Maximum Power
for
Two-Level Thermal Machines:

Optimality of Fast Cycles

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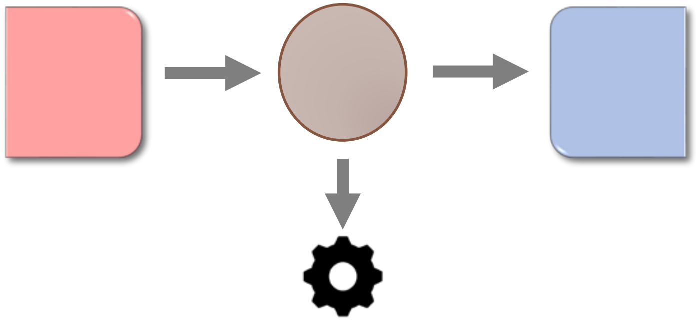
PAOLO ANDREA ERDMAN, VASCO CAVINA,

ROSARIO FAZIO, FABIO TADDEI, VITTORIO GIOVANNETTI

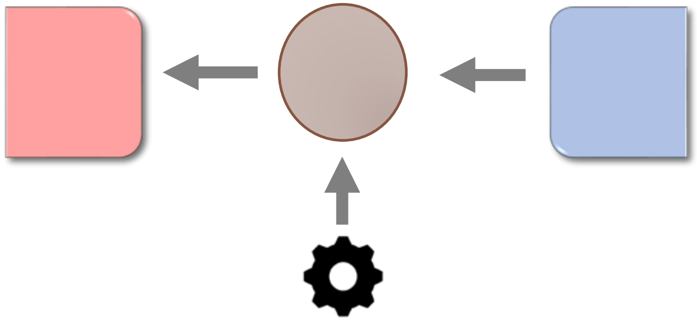
Thermal Machines

1) Heat engine



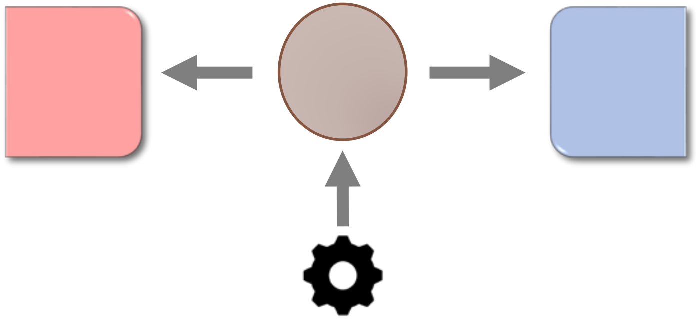
$$Q_H \geq 0$$
$$Q_C \leq 0$$
$$W \geq 0$$

2) Refrigerator



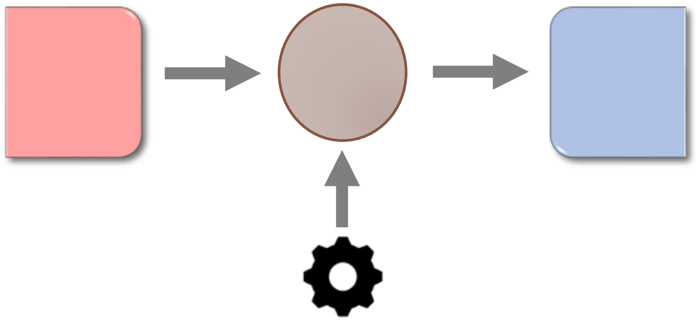
$$Q_H \leq 0$$
$$Q_C \geq 0$$
$$W \leq 0$$

3) Heater



$$Q_H \geq 0$$
$$Q_C \leq 0$$
$$W \geq 0$$

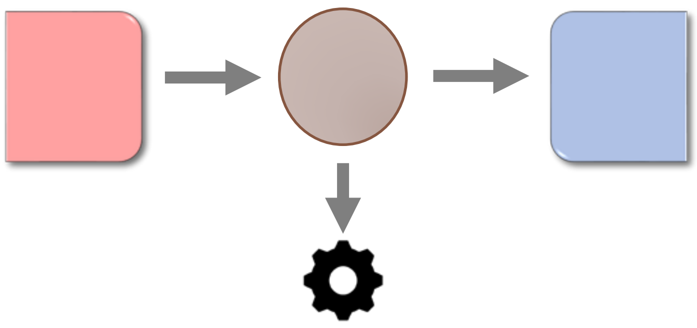
4) Thermal accelerator



$$Q_H \geq 0$$
$$Q_C \leq 0$$
$$W \leq 0$$

Thermal Machines: Characterization

Heat engine



Power: $$P_E = \frac{W}{\tau}$$

Efficiency: $$\eta = \frac{W}{Q_{\mathrm{H}}} \leq \eta^{(c)} = 1 - \frac{T_C}{T_H}$$

Thermal Machines: Optimization

Maximize ***Efficiency***:

Reversible Transformations

Infinitely slow

$\eta = \eta^{(c)}$

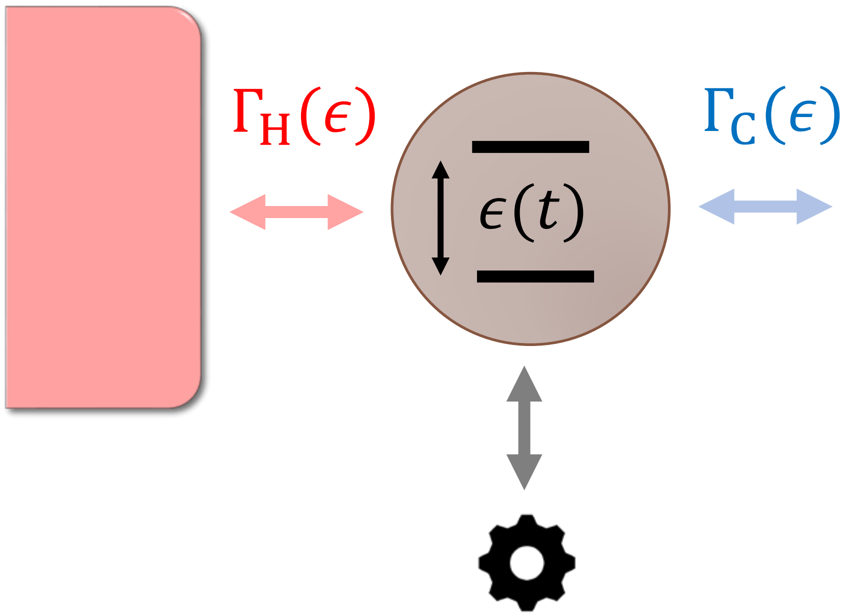
However, $P = 0$

Maximize ***Power***:

- Finite-time thermodynamics
- Microscopic model

- Universal strategy?
- Efficiency at maximum power?

The Model



$$\mathcal{H}_s(t) = \boxed{\epsilon(t)}\,\sigma^+\sigma^-$$

Control parameter

$$\frac{\mathrm{d}}{dt}\hat{\rho} = -\frac{i}{\hbar}[\mathcal{H}_s, \hat{\rho}] + \mathcal{D}_{\mathrm{H}}[\hat{\rho}]$$

$$\frac{d}{dt}p(t) = -\sum_{\alpha} \lambda_\alpha(t)\Gamma_\alpha[\epsilon(t)]\,(p(t) - p_{eq}^{\alpha}[\epsilon(t)])$$

$\mathrm{Tr}[\sigma^+\sigma^-\hat{\rho}(t)]$

Bath switch

Dissipation Rate Hot Bath

$$p_{eq}^{\mathrm{H}} = \frac{1}{1 + \mathrm{e}^{\beta_{\mathrm{H}}\epsilon}}$$

Power Maximization

Given a fixed physical system, i.e. fixed $\Gamma_\mathrm{C}(\epsilon), \Gamma_\mathrm{H}(\epsilon)$

Arbitrary input *periodic* control $\epsilon(t)$

$\downarrow$ Eq. of motion

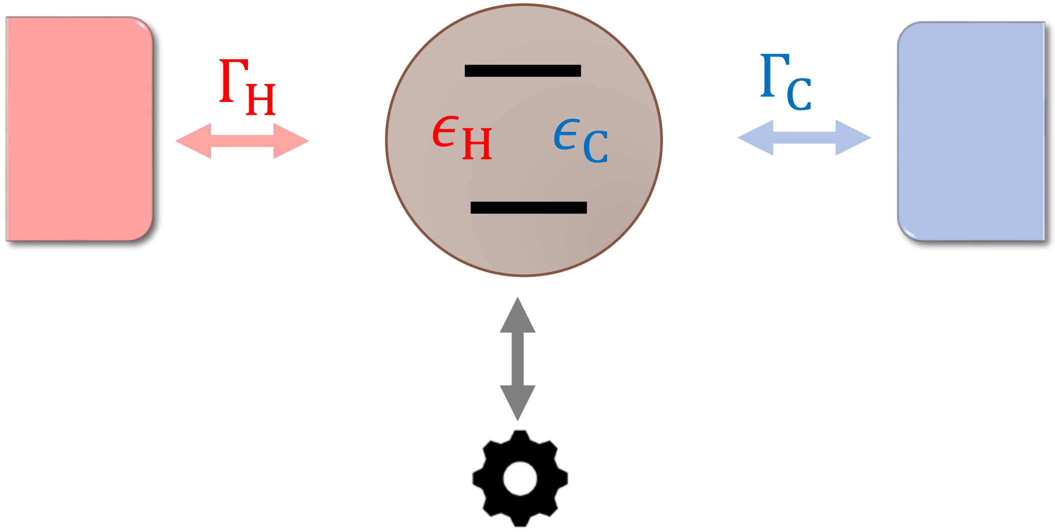
Compute periodic state $p(t)$

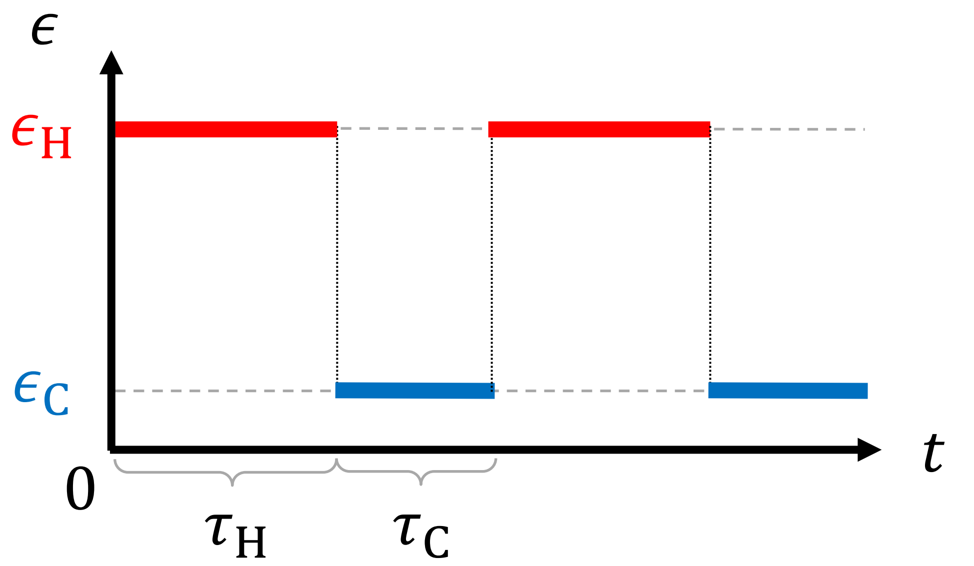
$\downarrow$

Average Power: $P_E = -\frac{\int_0^\tau \dot{\epsilon}(t)\, p(t)\mathrm{dt}}{\tau}$

Maximize over $\epsilon(t)$

Maximum Power: Infinitesimal Otto Cycle





Two Stroke Cycle:

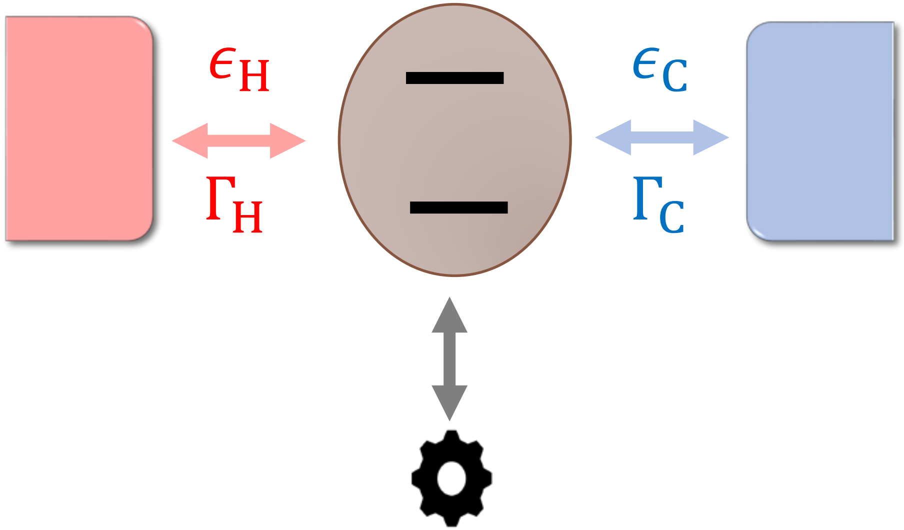
- Contact with HOT: ϵ_H
- Contact with COLD: ϵ_C
- Repeat

$$\boxed{\tau_H + \tau_C \to 0}$$

$$\boxed{\frac{\tau_H}{\tau_C} = \sqrt{\frac{\Gamma_C}{\Gamma_H}}}$$

Power of Optimal Cycle

$$P_{[\nu]}^{(\mathrm{max})} = \max_{\epsilon_\mathrm{H},\epsilon_\mathrm{C}} \frac{\Gamma_\mathrm{H}\Gamma_\mathrm{C}}{\left(\sqrt{\Gamma_H}+\sqrt{\Gamma_C}\right)^2}\,\tilde{\epsilon}_{[\nu]}\left[p_\mathrm{eq}^{(\mathrm{H})} - p_\mathrm{eq}^{(\mathrm{C})}\right]$$



Where:

$$\tilde{\epsilon}_{[\mathrm{E}]} = \epsilon_\mathrm{H} - \epsilon_\mathrm{C}, \qquad \tilde{\epsilon}_{[\mathrm{R}]} = -\epsilon_\mathrm{C},$$
$$\tilde{\epsilon}_{[\mathrm{A}]} = +\epsilon_\mathrm{C}, \qquad \tilde{\epsilon}_{[\mathrm{H}]} = \epsilon_\mathrm{C} - \epsilon_\mathrm{H},$$

Optimality of fast cycles

Fast driving regime little attention, but:

-V. Cavina, A. Mari, A. Carlini, and V. Giovannetti, Phys. Rev. A **98**, 012139 (2018)

- P. Abiuso, M. Perarnau-Llobet, arXiv:1907.02939 (2019)

- J. P. Pekola, B. Karimi, G. Thomas, and D. V. Averin, Phys. Rev. B **100**, 085405 (2019)

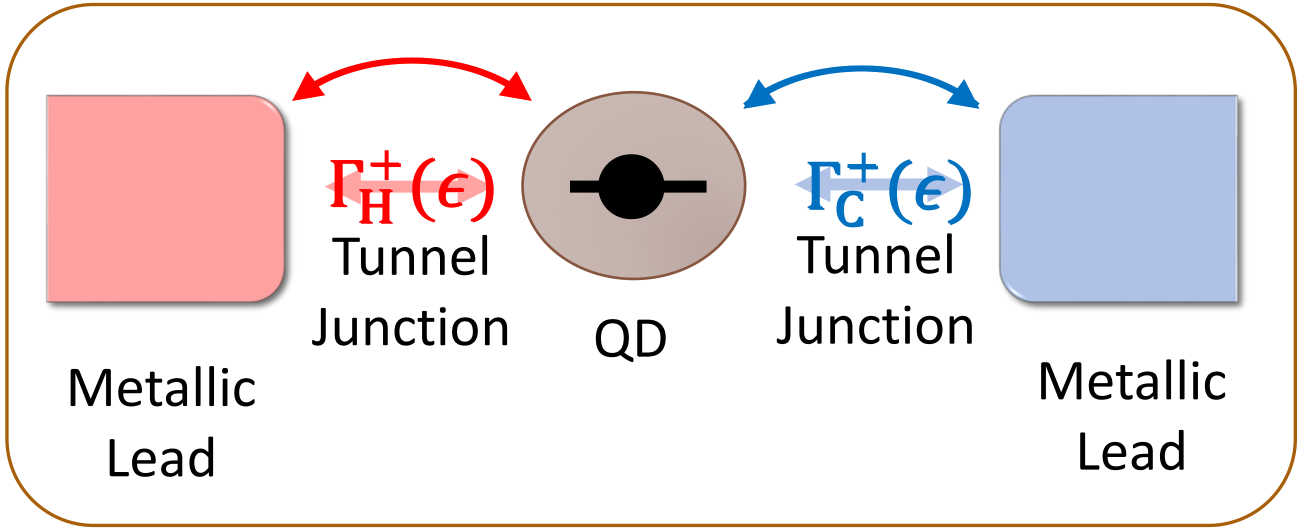
- O. Abah, M. Paternostro, E. Lutz, arXiv:1911.00373 (2019)

- P. Menczel, T. Pyhäranta, C. Flindt, and K. Brandner, Phys. Rev. B **99**, 224306 (2019)

…

Relevant Physical Systems

Single level ***Quantum Dot***:



States: occupation of QD (n=0,1)

1. $\Gamma^+ = k\, f(\beta\,\epsilon)$
2. $\Gamma^- = k\,[1 - f(\beta\,\epsilon)]$

$$\boxed{\Gamma \equiv \Gamma^+ + \Gamma^- = k}$$

Two-level ***Atom in Dissipative EM Environment***:

$$\boxed{\Gamma = k \coth(\beta\,\epsilon/2)}$$

Efficiency at Maximum Power (EMP)

Many upper bounds in literature:

- Curzon-Ahlborn EMP:

$$\eta_{CA} = 1 - \sqrt{1 - \eta_C}$$

[F. L. Curzon, B. Ahlborn, Am. J. Phys. **43**, 22 (1975)] *Endoreversible Engines*

[C. Van den Broeck, Phys. Rev. Lett. **95**, 190602 (2005)] *Linear Irreversible Thermodynamics*

- Schmiedl-Seifert:

$$\boxed{\eta_{SS} = \frac{\eta_C}{2 - \eta_C}}$$

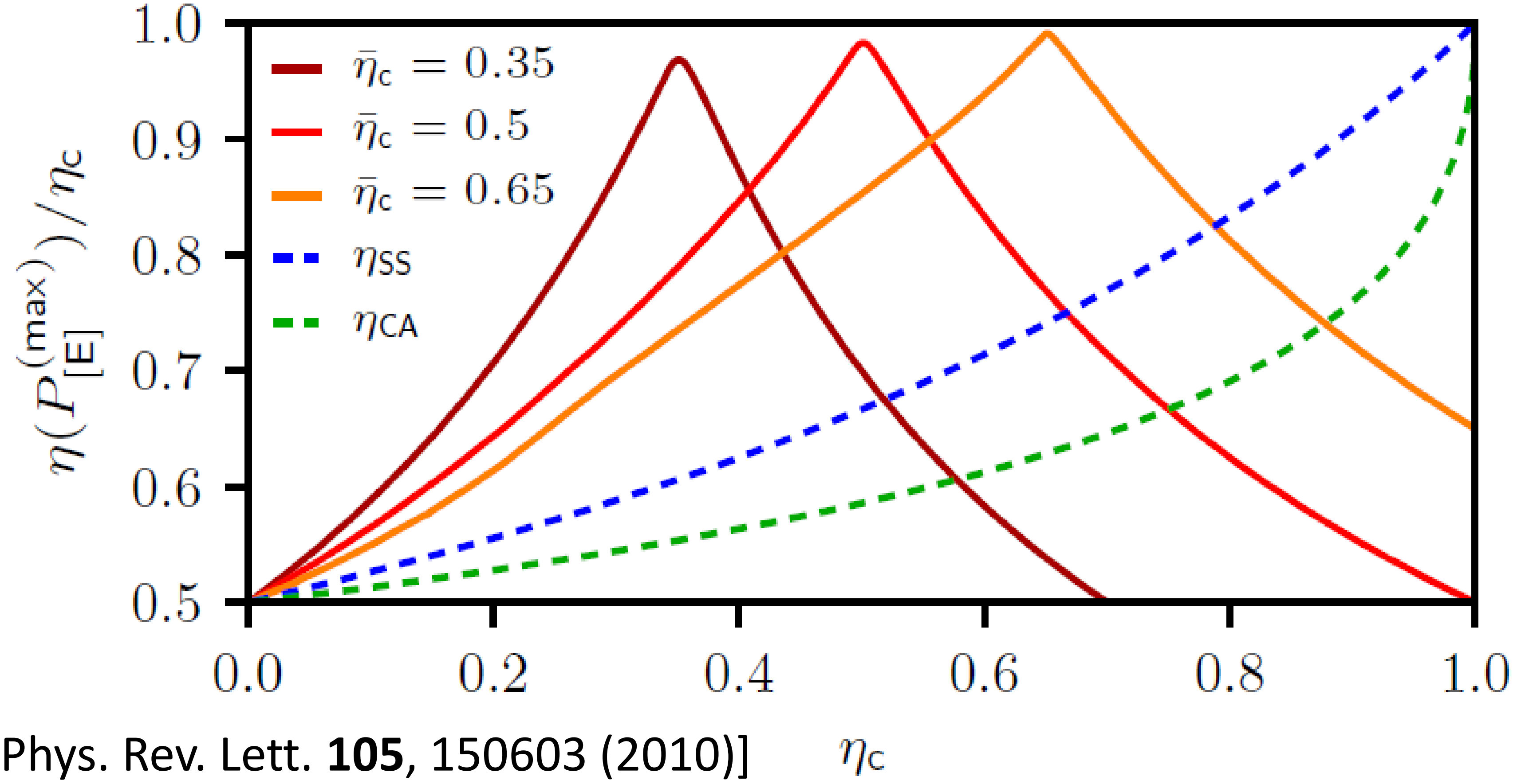
[T. Schmiedl and U. Seifert, EPL **81**, 20003 (2008)] *Brownian Heat Engine*

[M. Esposito et al., Phys. Rev. Lett. **105**, 150603 (2010)] ***Universal in low dissipation regime***

Carnot Efficiency at Maximum Power

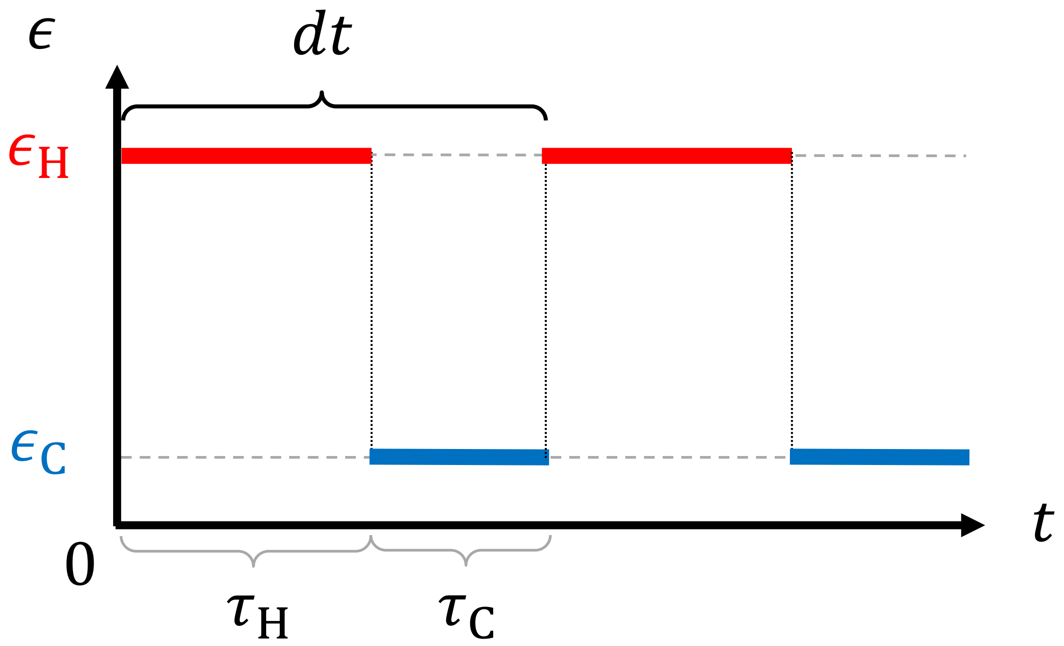
Low dissipation / slow driving : $\eta_{SS} = \dfrac{\eta_C}{2 - \eta_C}$

Fast driving / far from equilibrium:

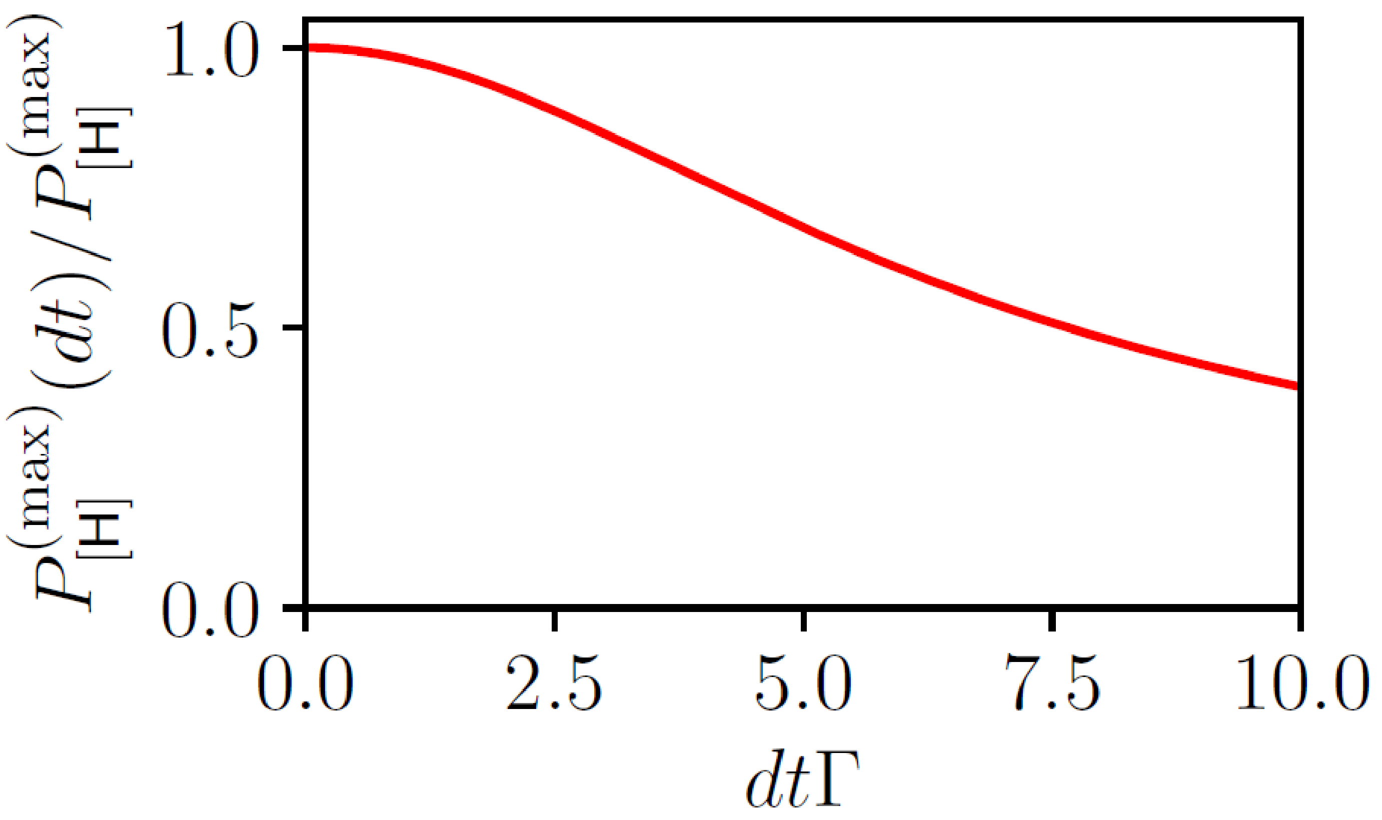


[Esposito et al., Phys. Rev. Lett. **105**, 150603 (2010)]

Feasibility



What happens for *finite* dt?



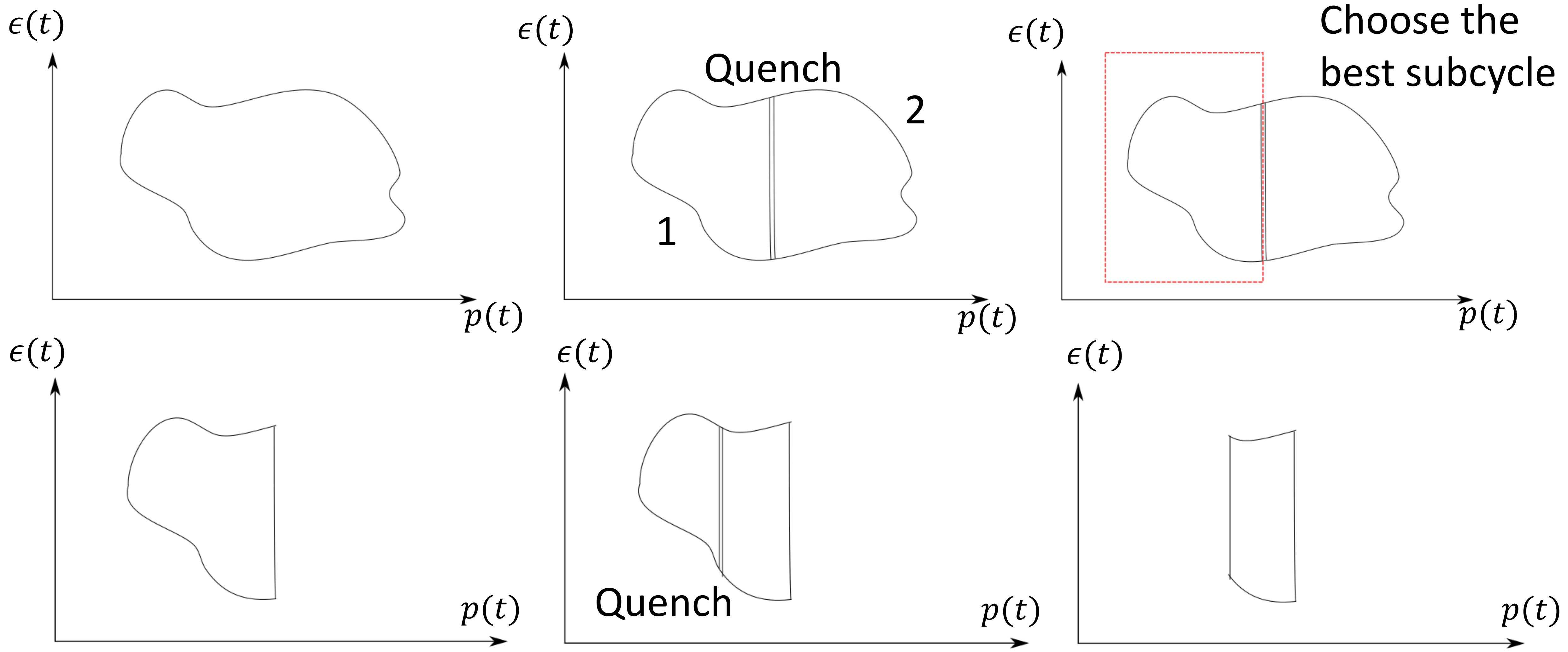
Conclusions

- Maximum Power in Two-Level Systems: Optimality of Fast Cycles
- “Duality” power - efficiency
- There is no fundamental upper bound for the EMP (Heat Engine and Refrigerator) thanks to the Fast Driving Regime.
- Applicable to Relevant Systems (Fermionic, Bosonic, etc).
- Feasibility of the Protocol.

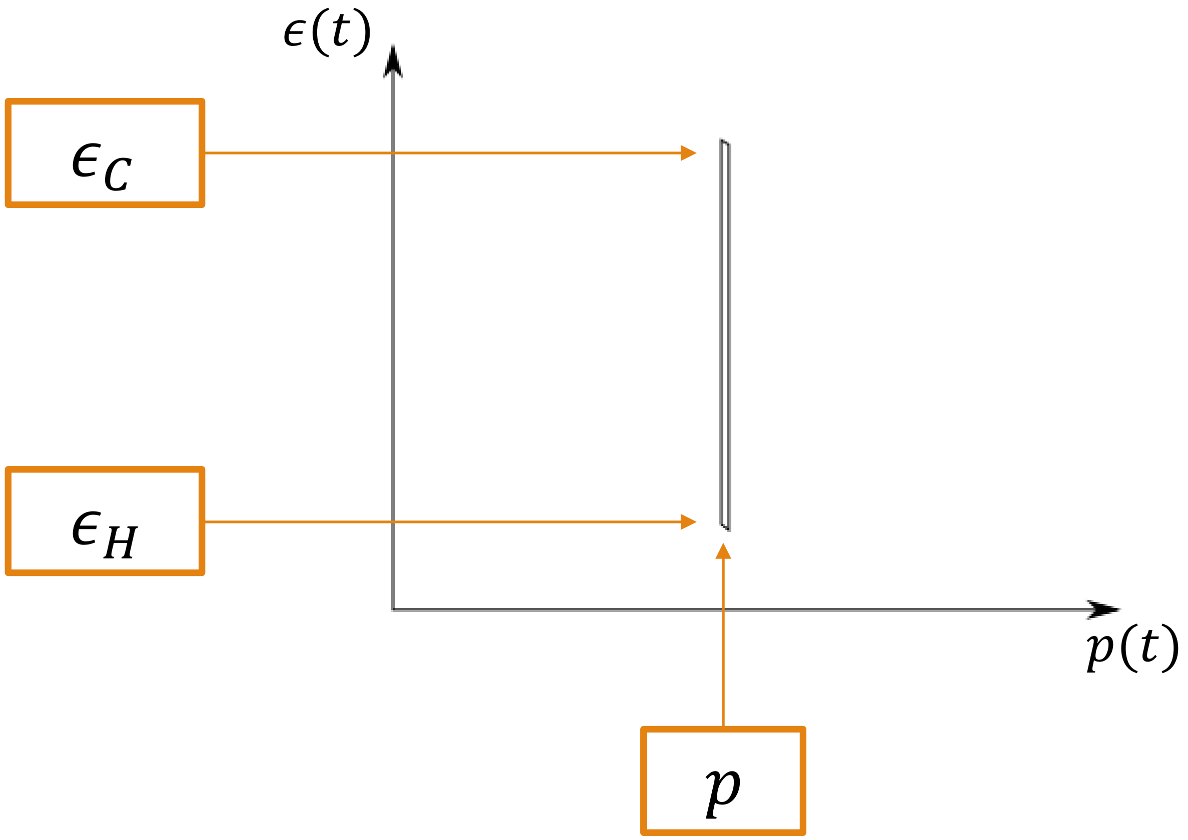
Thanks for your attention

[P. A. Erdman et al., New J. Phys. **21**, 103049 (2019)]

From asymptotic to infinitesimal



From asymptotic to infinitesimal



- $p(t) \approx p$ is almost costant during a cycle
- $\epsilon(t)$ switches between two extremal values ϵ_C and ϵ_H

- $p(t) \approx p$ needs a very fast driving: the population does not have time to relax

Fast Driving Regime

- t_b: Bath correlation time timescale
- t_s: System timescale
- t_r: Relaxation timescale (induced by bath)
- t_d: Driving timescale

$$\boxed{t_b \ll t_s \ll t_d \ll t_r}$$

Markovian Approx.

Secular Approx.

Fast Driving

[R. Dann, A. Levy, and R. Kosloff, Phys. Rev. A **98**, 052129 (2018)]