Shortcuts-to-adiabaticity protocols for a robust generation of non-classical states in spin-bosons systems

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arXiv:1912.05264

Bad Honnef, 6th February 2020
Quantum Thermodynamics for Young Scientists

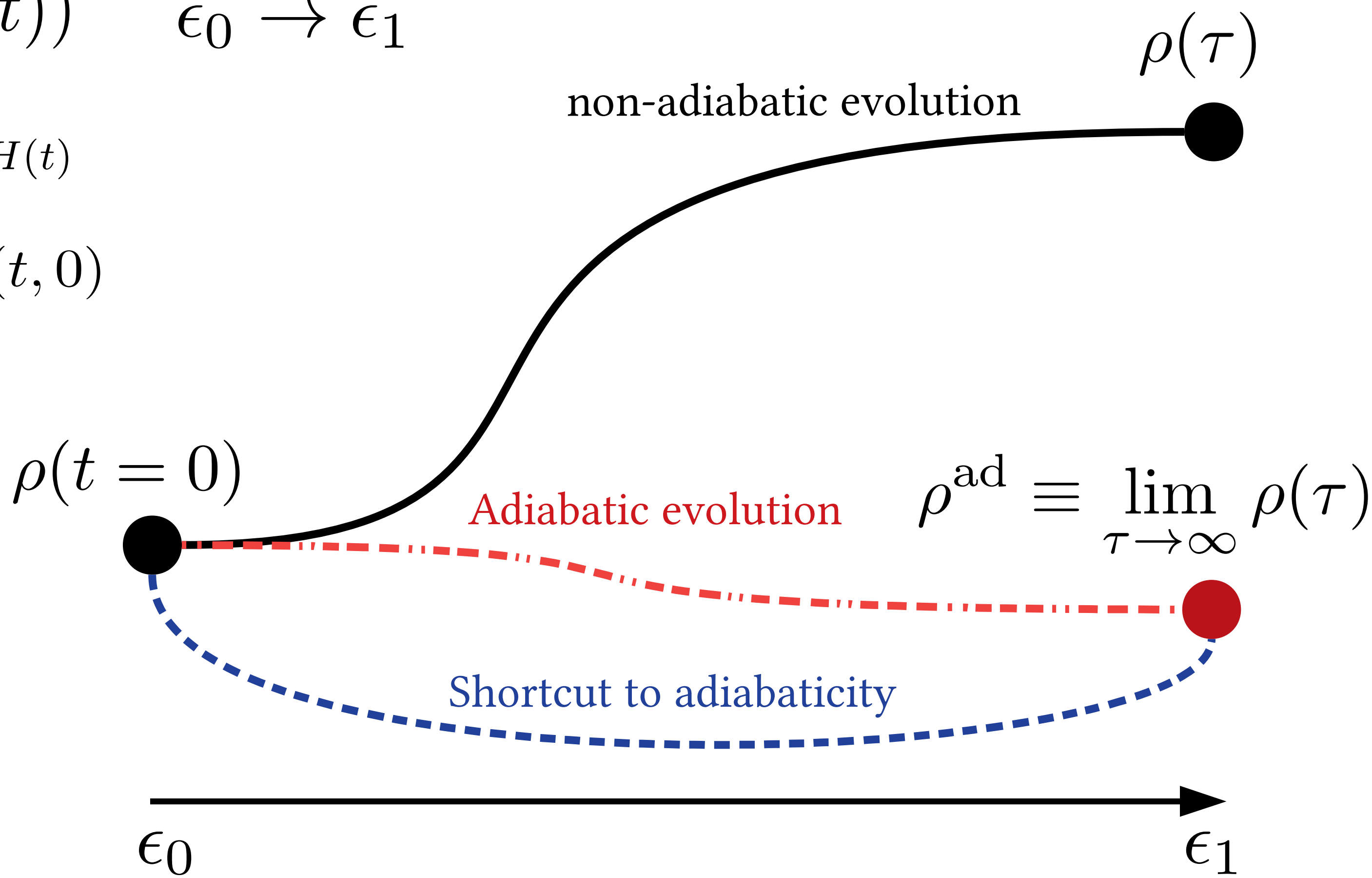
Shortcuts to adiabaticity (STA)

- **Time-dependent transformation**

$$H(t) \equiv H(\epsilon(t)) \qquad \epsilon_0 \xrightarrow{\tau} \epsilon_1$$

$$U(t_2, t_1) = \mathcal{T} e^{-i \int_{t_1}^{t_2} dt H(t)}$$

$$\rho(t) = U(t,0)\rho(0)U^\dagger(t,0)$$



$$H(t) \to H_{\text{STA}}(t)$$

M. Berry, J. Phys. A: Math. Theor. **42** 365303 (2009)
A. del Campo, Phys. Rev. Lett. **111** 100502 (2013)
E. Torrontegui *et al.*, Adv. At. Mol. Opt. Phys. **62** 117 (2013)

Shortcuts to adiabaticity (STA)

- **Counterdiabatic driving**

$$H_{\mathrm{STA}}(t) = H(t) + H_{\mathrm{CD}}(t) \qquad \epsilon_0 \xrightarrow{\tau} \epsilon_1$$

System's Hamiltonian: $$H(t) = \sum_n E_n(t)|n_t\rangle\langle n_t|$$

Counterdiabatic term: $$H_{\mathrm{CD}}(t) \equiv i\sum_n \left[|\partial_t n_t\rangle\langle n_t| - \langle n_t|\partial_t n_t\rangle|n_t\rangle\langle n_t|\right]$$

$\longrightarrow$ Time-evolution leads now to the adiabatic state aided by $H_{\mathrm{CD}}(t)$

$$\rho(\tau) = U_{\mathrm{STA}}(\tau,0)\rho(0)U^{\dagger}_{\mathrm{STA}}(\tau,0) \equiv \rho^{\mathrm{ad}} \qquad \forall\tau$$

M. Berry, J. Phys. A: Math. Theor. **42** 365303 (2009)
A. del Campo, Phys. Rev. Lett. **111** 100502 (2013)
E. Torrontegui *et al.*, Adv. At. Mol. Opt. Phys. **62** 117 (2013)

Shortcuts to adiabaticity (STA)

- **STA has been succesfully applied to many situations:**

→ State manipulation

B. Julia-Diaz *et al.*, Phys. Rev. A **86** 063623 (2012)
A. del Campo and M. G. Boshier, Sci. Rep. **2** 648 (2012)
⋮

→ Many-body systems

A. del Campo, M. M. Rams and W. H. Zurek, Phys. Rev. Lett. **109** 115703 (2012)
⋮

→ Transport

X. Chen *et al.*, Phys. Rev. A **84** 043415 (2011)
⋮

→ Quantum heat engines

A. del Campo, J. Goold and M. Paternostro, Sci. Rep. **4** 6208 (2014)
B. Çakmak and Ö. E. Müstecaplıoğlu, Phys. Rev. E **99** 032108 (2019)
⋮

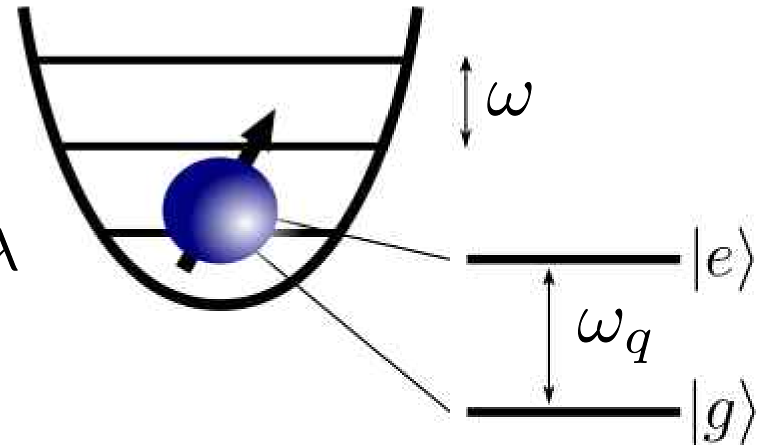
→ Open quantum systems

L. Dupays *et al.*, arXiv:1910.12088
R. Dann, A. Tobalina, and R. Kosloff, Phys. Rev. Lett. **122** 250402 (2019)

→ Today's talk: **STA to obtain non-classical states**

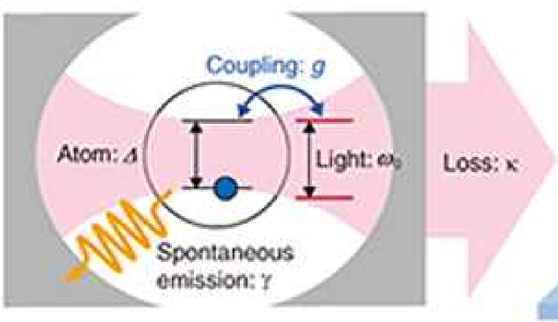
Jaynes-Cummings model

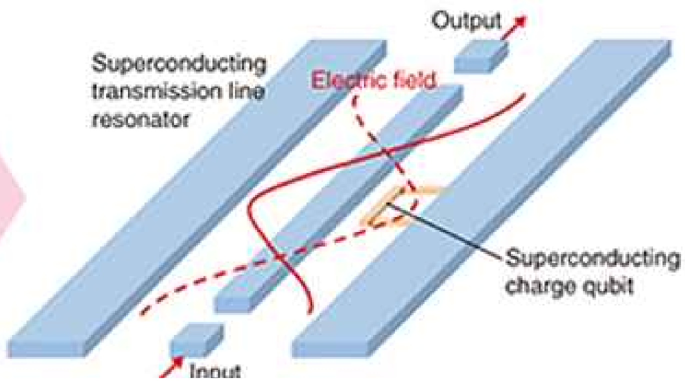
$$H_{\rm JC} = \frac{\omega_q}{2}\sigma_z + \omega a^\dagger a + \lambda(a\sigma^+ + a^\dagger\sigma^-)$$

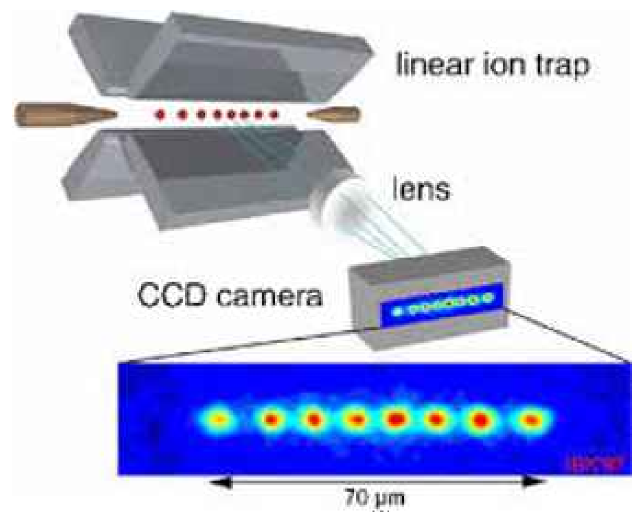


$\longrightarrow$ Qbit interacting with a bosonic field with strength λ

- **Relevant in different platforms:**







P. Forn-Diaz *et al.*, Phys. Rev. Lett. **105** 237001 (2010)
D. Lv *et al.*, Phys. Rev. X **8** 021027 (2018)
A. F. Kockum *et al.*, Nat. Rev. Phys. **1** 19 (2019); P. Forn-Díaz *et al.*, Rev. Mod. Phys. **91** 025005 (2019)

Jaynes-Cummings model

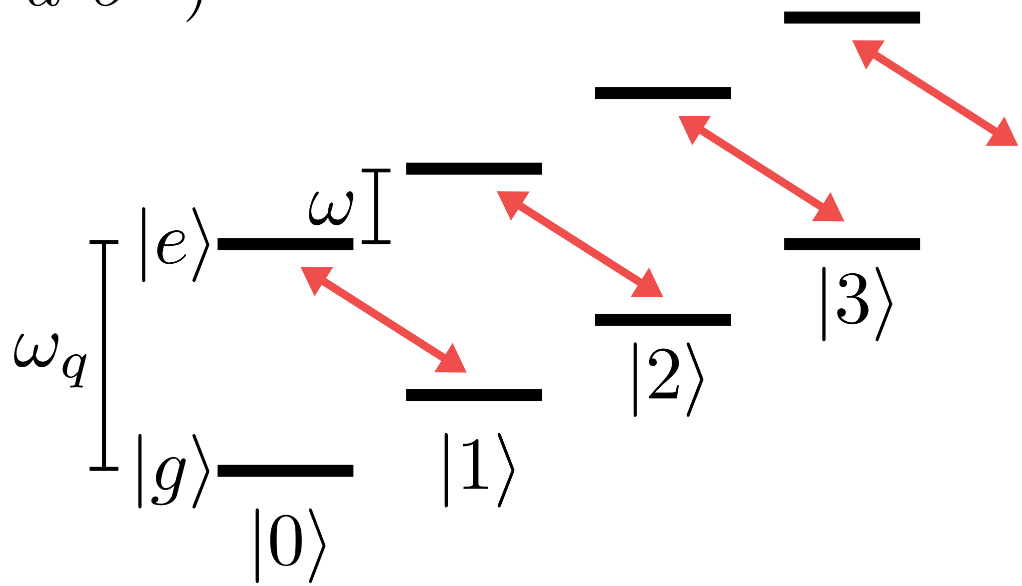
$$H_{\mathrm{JC}} = \frac{\omega_q}{2}\sigma_z + \omega a^\dagger a + \lambda(a\sigma^+ + a^\dagger\sigma^-)$$

- **At resonance ω_q=ω**
  there is a complete population transfer

$$|e, n\rangle \leftrightarrow |g, n+1\rangle$$

$$T_n = \frac{\pi}{2\lambda\sqrt{n+1}}$$

$\longrightarrow$ Not possible for all *n* at the same time



- **One way to overcome this: adiabatic evolution under a time-dependent H_{JC}**

$$H_{\mathrm{JC}}(t) = \frac{\omega_q(t)}{2}\sigma_z + \omega a^\dagger a + \lambda(t)(a\sigma^+ + a^\dagger\sigma^-)$$

Jaynes-Cummings model

$$H_{\mathrm{JC}}(t) = \frac{\omega_q(t)}{2}\sigma_z + \omega a^\dagger a + \lambda(t)(a\sigma^+ + a^\dagger\sigma^-)$$

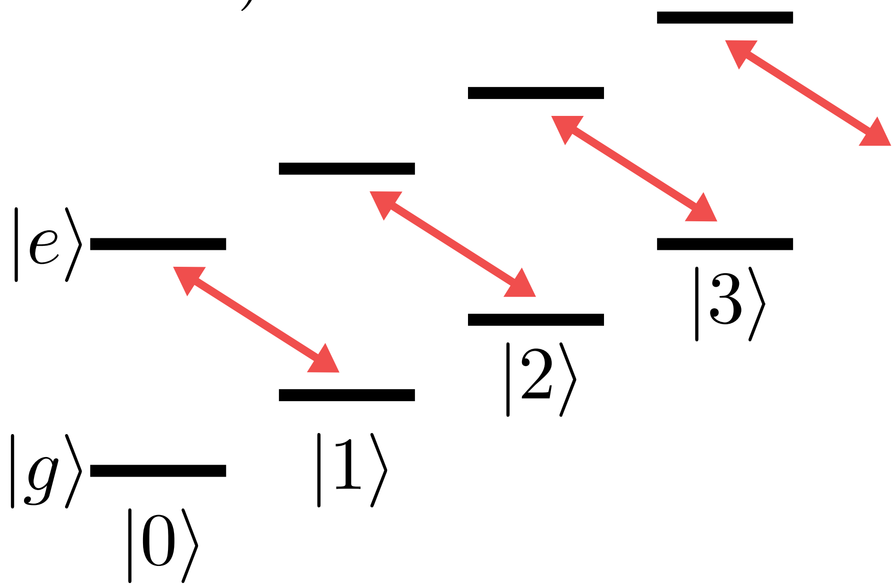
- **Split H_{JC} in a set of Landau-Zener problems**

$$H_{\mathrm{JC}}(t) = -\frac{\omega_q(t)}{2}|g,0\rangle\langle g,0| + \bigoplus_{n=0} H_n(t)$$

$$H_n(t) = \frac{\omega_q(t)-\omega}{2}\tilde{\sigma}_{z,n} + \lambda(t)\sqrt{n+1}\tilde{\sigma}_{x,n}$$

$$\tilde{\sigma}_{z,n} = |e,n\rangle\langle e,n| - |g,n+1\rangle\langle g,n+1|$$

$$\tilde{\sigma}_n^+ = |e,n\rangle\langle g,n+1|$$



- **Requirements for the time-dependent parameters**

$$\omega_q(0) < \omega \qquad \lambda(0) = \lambda(\tau) = 0$$

$$\omega_q(\tau) > \omega \qquad \lambda(0 < t < \tau) \neq 0$$

Counterdiabatic driving

- **Find the STA:**

$$H_{\mathrm{CD}}(t) \equiv i \sum_n \left[|\partial_t n_t\rangle\langle n_t| - \langle n_t|\partial_t n_t\rangle |n_t\rangle\langle n_t| \right]$$

$$\longrightarrow \quad H_{\mathrm{STA}}(t) = H_{\mathrm{JC}}(t) + i\theta(t)(a^\dagger\sigma^- - a\sigma^+)$$

$$\theta(t) = \frac{(\omega_q(t)-\omega)\dot{\lambda}(t) - \lambda(t)\dot{\omega}_q(t)}{(\omega_q(t)-\omega)^2 + 4\lambda^2(t)(n+1)}$$

- **Local counterdiabatic driving: bring it back to the original form of $\mathbf{H_{JC}}$**

It is possible to find a unitary transformation to obtain

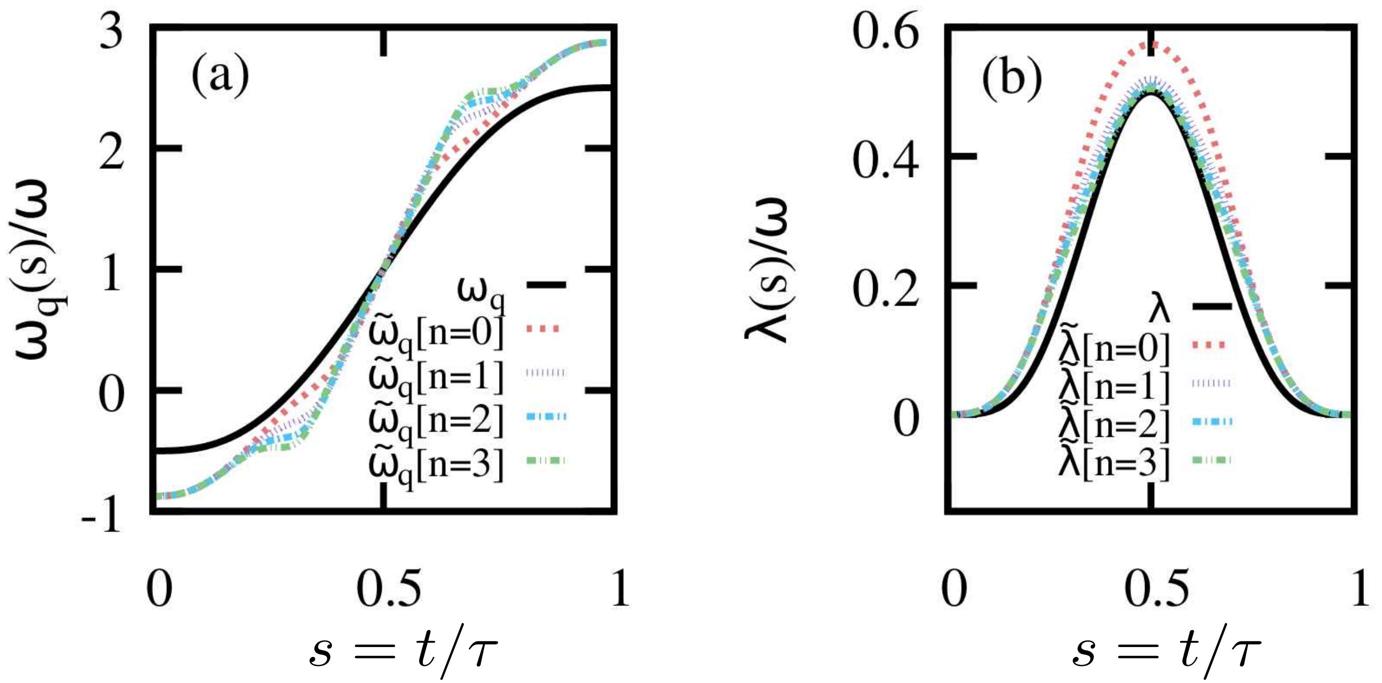
$$H_{\mathrm{STA}}(t) = \frac{\tilde{\omega}_q(t)}{2}\sigma_z + \omega a^\dagger a + \tilde{\lambda}(t)(a\sigma^+ + a^\dagger\sigma^-)$$

$$\tilde{\omega}_q(t) = \omega_q(t) - 2\sqrt{n+1}\,\frac{\lambda(t)\dot{\theta}(t) - \theta(t)\dot{\lambda}(t)}{\theta^2(t) + 4\lambda^2(t)(n+1)} \qquad \tilde{\lambda}(t) = \sqrt{\lambda^2(t) + \theta^2(t)}$$

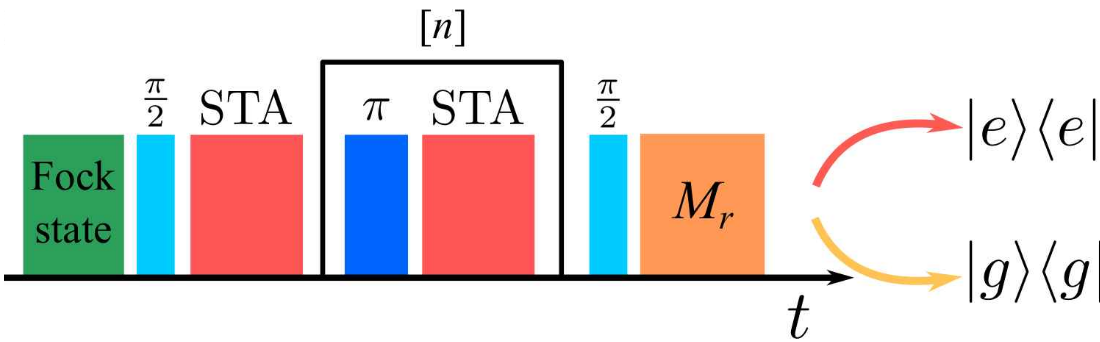
Local counterdiabatic driving

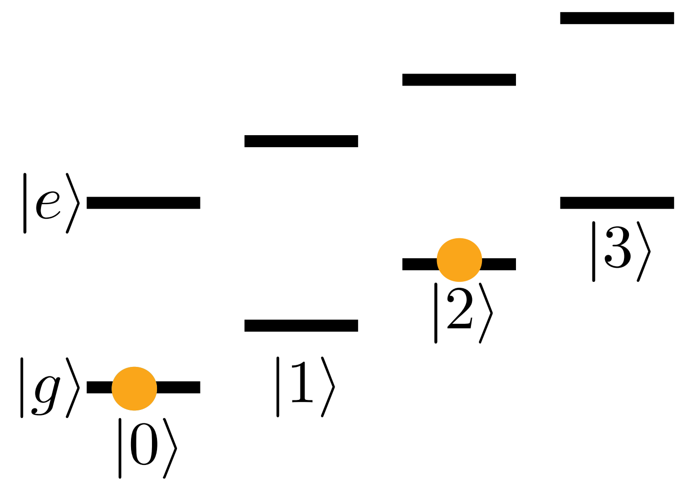
→ Choose protocols such that $H_{\mathrm{STA}}(t=0,\tau) = H_{\mathrm{JC}}(t=0,\tau)$

Smooth functions at the beginning and at the end



Cat states





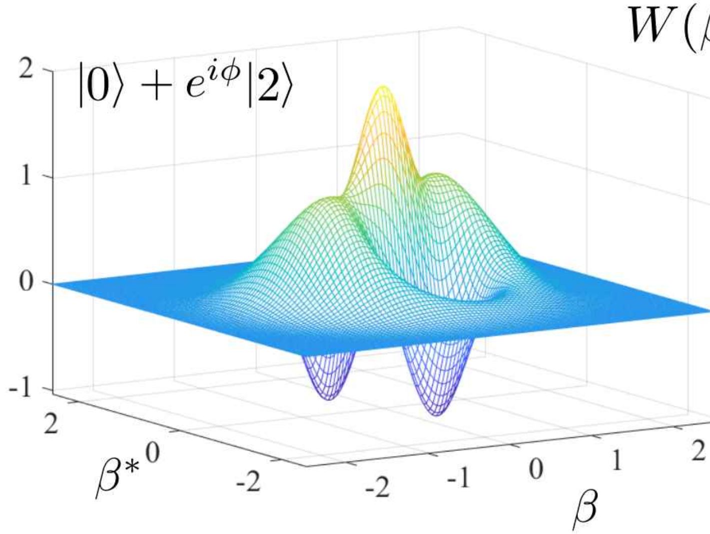
$$M_r = |r\rangle\langle r| \otimes I_b$$

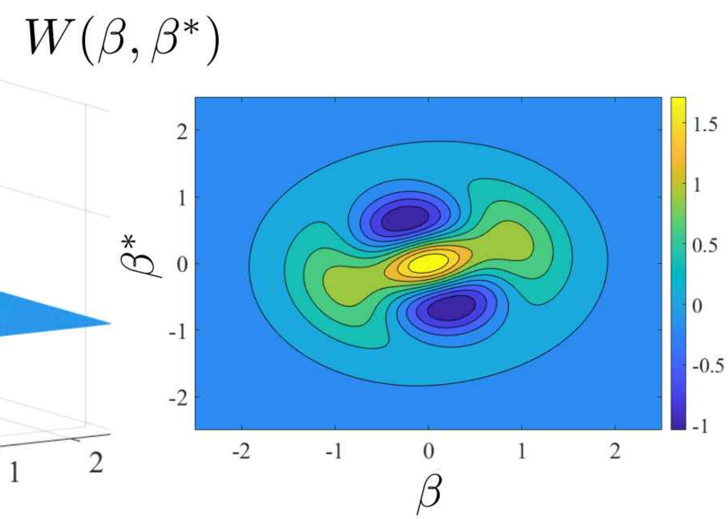
$$r \in \{e, g\}$$

$$F = \langle \psi_{\text{cat}} | \rho_b | \psi_{\text{cat}} \rangle$$

$$F_{\text{STA}} \gtrsim 0.999$$

$$F_{\text{bare,TI}} \approx 0.8$$





$\longrightarrow$ STA scheme offers a large improvement!

Photon-shifted states

- **Photon-addition/subtraction:**

$$|\psi\rangle_{\mathrm{ph-add}} \propto a^\dagger |\psi\rangle \longrightarrow$$

Non-classical state by removing the vacuum contribution

M. S. Kim, J. Phys. B: At. Mol. Opt. Phys. **41** 133001 (2008)
V. Parigi *et al.*, Science **317** 1890 (2007)

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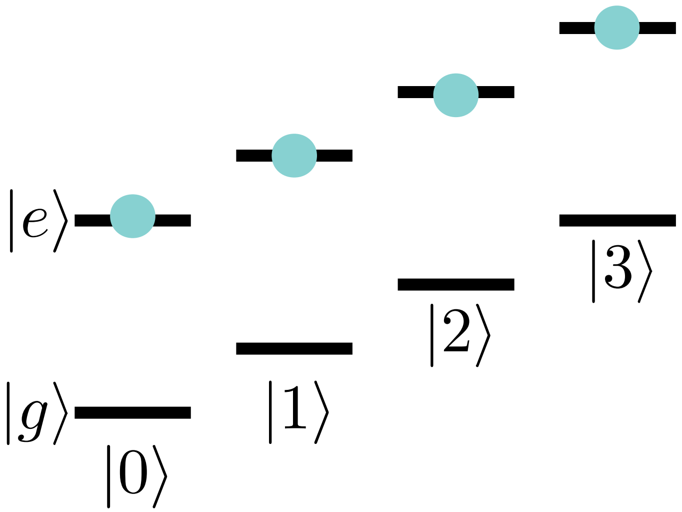
- **Photon-shifted states:**

$$|\psi\rangle_{\mathrm{ph-sh}} \equiv \left[\sum_{n=0} |g, n+1\rangle\langle e, n|\right]|\psi\rangle$$

STA protocols allow us to implement
(to a good approx.) this operation



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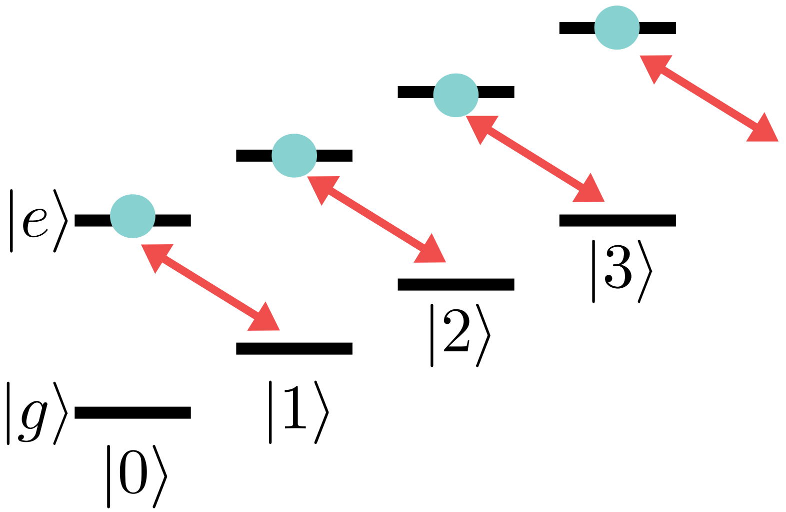

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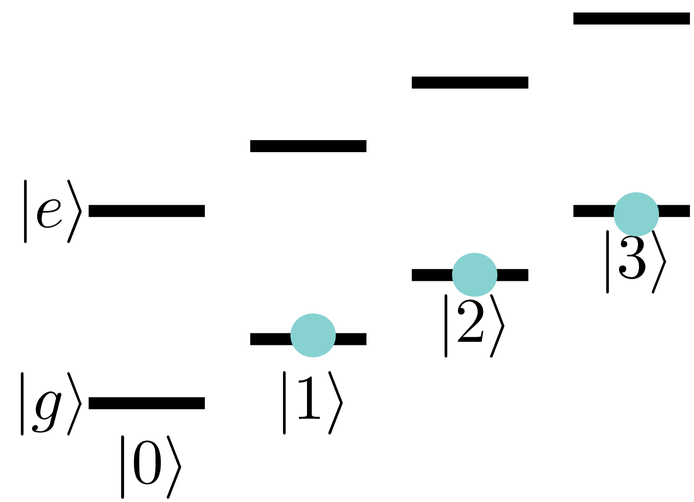

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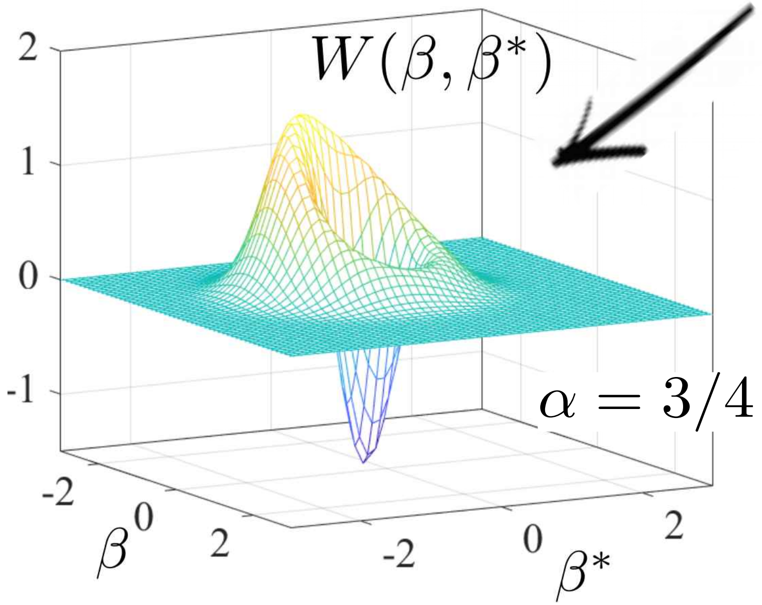
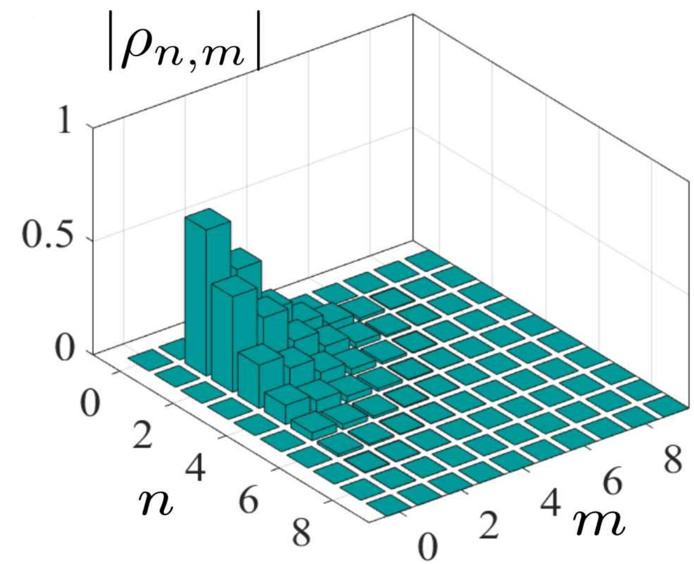
STA protocols allow us to implement (to a good approx.) this operation

$$|\psi(t=0)\rangle = |e\rangle \otimes |\alpha\rangle$$

$$|\alpha\rangle = D(\alpha)|0\rangle$$

Non-classical state by removing the vacuum contribution (make a hole in the Wigner distribution)





Photon-shifted states

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$$|\psi\rangle_{\mathrm{ph-sh}} \equiv \left[\sum_{n=0} |g, n+1\rangle\langle e, n|\right] |\psi\rangle$$

STA protocols allow us to implement (to a good approx.) this operation

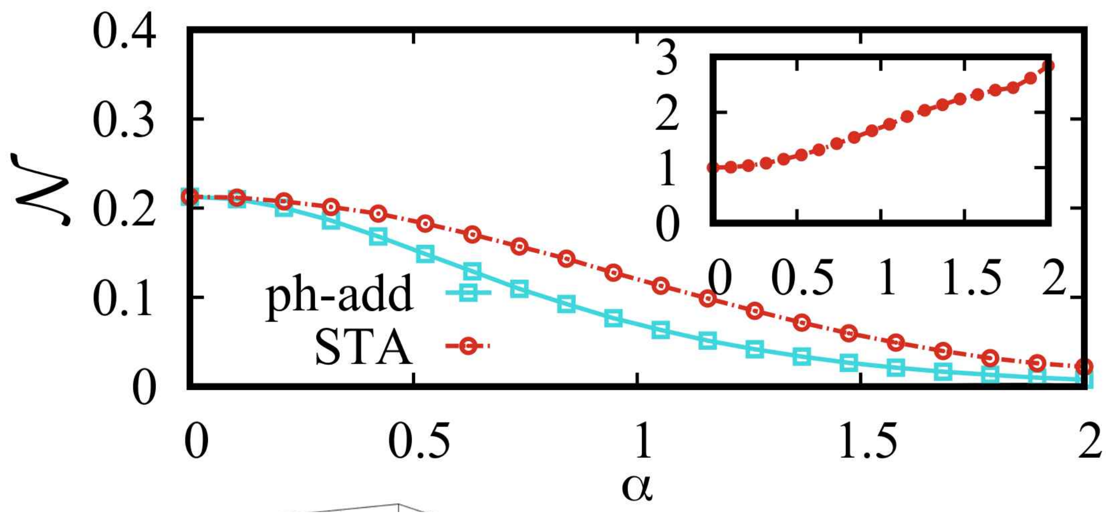
- **More non-classicality than photon addition**

$$\mathcal{N} = \frac{1}{2\pi}\int d^2\beta\,(|W(\beta,\beta^*)| - W(\beta,\beta^*))$$

(int. of the Wigner function where is negative)

$$|\psi(t=0)\rangle = |e\rangle \otimes |\alpha\rangle$$

Similar for initial thermal states



Summary

- **Protocols based on shortcuts-to-adiabaticity to generate non-classical states**

  $\longrightarrow$ Cat superposition of Fock states and photon-shifted states

- **The method is robust against noise (short evolution time)**

  $\longrightarrow$ Also robust against imperfect pulses

- **Feasible to implement in trapped ions: non-classical states of the vibrational mode**

- **More details in O. Abah, RP and M. Paternostro, arXiv:1912.05264**

Thank you!

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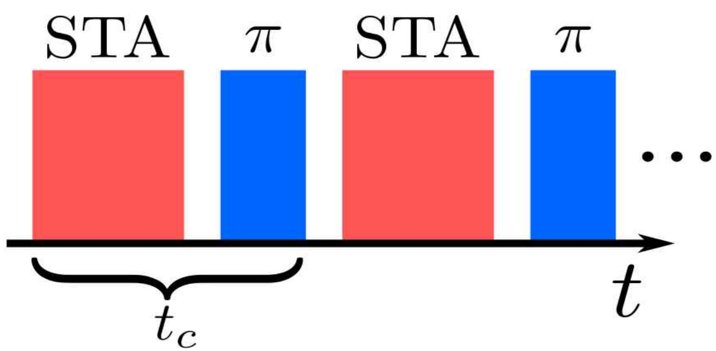
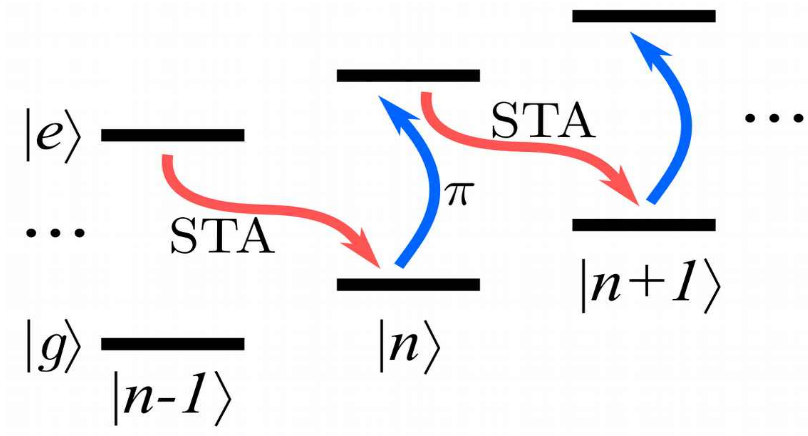
Extra slides

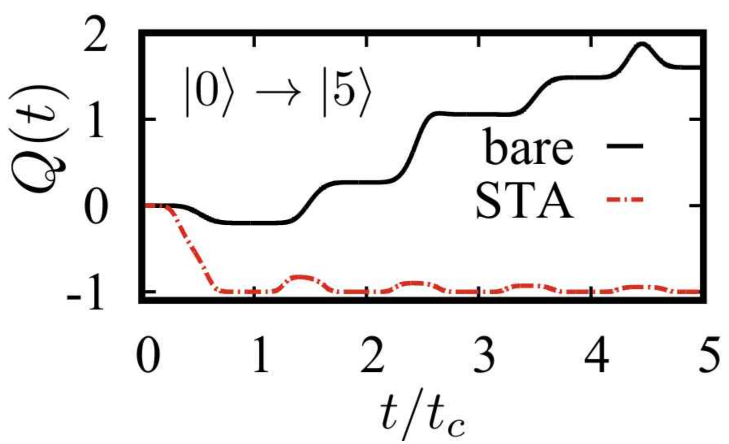
Fock state generation





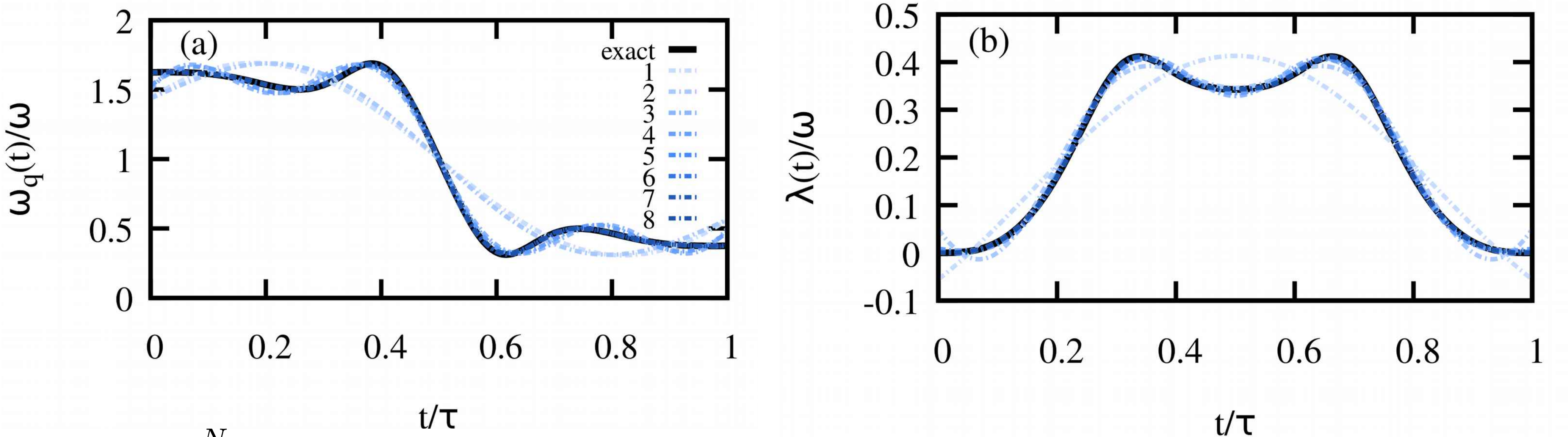
$$Q(t) = \frac{\langle \hat{n}^2(t) \rangle - \langle \hat{n}(t) \rangle^2}{\langle \hat{n}(t) \rangle} - 1$$

$$\hat{n} = a^\dagger a$$



→ However, a simple resonant Jaynes-Cummings will perform similarly: $T_n = \dfrac{\pi}{2\lambda\sqrt{n+1}}$
No advantage of STA in this case

Robustness: imperfect pulses

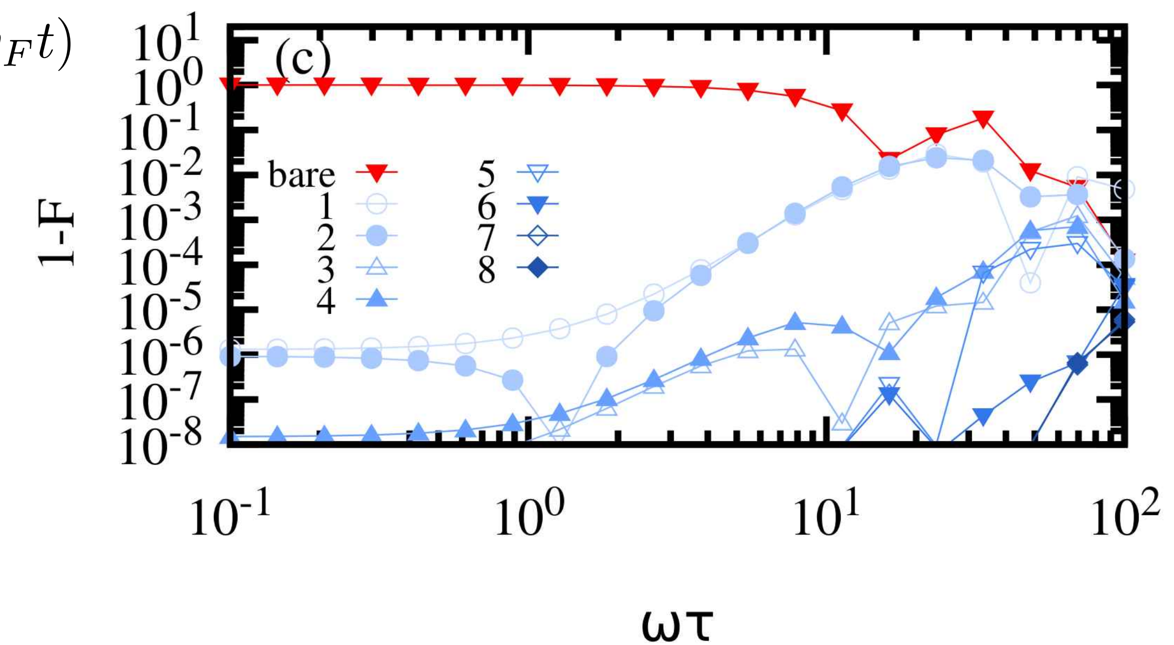


$$\tilde{x}_F(t) = \sum_{k=0}^{N_F} c_k \cos(k\omega_F t) + s_k \sin(k\omega_F t)$$

$$\tilde{x} \in \{\omega_q, \lambda\}$$

$$|\psi(t=0)\rangle = |e, 0\rangle$$

$$1 - F(\tau) = 1 - |\langle g, 1|\psi(\tau)\rangle|^2$$

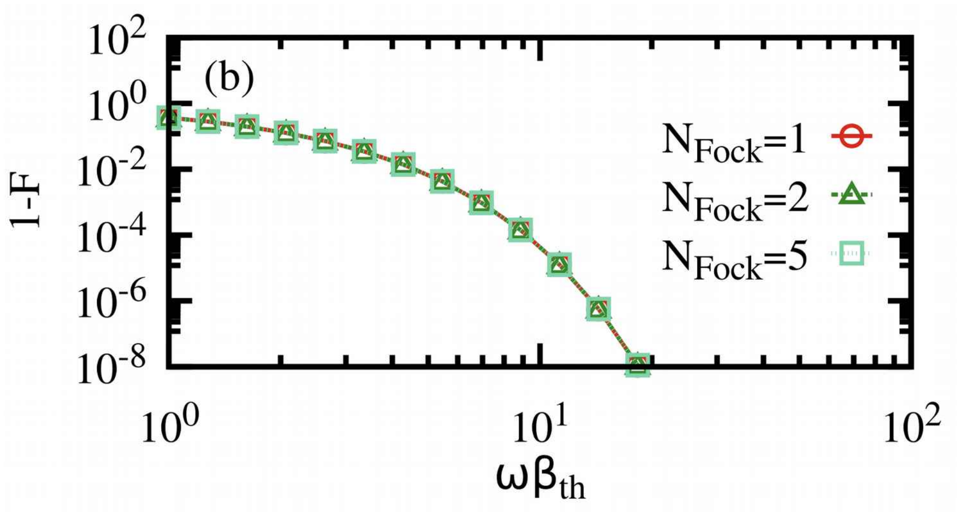


Role of initial thermal states

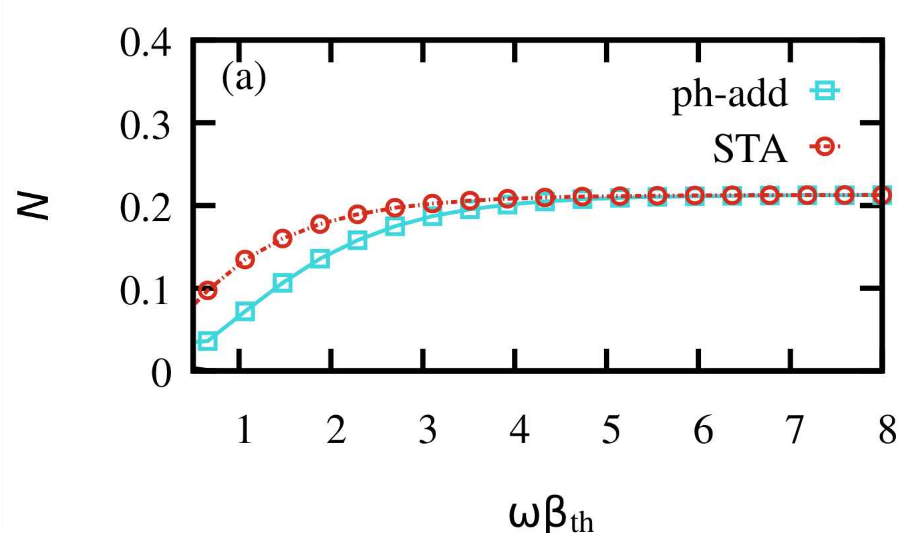
- **Fock and Cat state preparation limited by the population of the ground state**

$$\rho^{\text{th}} = \sum_{k=0} p_k |k\rangle\langle k|$$

$$p_k = \frac{\langle n\rangle_{\text{th}}^k}{(1+\langle n\rangle_{\text{th}})^{k+1}}$$

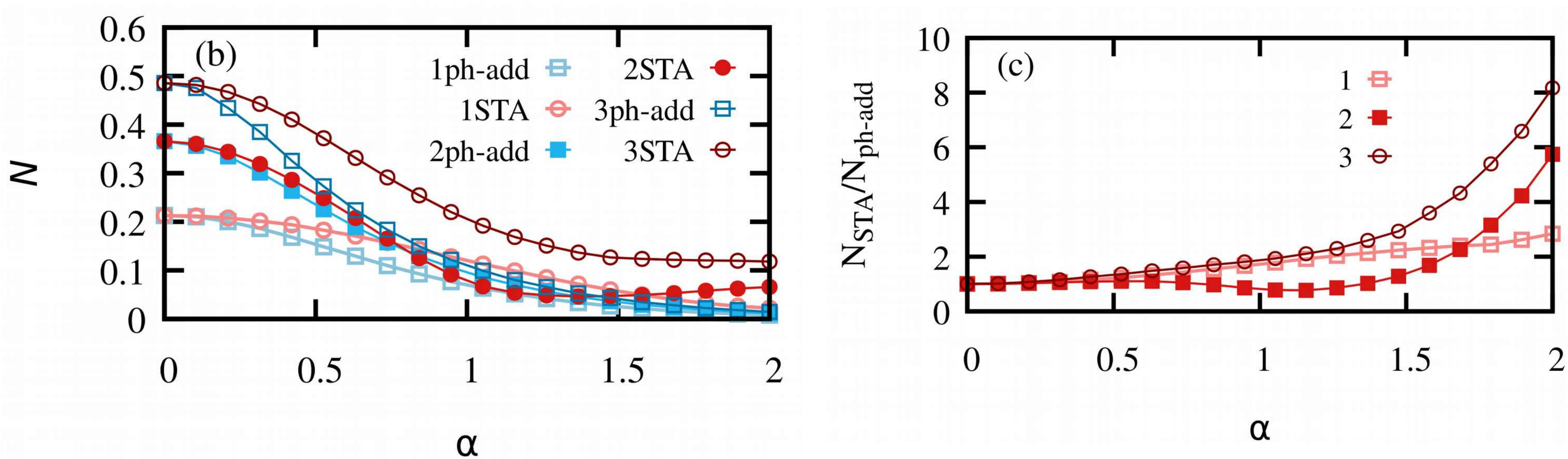


- **Photon-shifted thermal state similar behavior as for coherent states**



Repeating the photon-shifted operation

- **Comparison of photon-addition with photon-shifted (done with STA protocol)**



Photon-shifting a coherent state more times can lead to higher negativities, more than performing photon-addition