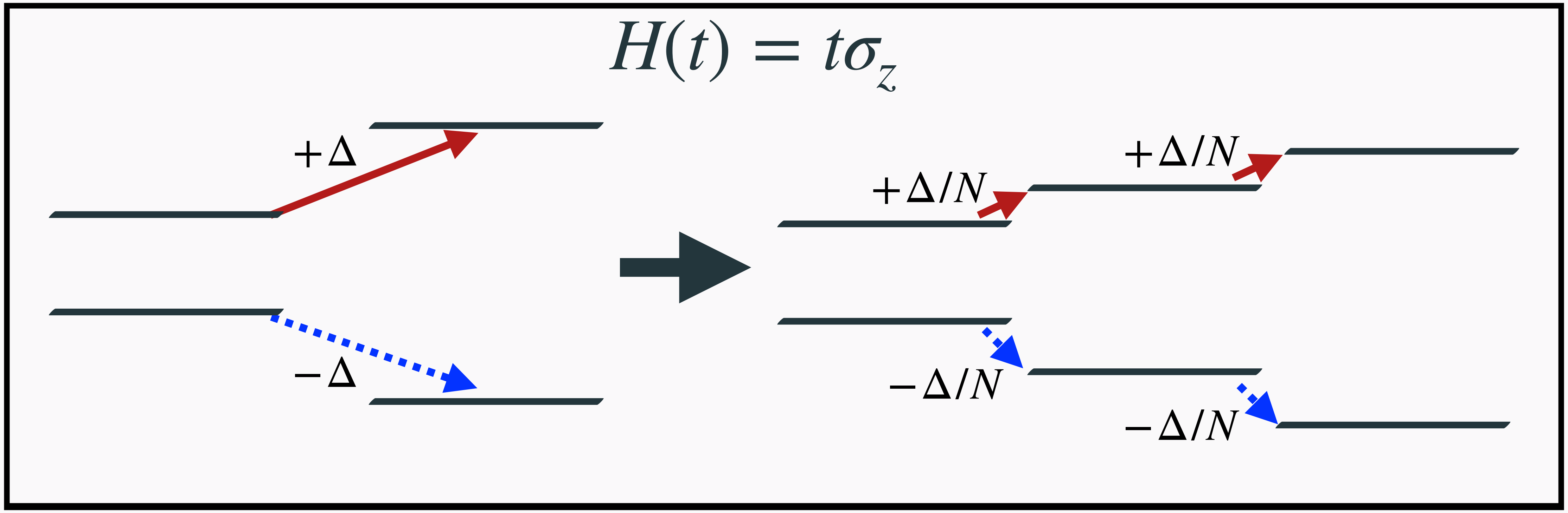
$T \gg \tau$

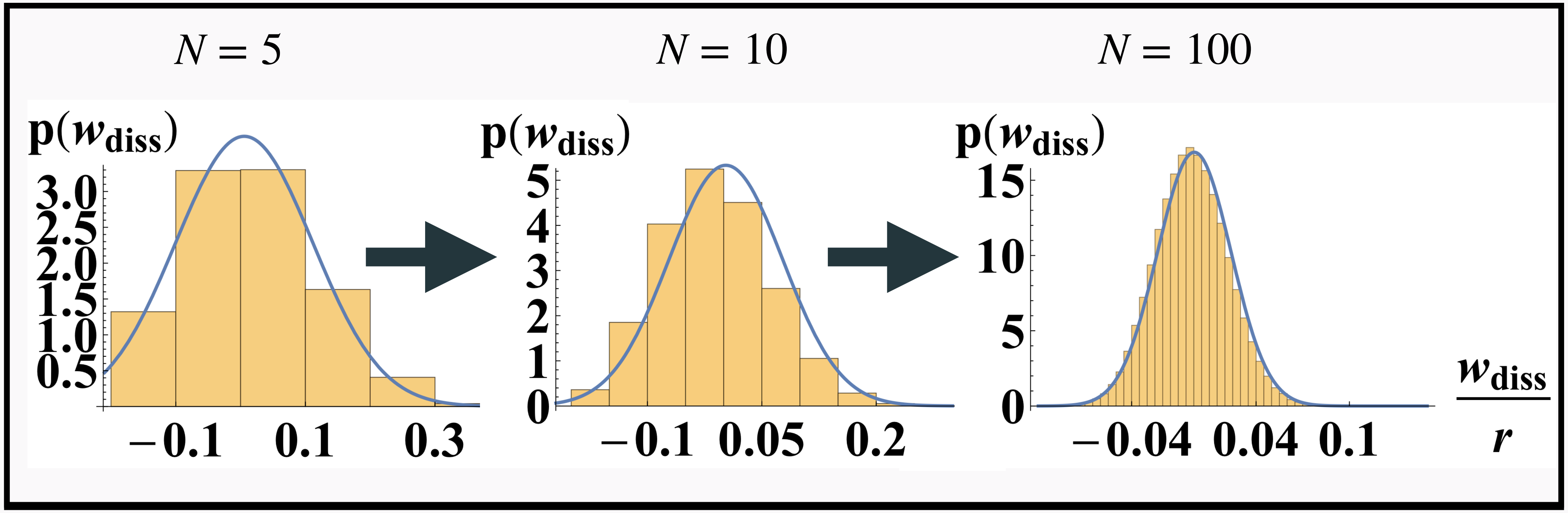
$$= \frac{\beta^2(\lambda^2 - \lambda)\tau}{2T} \int_\gamma \mathrm{Var}_{\pi_t}[\dot{H}_t] + \frac{\beta^2 \tau}{2T} \int_\gamma \int_0^\lambda \mathrm{d}x \int_x^{1-x} \mathrm{d}y \, I^y(\pi_t, \dot{H}_t)$$

Classical | **Quantum contribution**

[3] M. Scandi, H. J. D. Miller, J. Anders, M. Perarnau-Llobet. arXiv:1911.04306

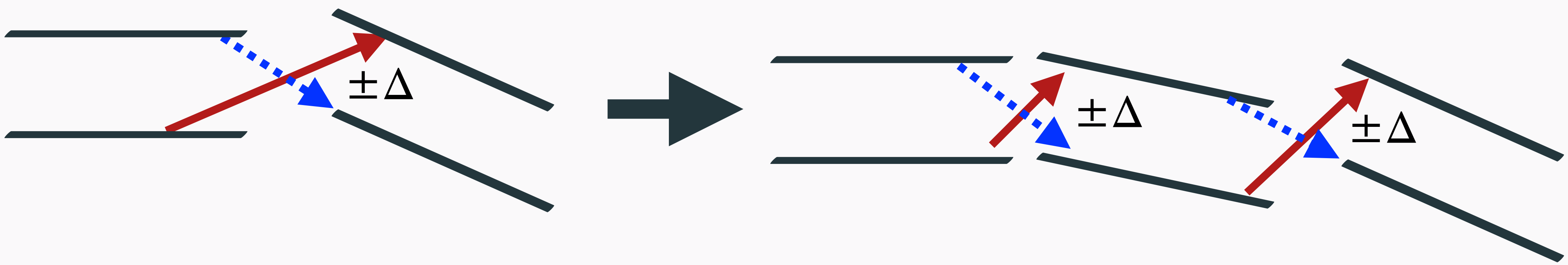
Qubit: diagonal driving

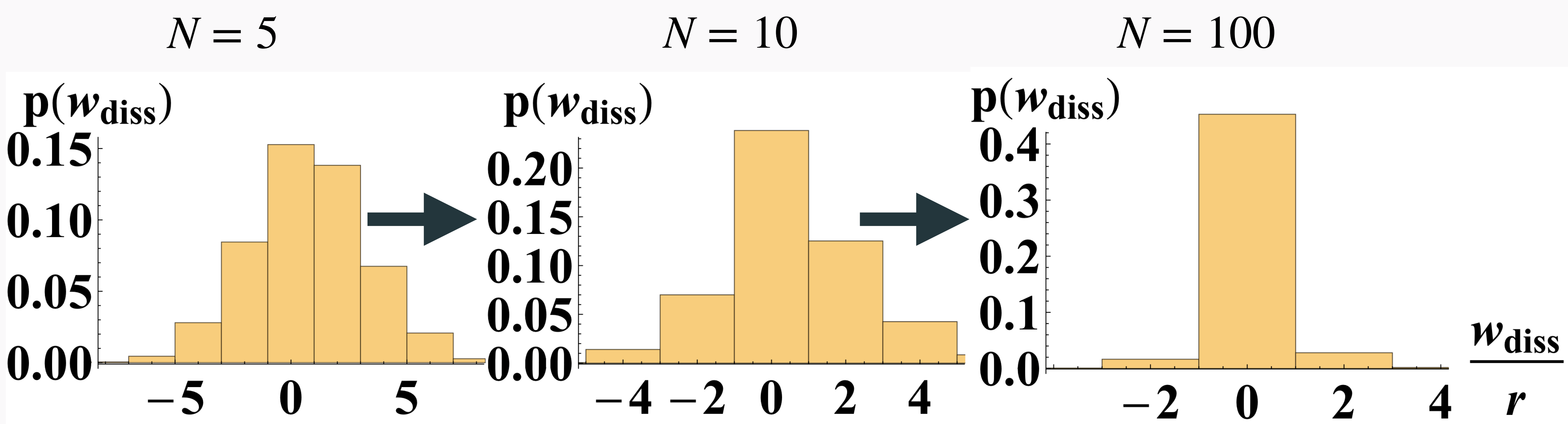




Qubit: coherent driving

$$H(t) = \cos(t)\sigma_z + \sin(t)\sigma_x$$





Simplifications

Time reversal

$$H_1 \xrightarrow{\Sigma} H_2 \xrightarrow{\Sigma} H_3 \longrightarrow \cdots\cdots \xrightarrow{\Sigma} H_N \longrightarrow p(w_{diss})$$

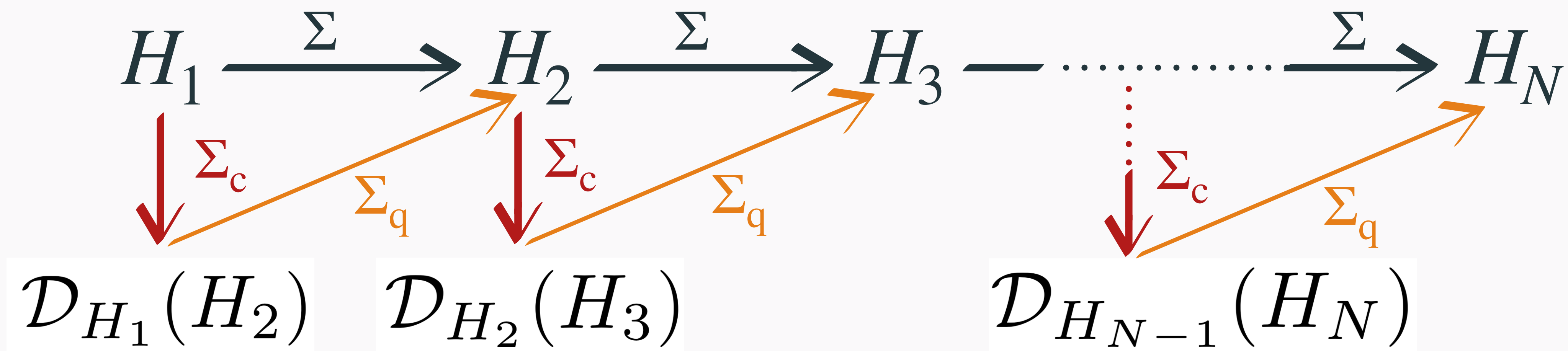
$$p_{\text{rev}}(w_{diss}) \longleftarrow H_1 \xleftarrow{\Sigma} H_2 \xleftarrow{\Sigma} H_3 \longleftarrow \cdots\cdots \xleftarrow{\Sigma} H_N$$

Crooks relations $\mapsto$ $p(w_{diss}) = p_{\text{rev}}(-w_{diss})e^{\beta w_{diss}}$

$\downarrow$ $T \gg \tau$

$p(w_{diss}) = p_{\text{rev}}(w_{diss}) \mapsto p(w_{diss}) = p(-w_{diss})e^{\beta w_{diss}}$

Channels of entropy production



$\rho_t \sim \pi_t$

Second laws

$S_\lambda(\rho \,||\, \pi)$ **decreases**

$S_\lambda(\rho \,||\, \mathscr{D}(\rho))$ **decreases**

Summary of the results

Quantum signatures	Consequences of Slow driving
$\frac{\beta}{2}\langle\sigma^2\rangle = \langle W_{diss}\rangle + \mathcal{Q}$	Time reversal symmetry
Non gaussianity	Decoupling of entropy production channels

Final remarks

- Quantum signatures survive the thermodynamic limit (FDR)

- Markovian dynamics define a family of metrics

- Extendible to strong coupling

Witnessing
Non Markovianity ?



Thanks for the attention!

Thermodynamic length: arXiv:1810.05583

Fluctuation dissipation relations: arXiv:1905.07328

Work statistics: arXiv:1911.04306